Choose

$x_1 = 1$ $F(x_1) = -1$ –ve

x2 = ½ $(x_0 + x_1)$ = ½ (0 + 1) = 0.5

F(0.5) = -0.25 is –ve

We neglect x_1 and we have

X3 = ½ $(x_0 + x_2)$ = ½ (0 + 0.5) = 0.25

F(0.25) = 0.3125 –ve

We neglect x_0 and we have

x_4 = ½ (x_2 + x3) = ½ (0.5 + 0.25) = 0.375

F (0.375) is +ve

We reject x_3 and we have

x_5 = ½ $(x_2 + x_4)$ = ½ (0.5 + 0.375) = 0.4375

F(0.4375) is –ve

We reject x_2, and have

x_6 = ½ $(x_4 + x_5)$ = ½ (0.375 + 0.4375)

$F(x_0) = +1$

$F(x_1) = -1$

$X_0 = 9_n$	$x_1 = b_n$	½ $(x_0 + x_1)$	F (c_n)
0	1	0.5	-0.25 –ve
0	0.5	0.25	0.3125 +ve
0.5	0.25	0.375	0.0156 +ve
0.5	0.375	0.4375	-0.12109 -ve
0.375	0.4375	0.40625	

This is linear function x(y) with $x(y_0) = x_0$ and $x(y_1) = x_1$

The line intersects the x – axis at the point obtained by putting y = 0 in (3.4)

$$x_2 \quad \frac{-y_1}{y_0 - y_1} x_0 + \frac{-y_0}{y_1 - y_0} x_1$$

$$= \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0}$$

This may be re-written on a linear interpolation form as $\frac{x_0 F(x_1) - x_1 F(x_0)}{F(x_1) - F(x_0)}$ … 3.5

This can also be written as

$$X_2 = x_1 - \frac{(x_1 - x0) F(x_1)}{F(x_i) - F(x_0)} \quad \dots 3.5$$

It may be repeated such that an iterative scheme results. As in the case of binary search (bisection rule) the two estimates slid give opposite signs in F(x).

The convergence in this method is faster than that in bisection rule.

Example $F(x) = x^2 - 3x + 1 = 0$

Choose $x_0 = 0$

$x_1 = 1$

Since $F(x_0)$ is +ve = 1

and F (x_1) is –ve = -1

$$x_2 = \frac{x_0 F(x_1) - x_1 F(x_0)}{F(x_1) - F(x_0)}$$

$$x_2 = \frac{0 - 1}{-1 - 1} = ½ = 0.5$$

$F(x_2) = F(0.5) = (0.5)^2 - 3(0.5) + 1 = 0.25 - 1.5 + 1 = -0.25$ +ve

So, we reject x_1

$$x_3 = \frac{x_0 F(x_2) - x_2 F(x_0)}{F(x_2) - F(x_0)}$$

$$x_3 = \frac{0 - 0.5 \text{ x } 1}{-0.25 - 1} = \frac{0.5}{1.25} = 0.4$$

$F(x_3) = F(0.4) = 0.4^2 - 3(0.4) + 1 = -0.04$ -ve

So, we reject x_2

$$x_4 = \frac{x_0 F(x_3) - x_3 F(x_0)}{F(x_3) - F(x_0)}$$

$$x_3 = \frac{-0.4}{-0.04 - 1} = \frac{0.4}{1.04} = 0.3846$$

$F(x_4) = (0.3846)^2 - 3(.3846) + 1 = -0.154$

And so on, we continue until convergence is achieved.

Assignment

Write a FROTRAN to illustrate the use of regula falsi to solve $x^2 - 5x + 1 = 0$.

4.0 ITERATIVE METHODS WITH DERIVATIVES

4.1 Order of Convergence:

Let α be a root of $F(\alpha) = 0$

$\alpha = F(\alpha)$

suppose the estimate α has error $\sum_n$

Comments:

1. Convergence is first order if $f'(\alpha) \neq 0$, $f''\alpha =, \ldots, = 0$
2. Convergence is second order if
   $f'(\alpha) = 0$ and $f''(\alpha)$
3. Convergence is third order if
   $f'(\alpha) = 0$ and $f''(\alpha)$
   $f'''(\alpha) \neq 0$

4.2 Newton-Raphson Method

From the figure, a is the point at which $f(x) = 0$ and x_0 is an estimate of a. the Newton Raphson method computes a new estimate, x_1 in the following way:

This is the second order convergence.

Let x be a root of equation (3.1) – $F(x) = 0$

If x_n is the nth estimate with error $\sum_n$,

then $x = x_n + \sum_n \ldots$ 4.1

$F(x) = F(x_n + \sum_n) = 0$

$= F(x_n) + n F'(x_n) + \frac{\sum_n^2}{2!} F''(x_n) + \ldots + = 0$

$\Omega\ F(x_n) + \sum_n F'(x_n) = 0p$

$= \sum_n = \frac{-F(x_n)}{F'(x_n)}$

The iteration scheme in 4.1 becomes

$x_n + 1 = x_n - \frac{F(x_n)}{F'(x_n)}$ - N – R method.

For example

$F(x) = x^2 - 4x + 2 = 0$

$F'(x) = 2x - 4$

Therefore $x_{n+1} = x_n - \frac{(xn^2 - 4x_n + 2)}{2x_n - 4}$

Using our former initial estimate

X_0	0	3

x_i	0.5	3.5
x_2	0.5833	3.4162
x_3	0.5858	3.4142
x_4	0.5858	3.4142
x_5		