CHAPTER TWO: SOLUTION OF SIMPLE PIPE FLOW PROBLEMS

The 3 simple pipe flow cases that are basic to solutions of the more complex problems are:

(i) **Given:** Discharge, Diameter, Length, Coefficient of dynamic/absolute of Kinematics' Viscosity, and absolute rough ness and **required to find Head loss due to friction**. (i.e. given Q, D, L, $\mu, \upsilon, \varepsilon$ and **required to find h_f**)

(ii) **Given:** $h_f, L, D, \mu, or\ \upsilon, \varepsilon$ **required to find Q**

(iii) Given: $h_f, Q, L, \mu, or, \upsilon, \varepsilon$ **required to find D**

CASE 1: EXAMPLE 1

(i) Calculate the loss head due to friction and the power required to maintain flow in a horizontal circular pipe 40mm diameter and 750m long when water with coefficient of dynamic viscosity equals $1.14x10^{-3}\ \frac{N.s}{m^2}$, flows at (a) 4liter/minute (b) 30Liter/minute. Assume that for the pipe the absolute roughness is $8x10^{-5}m$.

SOLUTION

- Establish whether the flow is Laminar or Turbulent:

$$\mathrm{Re} = \frac{\rho dv}{\mu} = \frac{vd}{\upsilon}$$

$$Note: \upsilon = \frac{\mu}{\rho}$$

$$Q = \frac{4x10^{-3}}{60} = 6.67x10^{-5} m^3 / s$$

$$A = \frac{\pi D^2}{4} = 1.26x10^{-3} m^2$$

$$V = \frac{Q}{A} = 0.053 m / s$$

$$\mathrm{Re} = \frac{\rho vd}{\mu} = \frac{10^3 x 0.053 x 0.04}{1.14 x 10^{-3}} = 1862$$

- The flow is Lamina Re<2000
- For Laminar flow the friction factor can be calculated thus: $f' = \frac{64}{\text{Re}} = 0.03436$
- Head loss due to friction, $h_f = f'\frac{LV^2}{D2g}$ normally referred to as Darcy Weisbach formula/equation
- $\therefore h_f = f'\frac{LV^2}{D2g} = 0.092m$
- Power required to maintain flow $P = \rho g h_f Q = \gamma H_f Q = 10^3 x 9.81 x 0.092 x 6.67 x 10^{-5} = 0.06 Watts$

$$Q = \frac{30x10^{-3}}{60} = 5x10^{-4} m^3 / s$$

(ii) $V = \frac{Q}{A} = 0.4 m/s$

$$\text{Re} = \frac{\rho VD}{\mu} = 14{,}035 = 1.4x10^4$$

$$\text{Re} > 2000$$

The flow is Turbulent

Calculate the relative roughness $\frac{\varepsilon}{D} = \frac{8x10^{-5}}{0.04} = 0.02$

Use Moody's Chart for $\text{Re} = 1.4x10^4$ **and** $\frac{\varepsilon}{D} = 0.02$

$$f' = 0.032$$

$$h_f = 4.89m$$

$$Power = 24.0 Watts$$

CASE 2 EXAMPE 2

2. Water at 15^{0}C flows through a 30cm diameter riveted steel pipe, absolute roughness of 3mm, with head loss of 6m in 300m. Determine the flow.

<u>SOLUTION</u>

$$\frac{\varepsilon}{D} = \frac{0.003}{0.3} = 0.01$$

Assume $f' = 0.04$

$$h_f = \frac{f' L V^2}{d 2g}$$

$$6 = 0.04 \frac{300}{0.3} \frac{V^2}{19.62}$$

$$V = 1.715 m/s$$

From table of physical properties of water (in any standard text book SI units) at 15^{0}C , Kinematic Viscosity is $1.139x10^{-6} m^2/s$.

$$\therefore \mathrm{Re} = \frac{Vd}{\upsilon} = 451712 \cong 4.5x10^5$$

from Mood's Chart for $\frac{\varepsilon}{D} = 0.01$ and $\mathrm{Re} = 4.5x10^5$

$f' = 0.038$ (this value is close enough to the assumed value) it is okay.

$$Q = AV = \pi r^2 \sqrt{\frac{h_f d 2g}{f' L}} = \pi\left(r^2\right)\sqrt{\frac{6x0.3x19.62}{0.038x300}} = 0.1245 m^3/s$$

CASE 3 EXAMPLE 3

In the third case with **<u>Diameter</u>** unknown:

(i) There are 2 unknowns in the Darcy-Weisbsch equation f', V and D.

$$h_f = \frac{f' L V^2}{d 2g} \;,\; f', V, d \text{ unknown}$$

(ii) There are 2 unknowns in the continuity equation V and d.

(iii) There are 3 unknowns in Reynolds Number equation V, D, Re

(iv) The relative roughness is also unknown

<u>SOLUTION</u>

Using the continuity equation to eliminate the **<u>velocity</u>** in darcy-Weisbach equation and in the expression for **Re** the problem will be simplified.

$$V = \frac{Q}{A}$$

$$h_f = f' \frac{L}{D} \frac{Q^2}{2g\left(\frac{\pi D^2}{4}\right)^2} \cdots\cdots\cdots\cdots 1$$

$$D^5 = \frac{8LQ^2}{h_f g \pi^2} f' = C_1 f'$$

Where $C_1 =$ the known quantities $= \frac{8LQ^2}{h_f g \pi^2}$

But $VD^2 = \frac{4Q}{\pi} \cdots\cdots\cdots$ continuity equation

$$\text{Re} = \frac{VD}{\upsilon} = \frac{4Q}{\pi \upsilon D} = \frac{C_2}{D} \cdots\cdots\cdots\cdots 2$$

$$C_2 = knownquantities = \frac{4Q}{\pi \upsilon}$$

The solution is now effected by the following procedure:

(i) Assume a value of f'

(ii) Solve equation 1 for D

(iii) Solve equation 2 for Re

(iv) Find the relative roughness

(v) Find new f' from moody's chart with the $\left(\text{Re}, \frac{\varepsilon}{D}\right)$

(vi) Use the new f' and repeat procedure

(vii) When the value of f' does not change in the first two significant figures all equations are satisfied and the problem is solved.

EXAMPLE 3

1. Determine the size of clean wrought iron pipe required to convey 260L/s of oil of kinematic viscosity of $9.26x10^{-6}m^2/s$, and 3048m length, with head loss of 22.8m. Absolute roughness is 0.046mm.

SOLUTION

$$D^5 = \frac{8LQ^2 f'}{h_f g\pi^2} = 0.745 f'$$

$$\text{Re} = \frac{4Q}{\pi\upsilon D} = \frac{35634}{D}$$

Assume f'=0.02

$$D = 0.431m$$

$$\frac{\varepsilon}{d} = 1.067x10^{-4}$$

$$\text{Re} = 8.268x10^4$$

From Moody's chart f'=0.019

$$\frac{\varepsilon}{D} = 0.00011$$

$$\text{Re} = 83{,}451$$

$$f' = 0.019$$

f' doesn't change significantly

2^{nd} trial for f'=0.019