CHAPTER THREE: MULTIPLE PIPE SYSTEMS ANALYSIS

3.1 Pipe in series: Discharge is constant i.e. Q=constant

The diagram and illustration as discussed in the class

$$h_f = \frac{f_1' L_1}{d_1}\frac{V_1^2}{2g} + \frac{f'_2 L_2 V_2^2}{d_2 2g} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$assume,\ f'_1 = f'_2 = f, thesame$$

Substitute
$$V = \frac{Q}{A}$$
$$\therefore h_f = \frac{f'_1 L_1}{2g}\frac{16Q^2}{\pi^2 d^5{}_1} + \frac{f' L_2 16Q^2}{2g\pi^2 d_2{}^5} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

But $\frac{f' LQ^2 x16}{19.62\pi^2 d^5} = \frac{f' LQ^2}{12d^5} = rQ^2$

Where r=pipe constant= $\frac{f' L}{12d^5}$

$$\therefore h_f = r_1 Q^2 + r_2 Q^2 + r_3 Q^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$h_f = Q^2 \sum_1^n r$$

or

$$Q = \sqrt{\frac{h_f}{\sum r}}$$

3.2 Equivalent Pipe Method for pipe in series:

An equivalent pipe is a pipe which will carry this particular flow rate and produce the same head loss as two or more pipes. If we are to replace this complex system with a single equivalent pipe;

$h_f = r_e Q^2$ where r_e=pipe constant for equivalent pipe

Hence in a series pipe system
$$r_e = \sum_1^n r$$
$$r_e Q^2 = Q^2 \sum_1^n r$$

3.3 Pipes in parallel: Head loss is a constant i.e. h_f=constant

The diagram and illustration as discussed in the class

$$h_{f1} = h_{f2} = h_{f3}$$

The head loss in each pipe between junctions where parallel pipes part and join again must be equal. $Q_T = Q_1 + Q_2 + Q_3$. The total flow rate will equal the s um of individual flow rates. $Q_T = \sqrt{\frac{h_f}{r_1}} + \sqrt{\frac{h_f}{r_2}} + \sqrt{\frac{h_f}{r_3}}$

$$Q_T = \sqrt{h_f} \sum_1^n \left(\frac{1}{\sqrt{r}} \right)$$

3.4 Equivalent Pipe Method for pipe in parallel

If we want to replace the system with a single equivalent pipe then: $h_f = r_e Q_T{}^2$

$$Q_T = \sqrt{\frac{h_f}{r_e}}$$

$$r_e = \left(\frac{1}{\sum_1^n \left(\frac{1}{\sqrt{r}} \right)} \right)^2$$

or

$$r_e = \frac{1}{\left(\sum_1^n \frac{1}{\sqrt{r}} \right)^2}$$

EXAMPLE 4: For pipe in series Q=constant.

Pipe in series as shown on the board. Find Q? Given total head loss as 26m, f'=0.01 kc=0.33, where kc is the coefficient of contraction. Consider all losses and use equivalent pipe method.

SOLUTION

(i) Consider all losses : Write Bernoulli's Equation from reservoir A to B

H_T=Entrance loss +head loss due to friction+ head loss due to contraction +head loss due to friction + Exit loss

$$H_T = \frac{0.5v_1^2}{2g} + f'\frac{Lv_1^2}{d2g} + \frac{0.33v_2^2}{2g} + f'\frac{Lv_1^2}{d2g} + \frac{v_2^2}{2g}$$

$$26 = 0.225v_1^2 + 0.468v_2^2$$

$$V_2 = V_1\left(\frac{A_1}{A_2}\right) = V_1\left(\frac{d_1^2}{d_2^2}\right) = 4V_1$$

$$26 = 0.225V_1^2 + 0.468(4V_1)^2 = 7.71V_1^2$$

$$V_1 = 1.83m/s$$

$$Q = A_1V_1 = A_2V_2 = 0.14m^3/s$$

(ii) Using equivalent pipe method

Neglecting minor losses and calculate pipe constants $r_1 = \frac{f'_1 L_1}{12d_1^5} = \frac{0.01x122}{12(0.31)^5} = 35.51$

$r_2 = 1136.37$

For pipe in series $r_e = \sum_1^2 r = 35.51 + 1136.37 = 1171.88$

$$h_f = r_e Q^2$$

$$Q = \sqrt{\frac{h_f}{r_e}} = \sqrt{\frac{26}{1171.88}} = 0.149m^3/s \cong 0.15m^3/s$$

Example 5 for pipe in parallel h_f=constant

Find the head loss across the system shown and discharges in each pipe.

SOLUTION

$r = \frac{f'L}{12d^5}$

D(mm)	r	$\sqrt{r}$	$\frac{1}{\sqrt{r}}$
305	785.8	28.03	0.036
200	3812.5	61.75	0.016
405	260.0	16.12	0.062
$\sum$			0.114

$$r_e = \left(\frac{1}{\sum \frac{1}{\sqrt{r}}} \right)^2 \text{ or } \frac{1}{\left(\sum \frac{1}{\sqrt{r}} \right)^2}$$

$$= \left(\frac{1}{0.114} \right)^2 \text{ or } \frac{1}{(0.114)^2} = 76.95$$

$r_e = 76.95$

$h_f = 76.95(0.34)^2 = 8.9m$

(i) To find the discharge in individual pipes, you have to consider individual pipe

$$h_f = \frac{f'LV^2}{d2g} = \frac{0.017x1464xV^2}{305x2x9.81} = 8.9m$$

$V = 1.46m/s$

Consider 305 mm diameter pipe $Q_{305} = 0.107m^3/s$

$Q_{200} = 0.049m^3/s$

$Q_{405} = 0.186m^3$

$Q_T = (Q_{305} + Q_{200} + Q_{405}) \cong 0.34m^3/s$

Using Equivalent pipe method

h_f=constant

$$h_f = r_e Q_T^{\ 2}$$

$$r_e Q_T^{\ 2} = r_1 Q_{305}^{\ 2}$$

$$76.95(0.34)^2 = 785.8 Q_{305}^{\ 2}$$

$$Q_{305} = 0.106 m^3 / s$$

$$Q_{200} = 0.048 m^3 / s$$

$$Q_{405} = 0.185 m^3 / s$$

$$Q_T = 0.339 \cong 0.34 m^3 / s$$