ENERGY PRINCIPLE IN OPEN CHANNEL FLOW

- Need to examine/determine BERNOULLI EXPRESSION

$$H = \frac{P}{\gamma} + \frac{V^2}{2g} + Z$$

- Define the terms
- What is Hydraulic Grade Line

EXAMPLE:

The depths a short distance upstream and down stream of a sluice gate in horizontal channel are 2.4m and 0.6m respectively. The channel is of rectangular section and 3m wide find the discharge under the gate. (to be discussed in class)

SPECIFIC ENERGY AND ALTERNATR DEPTHS

We can define the specific energy E as the energy referred to the channel bed as datum i.e.

$$E = y + \frac{V^2}{2g} \cdots\cdots 1$$

Consider a wide rectangular channel of width b

Let q denotes the flow per unit width of a wide rectangular channel

$$q = \frac{Q}{b} = vy = \frac{AV}{b}$$

$$V = \frac{Q}{A} = \frac{qb}{yb} = \frac{q}{y}$$

rewrite eqn 1

$$E = y + \frac{q^2}{2gy^2}$$

We can consider how E will vary with y for a given constant value of q.

We can construct a graph on the E-y plane

$$(E - y)y^2 = \frac{q^2}{2g} = \text{a constant}$$

$$q = y\sqrt{2g(E - y)}$$

THE GRAPH AS ILLUSTRATED IN THE CLASS AND IN THE TEXT.

NOTE:

- When two depths of flow are possible for a given E and q they are referred to as ALTERNATE DEPTHS.
- Alternatively we may say that the curve represents two possible REGIMES of flow i.e. slow and deep on the upper limb, fast and shallow on the lower limb meeting at the crest of the curve (**at critical depth)**

ANALYTICAL PROPERTIES OF CRITICAL FLOW

Let us drive equations defining critical flow

$$E = y + \frac{q^2}{2gy^2}$$

E is minimum at critical depth

We obtain minimum by differentiation

$$\frac{dE}{dy} = 0 = 1 - \frac{q^2}{gy^3}$$

$$1 - \frac{q^2}{gy^3} = 0$$

$$\frac{q^2}{gy^3} = 1$$

$$q^2 = gy^3$$

i.e. $y_c = \sqrt[3]{\frac{q^2}{g}} = \left(\frac{q^2}{g}\right)^{1/3}$

$y_c =$ Critical depth

The depth for minimum energy is called critical depth

NOTE:

$$q = vy$$

$$q^2 = gy_c{}^3 = v^2 y^2$$

$$V_c = gy_c$$

$\frac{V^2{}_c}{2g} = \frac{y_c}{2}$ or $V_c = \sqrt{gy_c}$

$$E_c = y_c + \frac{V^2{}_c}{2g} = \frac{3}{2} y_c$$

$$y_c = \frac{2}{3} E_c$$

The above equations are established by considering the variation of E with y for a given q.

IT IS ALSO OF PRACTICAL INTEREST TO STUDY HOW q varies with y for a given E i.e constant E

We can find the maximum by rearranging the specific energy and differentiate

$$E = y + \frac{q^2}{2gy^2}$$

differentiate with respect to y

$$2q\frac{dq}{dy} = 4gyE - 6gy^2 = 0$$
$$6qy^2 = 4qyE$$
$$y = \frac{2}{3}E$$

We have therefore established another important property of critical flow:

- It shows not only minimum specific energy for a given q
- Also a maximum q for a given E

SUMMARY:

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \left(\frac{q^2}{g}\right)^{1/3}$$
$$V_c^{\,2} = gy$$
$$V_c = \sqrt{gy_c}$$
$$\frac{V_c^{\,2}}{2g} = \frac{y_c}{2}$$
$$E_c = \frac{3}{2}y_c$$
$$y_c = \frac{2}{3}E_c$$

When $y>y_c$ subcritical flow exist

$y<y_c$ super critical flow exist