COURSE CODE: **PHS 312**

COURSE TITTLE: **Analytical Mechanics II**

NUMBER OF UNITS: **3 Units**

COURSE DURATION: **Three hours per week**

COURSE DETAILS:

Course Coordinator: **Dr. O.I. Olusola B.Sc., M.Sc. Ph.D.**

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Office Location: **Room C210, COLNAS**

Other Lecturers: **Nill**

COURSE CONTENT:

Newtonian mechanics of system of particles, D'Alembert's principle,; Degree of freedom, generalized coordinates and Lagrange's formulation of mechanics; simple applications. The calculus of variations and the action principle. Hamiltonian mechanics. Invariance and conservation laws. Small oscillations and normal modes.

COURSE REQUIEREMENTS:

This course is compulsory for all students pursuing a B.Sc. Degree program in Physics in the University. In this light, students are expected to participate in all the course activities and obtain minimum of 75% attendance to be able to write the final examination.

READING LIST:

1. Golstein G,Classical mechanics, McGrawhill, 1983
2. Plumpton, C., Tomkys, W.A., Theoretical Mechanics for sixth forms, 2nd Ed., Pergamon Press, New York
3. Harper, C., Introduction to Mathematical Physics, Prentice-Hall of India, New Delhi, 1989.
4. Chorton, F.A textbook of dynamics, Ellis Horwood Ltd. New York, 1988
5. Spiegel, M.R., Theory and Problems of Theoretical Mechanics, McGraw-Hill Book Company, New York, 1982.

LECTURE NOTES

1.0 ANGULAR MOMENTUM AND TORQUE FOR A PARTICLE

The angular momentum, L, for a single particle about a point O is defined as

$$L = r\Lambda p = mr\Lambda v \text{ --1.0}$$

where r is the position vector of the particle with respect to O and p=mv is the linear momentum.

The magnitude of L in Eq. 1.0 is given as

$$|L| = |r||p|\sin\theta \text{ --1.1}$$

where θ is the smaller angle between the positive directions of r and p.

The tendency for a force to produce rotation is proportional to the magnitude of the force and to the distance from the point of application to the center of rotation. The quantitave measure of this rotation tendency is the TORQUE produced by the force. The torque is the moment of the force and can be expressed as

$$\tau = r\Lambda f \text{ --1.3}$$

1.1 TWO-PARTICLE SYSTEM

Let us consider two particles with masses m_1 and m_2 located at positions identified by the vectors r_1 and r_2 respectively in some inertia frame. The equation of motion between the two particles can be obtained by solving the coupled equations that represent the Newton's second law of the form:

$$F_{12} = m_1 \frac{d^2 r_1}{dt^2} \text{ --1.4}$$

$$F_{21} = m_2 \frac{d^2 r_2}{dt^2} \text{ --1.5}$$

The center of mass (C.M) position vector that related the position vectors r_1 and r_2 to C.M position vector R is given as

$$R = \frac{1}{M}\sum_i m_i r_i \qquad \text{---1.6}$$

where i=1, 2 and M= $m_1 + m_2$

By using the definition of relative coordinate ($r=r_{12}=r_2-r_1$) in Eq. 1.6, the expressions for r_1 and r_2 can be given as

$$r_1 = R - \frac{m_2}{M} r \quad \text{---1.7}$$

$$r_2 = R + \frac{m_1}{M} r \quad \text{---1.8}$$

1.3 MANY-PARTICLE SYSTEM

The most convenient way to describe systems that consists of many particles is to identify the center of mass (C.M) coordinate R and to specify the position of each particle with respect to center of mass. Thus, the particle labeled i has a position vector r_i in some inertia frame and a coordinate r_{ci} with respect to C.M and r_i is given as

$$r_i = R + r_{ci} \quad \text{---1.9}$$

The total force for a many particle system is given as

$$F_t = F_i^e + \sum_{\substack{i=1 \\ i\neq j}}^{n} F_{ij} = \frac{dp_i}{dt} \quad \text{--1.10}$$

where the index j runs from I to n but excluding i=j, thereby eliminating unphysical self -interaction force F_{ii}.

The total kinetic energy of a system of particles is given as

$$T = \frac{1}{2} M V^2 + \frac{1}{2}\sum m_i v_{ci}^2 \quad \text{--1.11}$$

where $V = \frac{dR}{dt}$ and $v_{ci} = \frac{dr_{ci}}{dt}$.

The total angular momentum, L, of a system of particles is the sum of the individual angular momenta of the particles relative to the same origin i.e.

$$L = \sum_i l_i = \sum_i r_i \times p_i \quad \text{--}1.12$$

By applying particle coordinates, the total angular momentum of a system of particles is given as

$$L = R\Lambda P + \sum_i r_{ci}\Lambda p_{ci} \quad \text{--}1.13$$

2.0 DYNAMICS OF A PARTICLE

2.1DYNAMICS – it involves the study of relationships between kinematics and force.
→ Free particle, a particle not subjected to any interaction. This is a theoretical concept.
In practice, a practice may be considered free either became it is sufficiently for away from others for their interaction to be negligible or because the interaction with other particles giving zero net interaction.
The law of inertia or Newton's 1^{st} law states that a free particle always move with constant velocity or no acceleration.
NOTE: the observer himself must be a free particle or system i.e. without acceleration (inclined rotation). Such an observer is called an inertia frame of reference.
The earth and the sun are approximately inertia frames of reference (i.e. neglecting their rotation & orbital motion).

2.2 LINEAR MOMENTUM – linear momentum ($\underline{p}$) of a particle is given by

$$\underline{P} = m\underline{v} \text{ ———— } (2.1)$$

S.I. unit of $\underline{p}$ is $kgms^{-1}$.
Hence the law of inertia may be restated – a free particle always moves with a constant momentum.

2.3 ISOLATED SYSTEM → A system that has no interaction c others

2.4 PRINCIPLE OR LAW OF CONCERVATION OF LINEAR MOMENTUM states that the total momentum of an isolated system of particles is constant.

Mathematically:

$$\underline{P} = \sum p_l = \underline{p}_1 + \underline{p}_2 + \underline{p}_3 + \ldots\ldots\ldots\ldots\ldots.. + \text{constant}$$

$$\underline{p}_1 + \underline{p}_2 = \underline{p}_1^{\,1} + \underline{p}_2^{\,1}$$

$\Rightarrow \underline{p}_1^{\,1} + p_1 = \underline{p}_2 + \underline{p}_2^{\,1}$

$= - (\underline{p}_2^{\,1} + \underline{p}_2)$

$\Rightarrow \Delta \underline{p}_1 = - \Delta \underline{p}_2$ ———————————————— (2.2)

Newtons 1st & 2nd laws follow from principle of concervation of momentum.

Hence we have the following definition of mass.

From (1) $\underline{p} = m\underline{v}$

$\Rightarrow \Delta \underline{p} = - m \,\Delta\, \underline{v}$

Thus from conservation of momentum for a system of two particles

We have $m_1 \,\Delta\, v_1 = - m_2 \,\Delta\, v_2$

$\Rightarrow m_2/m_1 = - \left| \Delta v_1/\Delta v_2 \right|$ ———————————— (2.3)

Newton's third law: let the time interval involved in (2.2) be Δt. then we have from (2.3) above,

$\frac{\Delta p1}{\Delta t} = \frac{\Delta p2}{\Delta t}$ or from $\frac{1m}{\Delta t \to 0} \frac{\Delta p}{\Delta t} = \frac{\Delta p}{\Delta t}$ ———————————————————————— (2.4)

We have $dp_1/dt = - dp_2/dt$ ———————————————————————— (2.5)

We define force, $\underline{F}$ as the time rate of change of momentum i.e.

$\underline{F} = dp/dt$ ——————————— (2.6)

Hence $\underline{F}_1 = - \underline{F}_2$ ——————————— (2.7)

Where $F_1 = dp_1/dt$ is the force on particle 1 due to its interaction with particle 2, and similarly for $\underline{F}_2$.

Thus (2.7) says:

When two particles interact, the force on one particle is equal and opposite to the force on the other. This is the Newton's 3rd law of motion w is sometimes called the law of action and reaction.

NOTE: In general, $\underline{F}, = \underline{F}\, (r_{1,2,}\, V_{1,2})$

i.e. relative position vector of the two particles & also their relative velocity

now $\underline{F}$ constant $\Rightarrow$ a = f/m constant, if m is constant. E.g bodies falling near the earth's surface, where

$\underline{F} = \underline{W} = m\, \underline{a} = mg$ ---------------------------------(2.8)

weight

$g = g_0 - w^2 r \cos$ -------------------------------------(2.9)

for a particle of mass m_1 interacting T particles m_1, m_2, m_3, …… then, $dp_1/dt + dp_{13}/\, dt$ +………

$= F_{12} + F_{13}$

$= F_1$

= resultant force acting on M_1

This result of course neglect interference effects i.e we have assumed that the interaction between M_1 and M_2 etc is not altered by the presence of M_3 and M_4 ………………….

For example, from (7), F= m dv/dt

= m d/dt

(dx/dt) in 1- dimension

$= md^2/dt^2$

i.e $d^2x/dt^2 \quad = f/m \quad = a$

i.e 2nd order differential equation of degree one.

2.5 FRICTIONAL FORCES

(a) Friction B/w Solids

Whenever two bodies are in contact, there is a resistance w opposes the relative motion of the two bodies. The force is called sliding friction, F_f.

Expt. shows that $\underline{F}_f \propto$ normal reaction, $|\underline{N}|$

i.e. $F_f \propto \underline{N} \Rightarrow F_f = FN$ ———————— (1)

└── Constant of prop called coefficient of friction

Thus, the equation of motion is

$$\underline{F} - fN\underline{v} = m\underline{a} \quad \text{———— (2)}$$

Two types of coefficient of friction exist, they are:

(i) Static coefficient of friction, F_s, defined as the minimum force required o set in relative motion, two bodies that are initially in contact and (a) ^ relative next.

(ii) Kinetic or dynamic coefficient of friction, F_k, defined as the force required to maintain the two boches in relative motion

□ Expt. shows that $F_s > F_k$ for all materials tested

2.5.1 FRICTION IN FLUIDS

When a body moves through a fluid, i.e. a gas or a liquid @ a relatively low velocity, the force of friction, $\underline{F}_f$ is proportional to the velocity, V, & tends to oppose the motion and K is a factor which depends on the shape of the body. For example, consider a sphere of radius R, it can be shown that

$K = 6\pi R$ ———— (6a) S.I. units in 'm'

so that $\underline{F}_f = -6\pi R\eta\underline{V}$ ———— (6b) ⇒ Stoke's law

η depends on the internal friction or viscosity of the fluid (i.e. the frictional force between different layers of the fluid moving the different velocities). The S.I. unit of η is $N.s.m^{-2}$ or $m^{-1}kgS^{-1}$. η is referred to as coefficient of viscosity.

Note the following:

(i) For liquids, the coefficient of viscosity, η decreases as tempt T increases.

(ii) For gases, η increases with increase in tempt T.

(iii)When a body moves through a viscovs fluid under the action of a force F, the equation of motion is:

F – K

Assuming F is constant, the acceleration a produces a continuous increase in V, and hence increases, so that eventually the L.H.S of (7) becomes zero.

$\Rightarrow$ "a" = zero also subsequently, the particle continue to move in the direction of the force with a constant velocity called limiting or for minal velocity, V_L, where

$$V_L = F / KV \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(8)$$

In free fall under gravity, E= mg

And $V_L = mg/KV$ ________________________________(9)

Equation 2 (a) has to be corrected for the buoyant force exerted by the fluid which (by Archimede's principle) is equal to the weight the fluid displaced by the body. If m_f = mass of fluid displaced, then its weight = m_f g and the upward buoyant force = - m_f g

Hence, net downward force = $(m-m_f)$ g and

$$V_2 = (M - M_f)g \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(10)$$

Note:- for large bodies and large velocities F_f & V^n, where n>I, and the above treatment is not applicable

2.7. SYSTEM WITH VARIABLE MASS

Definition, momentum, P = mv

$$F = {dp}/{dt} = m = {dv}/{dt} + V{dm}/{dt}$$

in a system where mass is not constant e.g rain drop while it falls, moisture may condense on its surface or very evaporate resulting in a charge of mass

Suppose mass of drop is in when it is velocity V and that of moisture velocity V_0 condense on the drop at a rate ${dm}/{dt}$, the equate of motion of the drop is

$$[M \, {dV}/{dt}] + [{dm}/{dt}]$$

Correspond to the acceleration of the drop rate of change of momentum of the moisture

(iii)a conveyor on W material is dropped (a) one and or discharged (a) the other end. Suppose the material is dropped at the state ${dm}/{dt}$

The conveyor is moving at constant velocity V and a force F is applied to move it. If M is the mass of the belt and m is the mass of material already dropped at time, t, the total momentum of the system at time t is

$$P = (M + M)\ V$$

The force applied to belt is

$F = {dp}/{dt} = V\, {dm}/{dt}$ since M= constant, V= constant

(iv)Rocket principle:- the mass of the rocket decreases because it consumes the fuel it carries

Let $\underline{V}$ = velocity of rocket relative to earth surface

Let $\underline{V}^1$ = exhaust velocity of the gases relative to the earth surface

Then, $V^1 - \underline{V} = r_e$ = exhaust velocity of gases relation to the rocket.

NOTE: V_e (or V^1) and V are usually oppositely directed. Let m = mass of the rocket, including its fuel at any time, t.

At time t, $\underline{p} = mv$

At time tidt, $p^1 = \frac{(m+dm)(V+dv)}{rocket} + \frac{(-dm)V}{Gases}$

$= mv + mdv - (V^1 - v)\, dm$ neglecting 2nd order term $dmd\underline{V}$

$= mv + mdv - V_e dm$

$dP = p^1 - p = MdV - V_E dM$

$\frac{dp}{dt} = \frac{mdr}{dt} + V_e \frac{dm}{dt}$ = F, external force

Acting on the rocket i.e the equation of motion is

$F = \frac{mdr}{dt} - V_e \frac{dm}{dt}$ ____________________________________()

Is known as the thrust of the rocket since it is equal to the force due to escaping gases assuming V_e is constant and neglecting air resistance.