

**UNIVERSITY OF AGRICULTURE, ABEOKUTA
COLLEGE OF NATURAL SCIENCES
DEPARTMENT OF COMPUTER SCIENCE**

**MODELING AND SIMULATION
CSC 421- 3 UNITS**

A MATERIAL DEVELOPED

BY

DR. O. R. VINCENT

COURSE CODE: CSC 351
COURSE TITLE: MODELING AND SIMULATION
NUMBER OF UNITS: 3 UNITS
COURSE DURATION: THREE HOURS PER WEEK

COURSE DETAILS:

Course Coordinator: Dr. A. T. Akinwale B. Sc., M. Sc., PhD
Email: vincent.rebecca@gmail.com
Office Location Room B201, COLNAS
Other Lecturers Dr. (Mrs.) O. R. Vincent B. Sc., M. Sc., PhD

COURSE CONTENT:

The concept and techniques used in modeling and simulation; the simulation methodology and suitable language; modeling generation of random variables; transformation of random numbers; parameter estimation, design of experiment, factorial design and optimization.

COURSE REQUIREMENTS:

This is an elective course for students in Computer Science

READING LIST:

MODELS

A model is a simplified representation of a system intended to enhance our ability to understand, predict and possibly control the behaviour of the system.

Forms of Models

- Mental - personal decision-making process.
personal view of an object or an event.
personal view of a foreign country or ideology.
- Physical - model of a house.
model of a bridge.
Wax model of a person.
- Symbolic - $H_2 + O \rightarrow \text{water}$ (Hydrogen + Oxygen = Water).
 $f = ma$ [Force = mass x acceleration]
electric circuit diagram.

Model has one major characteristic in common, they help us to evaluate the outcome of an action in a real-world situation without actually taking that action .

A model is only a representative of a system and not the system itself. For example, a road map is only a man-made model of the road networks of a region . An ordinary photograph is simply an analogue representation of a scene or an object and not the real thing itself.

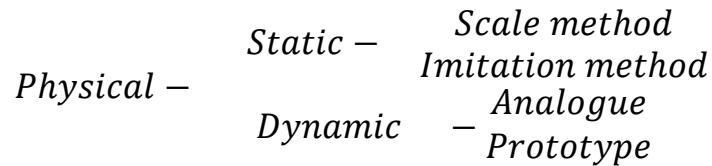
Types of Models

- Physical models (they are made of tangible components).
- Symbolic models (mathematical, symbolic or logical operations e.g maps, graphs, words, pictures).
- Mental models (exists only in the mind of individuals).

Physical – They are made of tangible components e.g. electrical, mechanical, fluid. They are described by measurable variables such as Voltages, Current, temperature, heat, length, weight, pressure, flow, force, velocity etc.

The construction of physical models could be expensive, time consuming, impractical and impossible.

Physical model can be divided into **Static or Dynamic** models.



Scale models:- They are accurate. E.g. wax statues, models of cars, bridges aircraft, ships.

Imitation models (Iconic model) :- puppets, dolls, shop window models, cartoons.

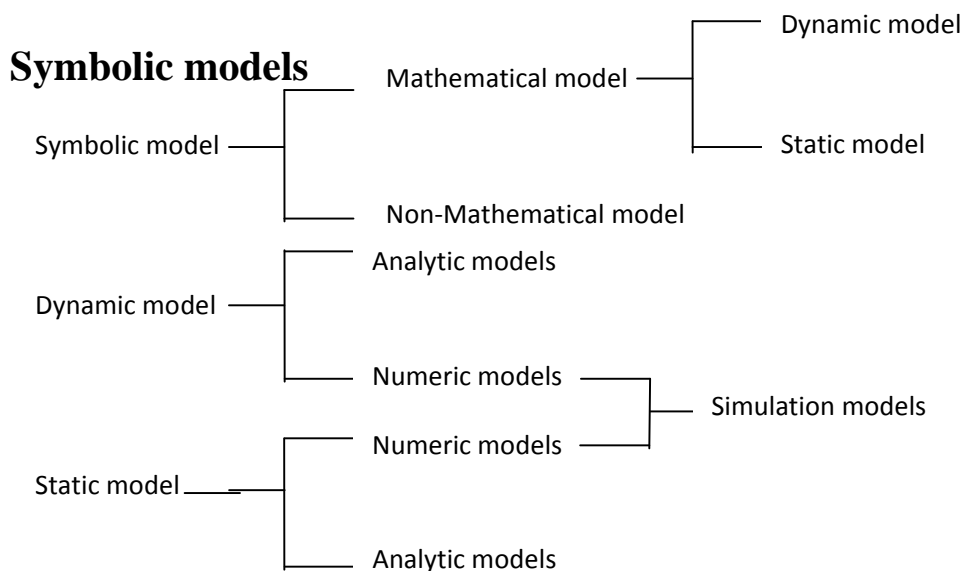
Analogue models:- circuit to study car suspension system, mercury or alcohol to measure temperature, water flow to study traffic flow, blood flow, money flow, rats or monkeys to test new medicines.

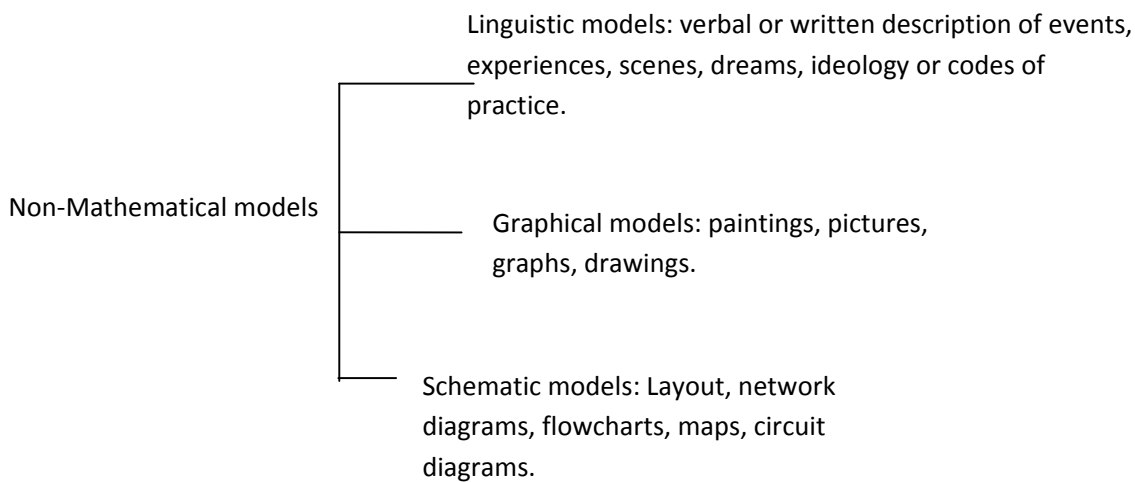
Prototype models:- prototypes of chemical reactions, experimental management information system.

The variable associated with the static models remains constant and those of dynamic models may undergo changes in time.

Imitation models:- are not very accurate but they are easily understood.

Scale models:- are accurate to scale , difficult to build and some of the features may not be easily understood.





A mathematical model is a set of mathematical and logical relations between various system elements. A theory expressed in mathematical form is a mathematical model. Mathematical models are expensive.

Note: Dynamic models are described by differential equations.

$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Static models are described by algebraic equations

$$\begin{aligned} 3x + 4y &= 39, & x &= 5 \\ 6x + 5y &= 60, & y &= 6 \end{aligned}$$

Mental models: they are heuristic or intuitive models which exist only in our minds. They are fuzzy, imprecise and difficult to communicate. Human beings accumulate experience, thus, stored experience serve as a mental model.

Examples of Mental models:

- process of decision making in complex situations.
- The ability to understand the operation of various loops in a computer program.

CLASSIFICATION OF MODELS

A model may be:

- **Continuous:** the variables describing continuous models undergo continuous change. E.g. models of chemical plants, steel mills, oil refineries, electricity etc.
- **Discrete:** the variables describing discrete models take values only in discrete steps or jumps e.g. models of telephoning networks connecting several computers, models of supermarkets, models of telephone exchanges.
- **Hybrid:** both continuous and discrete model properties e.g. at times we treat them as either discrete or continuous and use hybrid (analogue or digital) computers to solve their problems.
- **Linear:** this model exists between input and output Linear models e.g. the price versus quantity of petrol at a filling station, the applied weight versus extension of a spring of a balance.
- **Non-Linear:** it binds the input and output of non-Linear models e.g. the fare versus distance travelled (by air, land, or sea).
- **Deterministic:** deterministic models have predictable relationships between input and output variables e.g. modern electricity and gas supply networks can be considered to be deterministic systems.
- **Stochastic:** Stochastic models have no predictable relationships between input and an output variable, for the same input stimulus, the output of a stochastic system is unpredictable. The interaction between the variables in a stochastic model is expressed in terms of statistical probabilities. e.g. the model of a reader system receiving signals corrupted by noise, a model of several species of insects in a region, are both stochastic.

RANDOM VARIABLES

A random variable is a variable whose value is determined by the outcome of a random experiment. A random variable can be discrete or continuous. A discrete random variable assumes values that can be counted. In other words, the consecutive values of discrete random variables are separated by a certain gap.

Examples of discrete random variables are:

- The number of cars sold at a dealership during a given month.
- The number of people coming to a theatre on a certain day.
- The number of shoe pairs a person owns.
- The number of customers arriving at a bank during an hour.
- The number of heads obtained in three tosses of a coin.

A random variable whose values are not countable is called a continuous random variable. A continuous random variable can assume any value over an interval(s). Because the number of values contained in any interval is infinite. The possible number of values that a continuous random variable can assume is infinite. We cannot count these values. Consider the life of a battery for instance, the life of this battery may be 40 hours, 40.25 hours or 40.247 hours. Assume that the maximum life of such a battery is 200 hours.

Every point on this line represents a possible value x , that denotes the life of a battery. There are infinite number of points on the line. The value represented by all points on this line is uncountable. Example of continuous random variables:

- The height of a person.
- The time taken to complete an examination.
- The weight of a baby.
- The time to play a football game.

Variation $\sigma^2 = \sum[(x - \mu)^2 \cdot P(x)]$

Standard deviation $\sigma = \sqrt{\sum[(x - \mu)^2 \cdot P(x)]}$ or $\sigma = \sqrt{\sum x^2 P(x) - \mu^2}$

x	P(x)	xP(x)	x ²	x ² P(x)
0	.02	0.00	0	0.00
1	.20	0.20	1	0.20
2	.30	0.60	4	1.20
3	.30	0.90	9	2.70
4	.10	0.40	16	1.60
5	.08	0.40	25	2.00
Σ		2.50		7.70

$$\begin{aligned} \sigma &= \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{7.70 - (2.50 * 2.50)} \\ &= \sqrt{7.70 - 6.25} \\ &= \sqrt{1.45} \\ &= 1.20 \end{aligned}$$

x	μ	$x - \mu$	$(x - \mu)^2$	P(x)	$(x - \mu)^2 \cdot P(x)$
0	2.50	-2.50	6.25	.02	0.125
1	2.50	-1.50	2.25	.20	0.450
2	2.50	-0.5	0.25	.30	0.075
3	2.50	0.50	0.25	.30	0.075
4	2.50	1.50	2.25	.10	0.225
5	2.50	2.50	6.25	.08	0.500
Σ			17.50		1.450

$$\sigma^2 = \sum[(x - \mu)^2 \cdot P(x)] = 1.450$$

$$\sigma = \sqrt{1.45} = 1.20$$

PROBABILITY DISTRIBUTION OF A DISCRETE RANDOM VARIABLE

The probability distribution of a discrete random variable lists all the possible values that the random variable can assume and their corresponding probabilities.

The probability distribution of a discrete random variable possesses the following two characteristics:

1. The probability assigned to each value of the random variable x lies in the range 0 to 1, that is $0 \leq P(x) \leq 1$ for each value of x .
2. The sum of probabilities assigned to all values of x is equal to 1.0, that is $\sum P(x) = 1$

These two characteristics are also called “two conditions” that a probability distribution must satisfy.

(x) No of children	frequency	relative frequency	probability distribution of the no of children $P(x)$
0	200	.10	.10
1	600	.30	.30
2	800	.40	.40
3	240	.12	.12
4	160	.08	.08
	N=2000	Sum=1.0	$\sum P(x) = 1.0$

- The probability of any value of x can be read .
- The probability that a randomly selected family from this town has two children is .40. The probability is written as $P(x=2) = .40$

The probability that the selected family has more than two children is given by the sum of the probability of three and four children. This probability is $.12 + .08 = .20$. The probability can be written as $P(x>2) = P(x=3) + P(x=4) = .12 + .08 = .20$

The probability distribution of x describes how the probabilities are distributed over the possible values of x.

X	probability P(x)
0	.15
1	.20
2	.35
3	.30

Find the probability that the number of breakdown for this machine during a given week are

1. Exactly 2 = $P(x=2) = .35$
2. Zero to 2 = sum of 0, 1, 2 breakdown. $P(0 \leq x \leq 2) = P(x=0) + P(x=1) + P(x=2) = .15 + .20 + .35 = .70$
3. More than one= by adding the probability of 2 and 3 breakdowns=
 $P(x>1) = P(x=2) + P(x=3) = .35 + .30 = .65$
4. Less than 2 (by the sum of the probability of 0 and 1 breakdown)=
 $P(x<2) = P(x=0) + P(x=1) = .15 + .20 = .35$

THE MEAN OF A DISCRETE RANDOM VARIABLE

The Mean of a discrete random variable denoted by μ , is the mean of the probability distribution. μ is also called the expected value, α is denoted by $E(x)$.

To calculate the mean of a discrete random variable, r, we multiply its value of x by the corresponding probability and sum the resulting product. This sum gives the mean (or expected value) of the discrete random variable r.

The Mean is the value that is expected to occur per repetition on average if an experiment is repeated a large number of times.

$$\mu = \sum xP(x)$$

X	P(x)	xP(x)
0	.15	.00
1	.20	.20
2	.35	.70
3	.30	.60
		1.50

An average of 1.5 breakdowns are expected to occur per week over a period of time for this machine. If the machine is used for many weeks, we may observe no breakdown for some weeks, for some weeks we may observe 1 breakdown per week, for some weeks we may observe 2 or 3 breakdowns.

The mean of breakdowns is expected to be 1.50 per week.

STANDARD DEVIATION OF A DISCRETE RANDOM VARIABLE

Standard deviation of a discrete random variable, denoted by σ , measures the spread of its probability distribution. A higher value for the standard deviation of discrete random variable x , indicates that x can assume values over a large range above the mean.

A smaller value for the standard deviation indicates that most of the values that x can assume are clustered about the mean.

The basic formula to compute the standard deviation of a discrete random variable is $\sigma = \sqrt{\sum [(x - \mu)^2 \cdot P(x)]}$

The short formula is $\sigma = \sqrt{\sum x^2 P(x) - \mu^2}$

x	P(x)	xP(x)	x ²	x ² P(x)
0	.05	.00	0	.00

1	.15	.15	1	.15
2	.25	.50	4	1.00
3	.40	1.20	9	3.60
4	.15	.60	16	2.40
		2.45		7.15

$$\sigma = \sqrt{\sum [(x - \mu)^2 \cdot P(x)]}$$

$$\sigma = \sqrt{7.15 - (2.45)^2}$$

$$= \sqrt{1.1475} = 1.07$$

Interpretation: It is expected that this auto dealer will sell an average of 2.45 cars per week with a standard deviation of 1.07 cars.

Find the probability that the number of customers are served exactly 12 minutes

- From 10 to 15 minutes.
- More than 15 minutes.
- Less than 18 minutes.

Find the standard deviation for the probability distribution of service time, interarrival and waiting time.

x	Freq	Relative freq.	Cumm. Freq.	percentage	Probability Distribution P(x)	Mean μ $\sum xP(x)$	X ²	X ² P(x)
12	4	0.16	0.16	16%	0.16	1.92	144	23.04
13	2	0.08	0.24	8%	0.08	1.04	169	13.52
14	1	0.04	0.28	4%	0.04	0.56	196	7.84
15	3	0.12	0.40	12%	0.12	1.80	225	27.00
16	2	0.08	0.48	8%	0.08	1.28	256	20.48

17	4	0.16	0.64	16%	0.16	2.72	289	46.24
18	5	0.20	0.84	20%	0.20	3.60	324	64.80
19	3	0.12	0.96	12%	0.12	2.28	361	43.32
20	1	0.04	1.00	4%	0.04	0.80	400	16.00
	25	1.00				16		262.24

$$\text{Mean} = \mu = \sum xP(x) = 16$$

$$\begin{aligned} \text{Standard deviation } \sigma &= \sqrt{\sum [(x - \mu)^2 P(x)]} = \sqrt{\sum x^2 P(x) - \mu^2} \\ &= \sqrt{262.24 - 16^2} \\ &= \sqrt{262.24 - 256} \\ &= \sqrt{6.24} = 2.50 \end{aligned}$$

x	freq	P(x)	$\sum xP(x)$	X^2	$X^2P(x)$
12	4	0.16	1.92	144	23.04
13	2	0.08	1.04	169	13.52
14	1	0.04	0.56	196	7.84
15	4	0.16	2.40	225	36.00
16	1	0.04	0.64	256	10.24
17	4	0.16	2.72	289	46.24
18	5	0.20	3.60	324	64.80
19	3	0.12	2.28	361	43.32
20	1	0.04	0.80	400	16.00
			15.96		261.00

$$\begin{aligned} \alpha^2 &= 261 - 15.96^2 \\ &= 261 - 254.721 \\ \alpha^2 &= 6.2784 \end{aligned}$$

$$\alpha = 2.50$$

Waiting time

x	freq	P(x)	Cumm.	xP(x)	X ²	X ² P(x)
0	12	0.48	0.48	0.00	0	0.00
1	3	0.12	0.60	0.12	1	0.12
2	0	0.00	0.60	0.00	4	0.00
3	3	0.12	0.72	0.36	9	0.84
4	2	0.08	0.80	0.32	16	1.28
5	1	0.04	0.84	0.20	25	1.00
6	1	0.04	0.88	0.24	36	1.44
7	1	0.04	0.92	0.28	49	1.96
8	0	0.00	0.92	0.00	64	0.00
9	2	0.08	1.00	0.72	81	6.48
	25			2.24		13.12

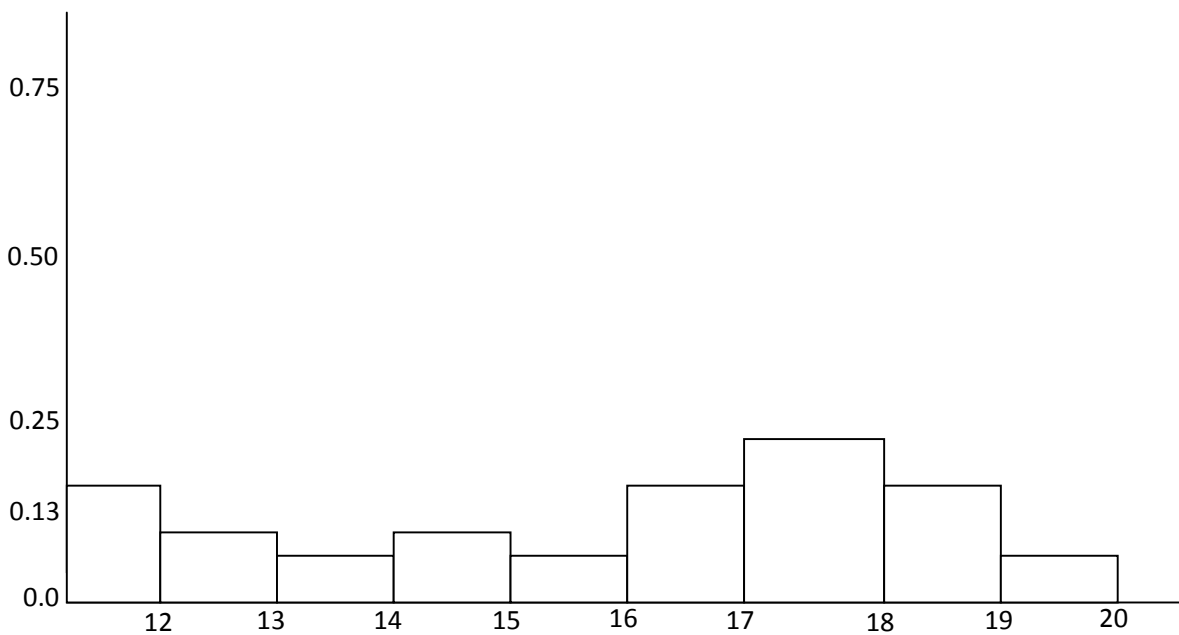
$$13.12 - (2.24)^2$$

$$13 - 5.0176$$

$$\sigma^2 = 7.9824$$

$$\alpha = 2.825$$

Standard deviation is greater than the expected average waiting time= no of customers that can assume values over a large range about the mean.



Inter-arrival	Frequency	Relative frequency	Cumulative frequency	%	Service probability distribution	μ Exp(x)	X^2	$X^2P(x)$
12	3	0.12	0.12			1.44	144	17.28
13	3	0.12	0.24			1.56	169	20.28
14	1	0.04	0.28			0.56	196	7.84
15	0	0.00	0.28			0.00	225	0.00
16	5	0.20	0.48			3.20	256	51.20
17	1	0.04	0.52			0.68	289	11.56
18	2	0.08	0.60			1.44	324	25.92
19	1	0.04	0.64			0.76	361	14.44
20	0	0.00	0.64			0.00	400	0.00
21	3	0.12	0.76			2.52	441	52.92
22	3	0.12	0.88			2.64	484	58.08
23	2	0.08	0.96			1.84	529	42.32
24	1	0.04	1.00			0.96	576	23.04
	25					17.6		324.88

Mean = μ = expected value = 17.6

$$***** = \sum X^2 P(x) - \mu^2 = 324.88 - 309.76$$

POISSON PROBABILITY DISTRIBUTION

- It is used after the French mathematician Simeon Dennis Poisson, who discovered this method in early 19 century.
- It is an important probability distribution of a discrete random variable that has a large number of applications.
- It is applied to experiments with random and independence occurrences

Conditions for Poisson Probability Distribution

- X is a discrete random variable
- The occurrences are random
- The occurrences are unpredictable
- The occurrences are independent.

If the average number of occurrences for a given interval are known, then by using the Poisson probability distribution. We can compute the probability of any number of occurrences X in that interval.

Note that the number of actual occurrences in an interval are denoted by x .

Examples of discrete random variable for which the occurrences are random are independent.

- a. The number of patients arriving at the emergency ward of a hospital during a 1-hour interval. The total number of patients who may arrive at the emergency ward during a 1-hour interval may be 0, 1, 2, 3, 4,.....
- b. Consider the number of defective items in the next 100 items manufactured on a machine. The occurrences are random because there may be 0, 1, 2, 3,...,100 defective items in 100 items.
- c. Consider the number of defects in a 5 foot long iron rod. The interval is a space (5 feet) interval ***** are random because there may be any ***** of defects in a 5 foot-long iron rod.

- d. The number of customer coming to a bank during an hour interval.
- e. The number of television sets sold at a departmental store during a given week.

On the other hand, arrival of patients at a physician's office will be non-random if the patients are to make an appointment to see the Doctor.

In the poisson probability distribution terminology, the average number of occurrences in an interval is denoted by λ (Greek letter lambda). The actual number of occurrences in that interval is denoted by x . We can find the probability of x given λ .

According to the Poisson probability distribution, the probability of x occurrence in an

interval is
$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Where $\lambda = \text{lambda} = \text{mean}$

$e =$ has the value of 2.71825

$$p(x = 12) = \frac{18^{12} e^{-18}}{12!}$$

$$\text{Average waiting time} = \frac{69}{26} \text{ or } \frac{56}{25}$$

$$= 2.65 \text{ or } 2.24$$

$$p(w = 0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{2.24^0 e^{-2.24}}{0!} = (.13534)(.7866) = 0.106$$

$$\approx 0.11$$

$$p(w = 1) = \frac{2.24^1 e^{-2.24}}{1!} = 2.24 \times 0.106 = 0.2374$$

$$\approx 0.24$$

The probability of non-waiting = $P(w=0) =$

$$\text{traffic intensity} = \frac{E(s)}{E(x)} = \frac{\text{service time}}{\text{inter arrival}}$$

exchange estimate of the waiting time =

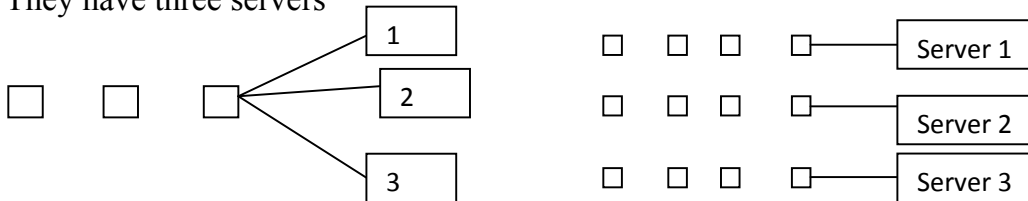
$$\bar{w}_n = \frac{\bar{w}_n + \bar{w}_n^e}{2}$$

Seed	Multiplier value	Eight digit product	Pseudo-random number
5167	3729	19,267,743	0.2677
5167	7743	40,008,081	0.0080

$$18 - 6 + (\text{RNI}) \times [2(6) + 1]$$

$$(0.000573)$$

They have three servers



5000 Customer	500 – waiting time
Single line: $P(w=0) = 0.066$	Multiple line: $P(w = 0) = 0.098$
Standard deviation = 0.013	Standard deviation = 0.011
Means = 12.1	Means = 13.0
Median = 8.11	Median = 9.01

estimated quantities of the waiting time distribution

Discipline	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.975	0.99
Single line	0.00	0.58	2.94	8.11	18.30	28.18	35.10	43.830	54.00
Multiple line	0.00	0.49	3.10	9.01	18.47	29.32	43.20	47.260	59.78

Traffic intensity $a = t = 0.30$
 $b = t = 0.60$
 $c = t = 0.75$

skewed means Gamma (0.5, 1)

regular means Gamma (5.0, 1)

The probability of non waiting (0.098) is higher for the multiple line system compare to $P(w = 0) = 0.0666$ for the single line system.

Looking at the table, indicate that waiting times are longer for the multiple line discipline.

Output Display

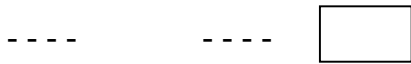
Queuing Data

Single

Multiple

----- -----

----- -----



Investigation – Our primary goal is to examine the stationary “waiting time” w in a multiserver queue to determine whether it is best to use a single-line or multiple-line queuing discipline.

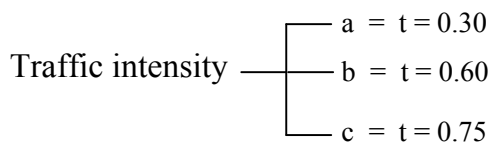
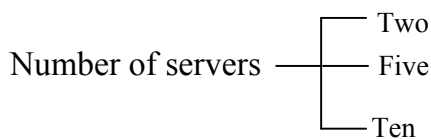
As an approximation to the stationary waiting time, we use the waiting time of the 5000th customer the queue. Notice that the waiting time w is a ran output from the simulation and that many quantities ions of this random variable may be relevant. For instance, we may consider - the mean time

- the median time
- the variance of the waiting time
- the upper extreme quantiles of the waiting distribution (0.95 or 0.99)
- the probability of not having wait, that is $P(w =)$

The waiting time distribution and the comparison between the two disciplines – multiple line or single line will be affected by a number of factors

- the queuing discipline – this is the *****
- the number of servers
- the service and inter-arrival time distribution
- the traffic intensity

Queuing discipline: multiple line and single line



- service and inter-arrival time distributions where “skewed” means Gamma (0.5, 1) and “regular” means Gamma (5.0, 1).

- a. skewed service and skewed interarrival times
- b. regular service and skewed interarrival times
- c. skewed service and regular interarrival times
- d. regular service and regular interarrival times

This 2 x 3 x 4 x 3 design gives us 72 different combinations of level of the four factors.

It is relatively easy to inspect the figures visually to determine the effects of the number of servers by looking at each subgroup of three.

- The effect of service discipline by comparing pair of triplets
- The effect of traffic intensity by comparing the three large clusters

$$\text{Traffic intensity} = \frac{E(w)}{E(x)}$$

Another possible improvements in the simulation of the waiting times in the G1/G1/1 queue would be to simulate delay time. $D_n = W_n + S_n$





We observe the effect of the ***** service and inter arrival time distribution, we need to use the four figures for the factor side by side.

Estimated means (standard deviations) for the distribution of waiting times (each estimate is based on 440 independent replication).

Service, interarrival distributions = traffic intensity	Multiple-line discipline			Single-line discipline		
	2 servers	5 servers	10 servers	2 servers	5 servers	10 servers
Skew, skew						
t = 0.30	0.28(0.82)	0.08 (0.50)	0.0008(0.02)	0.20(0.52)	0.03(0.22)	0.0005
t = 0.60	1.9 (2.9)	1.7 (3.8)	1.2(4.6)	1.6(2.6)	0.25(1.9)	0.46
t = 0.75	4.9 (6.9)	4.5(9.3)	4.5(11.3)	4.2(5.8)	2.6(4.6)	1.9
Regular, skew						
t = 0.30	0.18(0.37)	0.10(0.47)	0.0(0)	0.13(0.27)	0.05(0.22)	0.002(
t = 0.60	0.19(1.4)	0.82(1.7)	0.18(1.9)	0.97(1.2)	0.58(1.2)	0.42
t = 0.75	2.7(3.3)	2.4(3.5)	1.7(3.8)	2.3(2.7)	2.1(2.8)	1.4
Skew, regular						
t = 0.30	0.03(0.21)	0.01(0.17)	0.0(0)	0.03(0.18)	0.0(0)	0.0
t = 0.60	0.65(1.6)	0.45(2.3)	0.18(1.8)	0.62(1.4)	0.30(1.2)	0.07
t = 0.75	2.2(3.9)	2.1(5.4)	1.7(7.8)	1.8(3.2)	1.2(2.8)	0.59
regular, regular						
t = 0.30	0.001(0.1)	0.0(0)	0.0(0)	0.0002(0.004)	0.0(0)	0.01

t = 0.60	0.08(0.30)	0.06(0.34)	0.02(0.29)	0.06(0.19)	0.01(0.08)	0.0003
t = 0.75	0.37(0.70)	0.41(1.2)	0.22(1.1)	0.23(0.46)	0.18(0.47)	0.06

It is clear immediately that traffic intensity t has a major effect on the waiting time distribution. It appears to be generally true that the multiple line discipline has larger extreme upper quantity than the corresponding single line discipline quantity.

- The effect of the number of servers on extreme quartiles depends on both the traffic intensity a the service and interarrival time distribution and is not consistent between the two disciplines.
- It is possible to compare the means and standard deviations of multiple and single line discipline
 - Means and median
 - Variance, coefficient of variation
 - Skewness or kurtosis
 - Quartiles, percentile
 - Cumulative distribution function
- * empirical cumulative distribution function
- * histograms, boxplots
- * probability (p-p) plots
 - (Q-Q) plots
 - antithetic varieties for queuing system
 - control variable – interval control
 - systematic sampling – exchange queuing system

- The average waiting time $E(W_n)$ is not much different from queuing disciplines multiple line or single line
- The probability of waiting a long time is much longer in multiple line than the single line

EXPONENTIAL DISTRIBUTIONS

Exponential distributions are generally used to represent occurrences of events such as arrivals and departures of customers (super – markets, banks, computer centres), births, deaths, accidents etc.

If the events are independent and if the probability of occurrence of an event in a small interval of time is very small, then the time interval between occurrences of these events is exponentially distributed. This is known as a Poisson process.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } \lambda > 0 \\ & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$f(x) = \int_0^x \lambda e^{-\lambda t} dt = 1 - e^{-\lambda x}$$

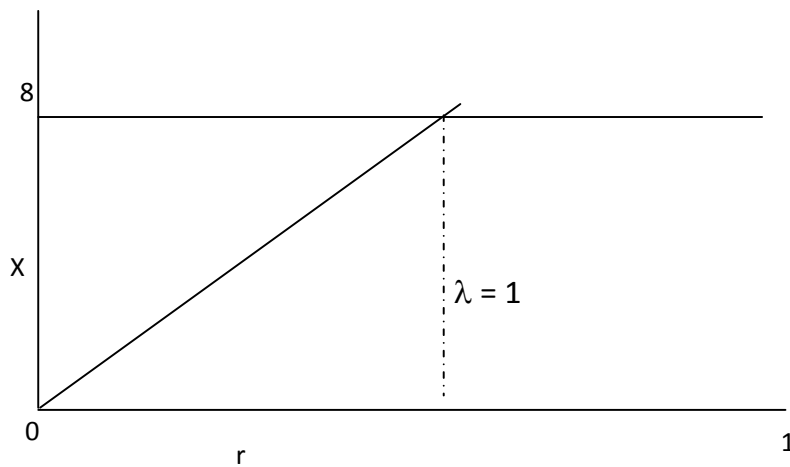
$$\text{Mean} = E(X) = \frac{1}{\lambda}, \text{ Variance} = \frac{1}{\lambda^2}$$

Using the inverse transformation technique

$$r = 1 - e^{-\lambda x} \quad 0 \leq r < 1$$

$$e = 2.71828$$

$$X = -\frac{1}{\lambda} \log(1 - r) = -(\text{mean}) \cdot \log(1 - r)$$



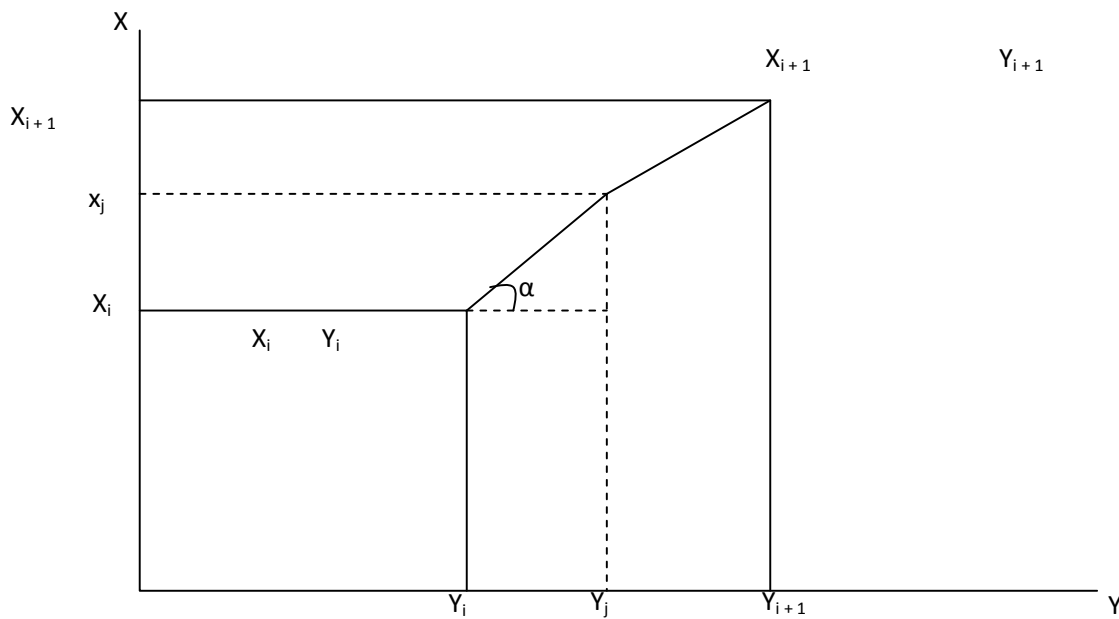
The inter event time x for an exponential distribution can be computed from the mean of the distribution

Methods for Random Number

- Von Neumann, midsquare method
- Linear congruential generators
- Multiplicative congruential generators

The statistical methods usually applied for testing the randomness of numbers are

- moments test
- frequency test – using chi-square method
- serial test
- runs tests
- gap test
- Kolmogorov – Smirnov test



$$\tan \theta = S_i = \frac{X_{i+1} - X_i}{Y_{i+1} - Y_i}$$

Any point X_j between X_i and X_{i+1} is given by

$$X_j = X_i + [(y_j - y_i)S_i]$$

$$X_i \quad Y_i = f(X_i)$$

$$0 \quad 0$$

$$2 \quad 0.3$$

$$x_i = \quad 0.4 = r = y_j$$

$$5 \quad 0.5$$

$$4 \quad 1$$

$$X_i = 2 \quad y_j = 0.3$$

$$X_{i+1} = 5 \quad y_{i+1} = 0.5$$

$$f(2) < r < f(5)$$

$$S_i = \frac{(5 - 2)}{(0.5 - 0.3)} = \frac{3}{0.2} = 1.5$$

$$X_i = 2 + [(0.4 - 0.3)1.5] = 3.5$$

$$X_i = 3.5$$

The service time of patients at a casualty department are shown below:

Service time (Minutes)	Frequency
1 – 2	3
2.1 – 3	5
3.1 – 4	15
4.1 – 5	23
5.1 – 6	33
6.1 – 7	25
7.1 – 8	37
8.1 – 9	17
9.1 – 10	11
10.1 – 11	9
11.1 – 12	1

Graph the cumulative distribution (probability versus time) and read out the service time when the random number generated by the simulation program was 0.732. Compare the result with the service time obtained analytically by interpolation techniques

Inter arrival time distribution of 50 customers in a car park is shown below. Determine the interarrival time X_j of a customer corresponding to the random number $r = 0.52$

Interarrival time (minutes)	X	Number of arrivals	Relative frequency	Cumulative frequency
0 – 9		14	0.28	0.28
10 – 19		10	0.20	0.48
20 – 29		12	0.24	0.72
30 – 39		14	0.28	1.00
		<u>50</u>	<u>1.00</u>	

$r = 0.52 = y_j$
 $y_i = 0.48$
 $y_{i+1} = 0.72$

$$x_i = 20$$

$$x_{i+1} = 30 \quad S_i = \frac{(30 - 20)}{(0.72 - 0.48)} = \frac{10}{0.24} = 41.67$$

$$X_j = 20 + [(0.52 - 0.48)41.67]$$

$$= 20 + (0.04)(41.67)$$

$$= 20 + 1.6668 = 21.66$$

$$\approx 22$$

Interarrival time (minutes)	X	Number of arrivals	Relative frequency	Cumulative frequency
12 – 15		7	0.28	0.28
16 – 19		9	0.36	0.64
20 – 23		8	0.32	0.96
24 – 27		1	0.04	1.00
		<u>50</u>		

$r = 0.52 = y_j$

$$y_i = 0.64 \quad x_i = 8 \quad 19$$

$$y_{i+1} = 0.96 \quad x_{i+1} = 12 \quad 23$$

$$S_i = \frac{(23 - 19)}{(0.96 - 0.64)} = \frac{4}{0.32} = 12.5$$

$$X_j = 19 + [(0.76 - 0.64)12.5]$$

$$= 20 + (0.12)(12.5) = 20.5$$

$$y_i = 0.28 \quad x_i = 14 \quad r = 0.38$$

$$y_j = 0.48 \quad x_{i+1} = 16$$

$$S_i = \frac{(16 - 14)}{(0.48 - 0.28)} = \frac{2}{0.20} = 10$$

$$X_j = 14 + [(0.38 - 0.28)10]$$

$$= 15$$

$$r = 22 = X_j$$

$$x_i = 20 \quad x_{i+1} = 30$$

$$y_i = 0.48 \quad y_{i+1} = 0.72$$

$$S_i = \frac{0.24}{10} = 0.024$$

$$y_j = 0.48 + [(22 - 20)0.024]$$

$$= 0.48 + 0.048$$

$$= 0.528$$

94 = Arrival time of the last customer = T

- Average arrival rate = $\frac{N}{T} = \frac{40}{94} = 0.4225$
- Average inter arrival time = $\frac{94}{40} = 2.35$
- Average service rate = $\frac{40}{221} = 0.1810$
- Average service time = $\frac{221}{40} = 5.525$

$$T_0 \quad T_1 \quad T_2 \quad T_3 \quad = 0.4225$$

$$S_0 \quad S_1 \quad S_2 \quad S_3 \quad = 0.1810$$

Customer arrives at $T_0 = 0$ has service time S_0

Customer arrives at $T_1 = 1$ has service time S_1

Theoretical (expected) frequency in an interval =

(Total no of occurrence) x (Theoretical probability of occurrence in that interval)

$$40 \quad x \quad 1 - e^{-\lambda(t)}$$

$$t = 0; f(t) = 0$$

$$1 - e^{-(0.4225)(0)}$$

$$1 - .6570 = 0.343$$

$$40 \times .343 = 13.72$$

$$t = 1 f(t) = 1 \quad 40 \times 1 - e^{-\lambda(t)} = 13.72$$

$$t = 2 \quad f(t) = 2 \quad 40 \times 1 - 0.6570^{(2)}$$

$$40 \times 0.5684 = 13.72$$

$$\text{Interval 1 - 2} \quad t = (1) \quad f(t) = 1 = 1 - e^{-0.4225(t)}$$

$$= 1 - .6570^{(2)}$$

$$= 1 - 0.4316$$

$$40 \times 0.5684$$

$$0.5684 - 0.343$$

$$40 \times (0.5684 - 0.343)$$

$$40 \times 0.2254$$

$$= 9.01$$

$$\text{Interval 2 - 3} \quad t = 2$$

$$\text{Interval 3 - 4} \quad t = 3 \quad 1 - (0.6570)^3$$

$$= 1 - 0.2836$$

$$0.7164 - 0.5684$$

$$40 \times 0.148 = 5.92$$

$$f(5) = 1 \quad 1 - e^{-0.181(1)}$$

$$1 - 0.8353$$

$$0.1647 \times 40 = 6.588$$

$$S = 2$$

$$f(5) = 2$$

$$1 - e^{-0.181(2)}$$

$$1 - (0.8353)^2$$

$$1 - 0.6977$$

$$= 0.3023$$

$$0.3023 - 0.1647$$

$$0.1376 \times 40 = 5.5$$

$$S(3)$$

$$f(5) = 3$$

$$1 - e^{-0.181(3)}$$

$$1 - (0.8353)^3$$

$$1 - 0.5827$$

$$0.4173 - 0.3023 = 0.115$$

$$0.115 \times 40 = 4.6$$

$$** =$$

$$f(t) = 11 - e^{-0.4225(1)}$$

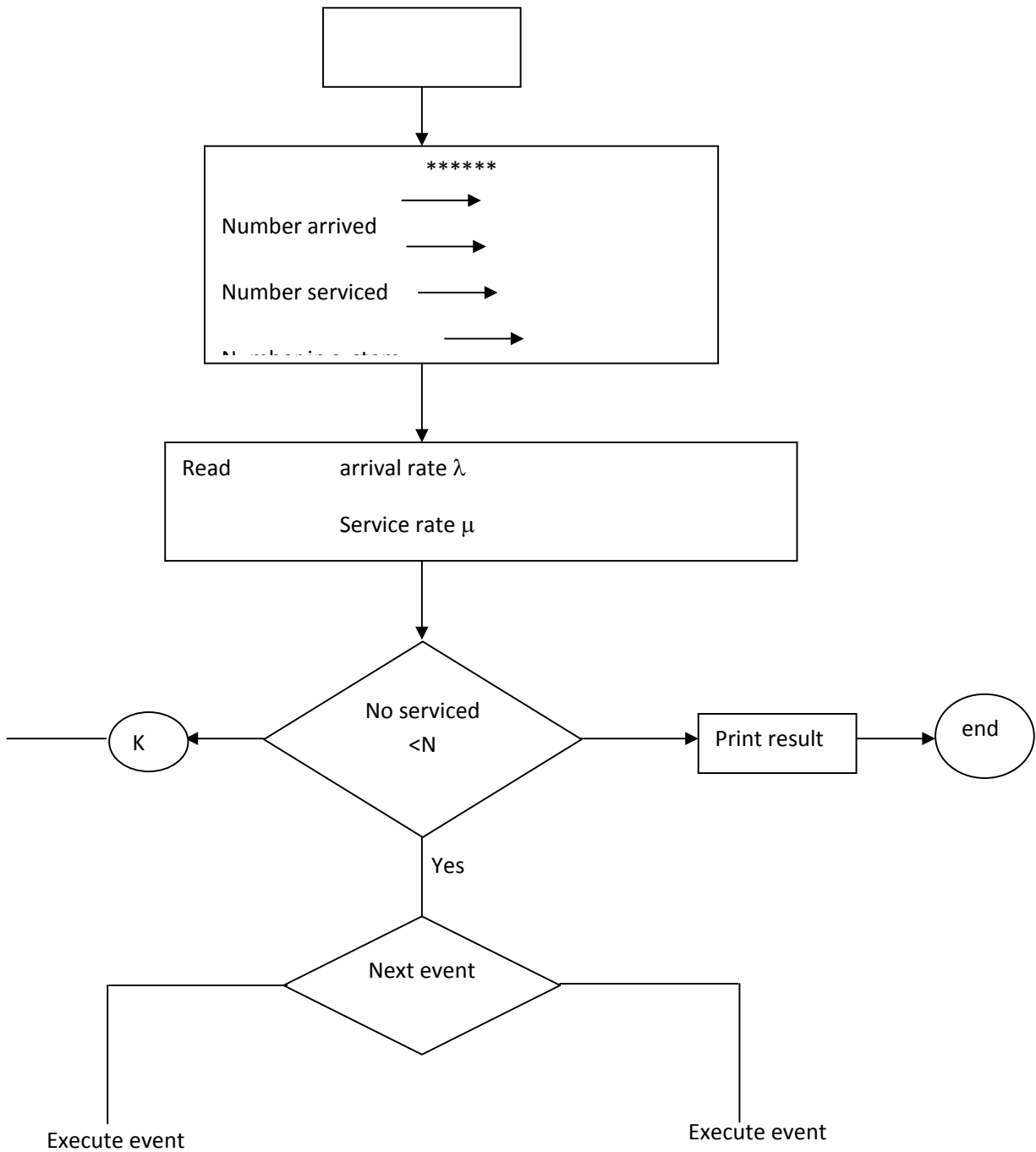
$$1 - 0.6570 = 0.343$$

$$0.343 \times 40 = 13.72$$

$$\begin{aligned}t = 2 \quad & 1 - e^{-0.4225(2)} \\ & 1 - (0.657)^2 \\ & 1 - 0.4316 = 0.5684 \\ & 0.5684 - 0.343 = 0.2254 \\ & 40 \times 0.2254 = 9.016\end{aligned}$$

$$\begin{aligned}t = 3 \quad & 1 - e^{-0.4225(3)} \\ & 1 - (0.657)^3 \\ & 1 - 0.2836 = 0.7164 \\ & 0.7164 - 0.5684 = 0.148 \\ & 40 \times 0.148 = 5.92\end{aligned}$$

$$\begin{aligned}t = 4 \quad & 1 - e^{-0.4225(4)} \\ & 1 - (0.657)^4 \\ & 1 - 0.1863 = 0.8137 \\ & 0.8137 - 0.7164 = 0.0973 \\ & 40 \times 0.0973 = 3.892\end{aligned}$$



- a Compute the length of time and the number of customers in the system between events
- b Update clock to next arrival time
- c Record arrival time of the customer
- d If the system is empty, then
 1. start service and record time
 2. generate service time from appropriate distribution and compute next discharge time
- e Increase number in system by one
- f Increase number arrived by one

- g Generate inter-arrival time from appropriate distribution and compute next arrival time

- a Compute the length of time and the number of customers in the system between events
- b Update clock to next discharge time
- c Record time service ends for the customer

- e Decrease number in system by one
- f If the system is not empty, then
 1. record time service begins for next customer
 2. generate service time from appropriate distribution and compute next discharge time
- g If the system is empty, then *** next discharge time to infinity

Probability of zero customer	= 11.2% = out of every 1 unit of time under consideration, cardphone have no customer to attend to or idle
Probability of 6 customers. They are five servers	= 0.0261 = The five servers are busy. One person is waiting in the system to be served in every 1 unit of the time is 2.61%
Probability that all servers are busy	= 0.105 = 10.5%. In every 1 unit of time under consideration, all server are busy is 10.5%. This shows under-utilization
Probability of queuing time longer than 1.5385	0.048 = 0.04% = 4.86% - whoever comes into the system has probability 4.8% chance of queuing longer than 1.5385 minutes
Average queuing time	= 0.2095 = 20.9% = Every customer comes into the system is likely to spend 0.2095 minutes waiting before service begins.
Average time spent in the system	= 5.609 = every customers comes into the system will spend about 5.609 minutes
Standard deviation	5.4719 = standard deviation is related to mean, n. considering the small sample used, the

	degree of **** between the average time spent in the system and standard deviation of the system response time is reasonably good
Average number of customers in the queue	= 0.0884. This means that the average number of customers that are queuing with none of them receiving service is 0.0884 which means nobody wait for service
Average number of customers in the system	= 2.367. This means that average number of the customers in the system with some of them receiving service from all the servers is 2.3675.

CSC351

that delivers a certain service to the transaction at a finite rate or speed. In this case, CPU and disks are examples of active resources.

CPU: executes a certain number of instructions per unit time.

Disks: read/write a transaction at a certain speed

: determine the number of 110 operations that can be performed per second.

(seek time, rotational speed,)

CPU ratio= path length / CPU speed= number of instruction

Passive resources= memory partition.

Once a transaction is in possession of a memory partition, it's execution progresses at a rate that is independent of any characteristic of the passive resources

Utilization= the utilization of any device is the fraction of time the device is busy.

For example, in a bank= job= Query transaction or Update transaction

Workloads in the banking system

- Cashier/teller on-line transaction
- Automated teller machine transaction
- Decision support system moved execution
- Payroll checks processing
- Statement processing

- Debit/credit transaction.

ARCHITECTURE OF A COMPUTING ENVIRONMENT

- Hardware
- Operating system
- Support software(interpreter, editors, compiler, DBMs, transaction monitors)
- DBMs transactions command(oracle, Informix, Sybase, ADABAS)
- User application

Time and rate are the basic measures of a components performance.

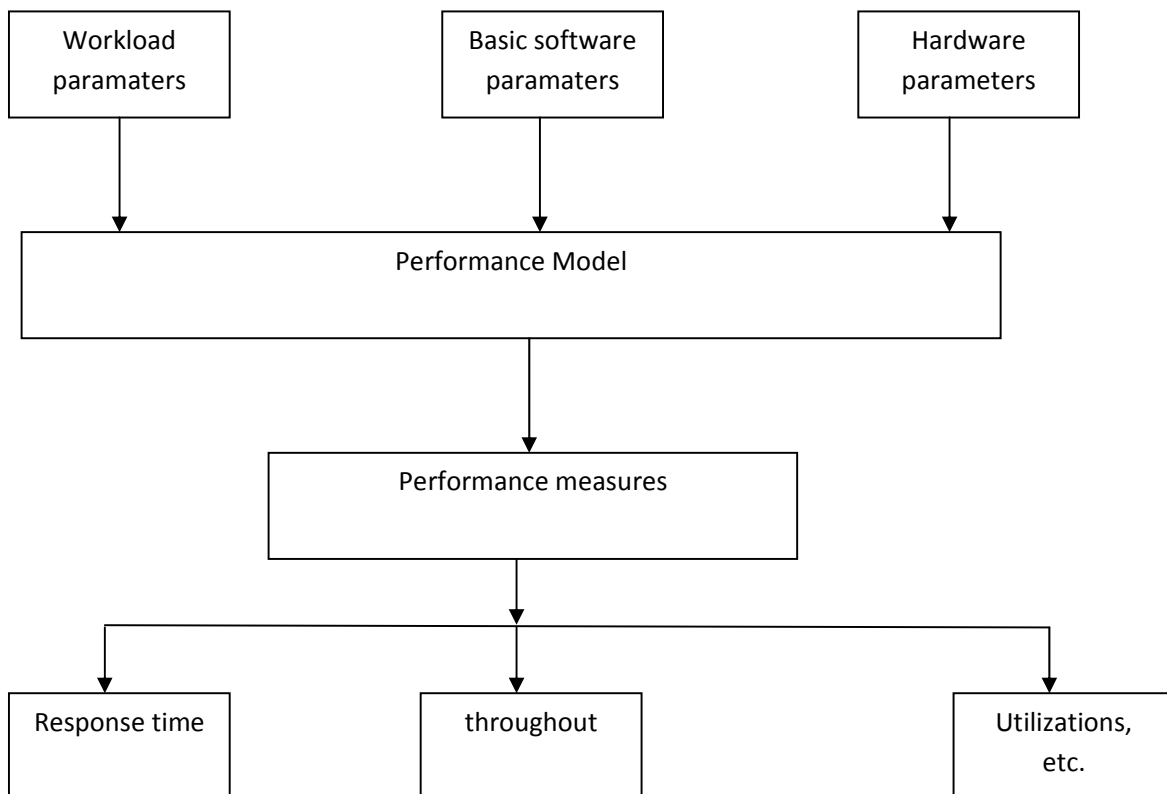
MIPS means Million of instruction per second =

$$\frac{\text{instructions}}{\text{cycle}} \times \frac{\text{cycles}}{\text{second}} \times 10^{-6}$$

RISC means reduced instruction set computer

CISC means complex instruction set computer

MFLOPS means million of floating point operation for scientific computer.



Workloads

- ◆ Jobs transactions into PC
- ◆ Average arrival times
- ◆ No of terminal in an interactive system
- ◆ Service demands placed on each of the systems resource by each type of job on transaction.

Basic software

- ◆ Operating system
- ◆ Maximum degree of multi-programming
- ◆ CPU dispatching priority

Hardware

- ◆ Processor speed
- ◆ Disk latencies
- ◆ Transfer rates
- ◆ Local area network speed

Performance matrices

Workload=

- ◆ Service demand
- ◆ Workload intensity

Service demand: is the total average time spent by a transaction at a particular server. It does not include the time waiting for the server

$D_i =$ service demand at device i .

Number of transaction processes, program or jobs that the content for the resources of computer system.

Service demand= $D_i = V_i S_i$ or

$$D_i = \frac{U_i T}{C_o}$$

V = average number of visit that a program makes to server i .

S = mean service time per visit to server i .

CPU device demand= $D_{CPU,i} = U_{CPU,prog} \times T / C_o$

$U_{CPU,prog}$ = Busy time of executing program instructions or application program= CPU utilization

T= monitoring interval

Co= total number of completed transactions during the monitoring interval.

During an observation period of 1800 sec. a hardware monitor measured a utilization of 40% for the CPU and counted 100 batch job completions.

CPU service demand= $(0.4 \times 1800)/100 = 7.2$ secs.

Execution of a transaction requires use of the CPU, a certain amount of time and the use of both disks for executing the I/O operations needed to answer the query. The following steps describe the behavior of a typical transaction T from its arrival at the system until its departure.

1. Transaction T has to acquire a memory partition to run on. If five transactions are already in execution, T has to queue for memory.
2. Once memory is obtained, transaction T will try to use the CPU to start the execution, T will have to queue for the CPU.
3. Once at the CPU the transaction will use it until either:
 - It requires an I/O operation. In this case, it will have to queue for the disk if the device is busy. After completing the I/O, T returns to the CPU queue.
 - Its time slice expires; in this case, transaction T is suspended and again placed in the CPU ready queue.
 - It complete execution; in this case, T release the memory partition it was holding and departs from the system.

Therefore, during its execution, a transaction will cycle between CPU and I/O a certain number of times until it completes execution. In the example above, a transaction may be found in one of the five following state:

- ❖ Queuing for the memory
- ❖ Queuing for the CPU
- ❖ Using the CPU
- ❖ Queuing for a disk
- ❖ Using a disk.

Classes of Two resources

- Active resources
- Passive resources

Active resources are also called SERVERS or DEVICES

It is equivalent to a 320MIPS single processor

3 multi-processor of 10MIPS≠30MIPS style processor

Standard programs known as bench marks that have been used to evaluate the performance of processors.

Examples of bench mark programs;

- ✓ Whatstone
- ✓ Dhystone
- ✓ Linpack
- ✓ Perfect club
- ✓ SPEC

The average time to read/write a disk block can be computed as;

Average access time= avg seek + avg rotational delay + block size/transfer rate + controller time.

Seek time= which corresponds to the time to move the arm of the disk to the destination track.

Rotational delay= which is the time for the requested section to rotate under the head.

Transfer rate= which represent the time to transfer the bits of a sector

Controller time= which corresponds to the overhead of the I/O controller.

- Most count disk rotate at 3600 ratio per minute
- Average seek time between 12m/sec and 20m/sec
- Typical rate s between 1 and 4mbytes/sec
- Assuming an average seek time of 20m/sec, a transfer rate 2mbytes/sec, a rotational delay of 8.33m/sec (half of the average rotational time) 2m/sec for the controller overhead.

The average time to read and write a 512-byte sector is

$$\begin{aligned} & 20 + 8.33 + 512/2m + 2 \\ & = 28.33 + 2 + 0.256 \\ & = 30.556 \end{aligned}$$

Computer system: each transaction requires an average 20 millisecond of processing from the central processor.

What is the current performance of the system, e.g.

- the processor utilization
- the system response time
- the system through put
- the average queue length

What is the predicted performance if the workload increases by 50% but the processor is upgraded by 50%?

- The primary workload = arrived rate of transactions = λ service rate = μ
 Service rate = $\mu = 1 \text{ transaction} / 20 \text{ milliseconds} = 50 \text{ tps}$
 $\lambda = 30 \text{ tps}$

0 1 2 k

$\mu = 50 \text{ tps}$

Utilization = $P_1 + P_2 + \dots = 1 - P_0 = \lambda / \mu = 30 / 50 = 60\%$

- System throughput = is the rate at which customers leave the system after being served. Total through put is the sum of the individual state through prts, weighed by the probability of the system is equal to the arrival rate.

Flow out equal to flow in...

$$= \mu (p_1 + p_2 + \dots) = \sum_{k=1}^{\infty} \mu k p_k$$

$$= \mu (p_1 + p_2 + \dots)$$

$$= \mu \times \text{utilization}$$

$$= \mu \times A / U$$

$$= \lambda = 30 \text{ transaction per second.}$$

This includes both the number of customers in the queue and the number being served.

$$\text{Queue length} = \lambda / \mu - \lambda = 30 / 50 - 30 = 30 / 20 = 1.5$$

Average response time: is the average time that a customer spends in the system, from arrival time until completion time. There are two components to a customer's response time, the time spent on waiting on the queue and the time spent in the service. The time that a customer spent waiting on the queue is equal to the time required to clear out the customers who are currently in the system when the customer arrives. Upon arrival, a customer "sees" steady-state. This is a consequence of the arrival theorem which states that an arriving customer sees steady-state of the system with itself removed.

An arriving customer sees mean queue length = $\lambda / \mu - \lambda$ = other customers ahead itself = mean queue length.

It requires $1 / \mu$ time units to flush each one of these earlier arrived customers out of the system.

- The time that a tagged customer spends waiting in the queue is $1/\mu \times \lambda/\mu-\lambda$
- Once the tagged customer gets to the head of the queue, it requires an additional $1/\mu$ time units to complete its own service.
- Therefore, the average response time is:
 $1/\mu \times \lambda/\mu-\lambda + 1/\mu = 1/\mu-\lambda = 1/50-30 = 1/20\text{secs} = 0.05\text{sec}$

From the above equation, we can deduce that;

$1/\mu$ is the time to finish those ahead of me.

$\lambda/\mu-\lambda$ is the no 1sec at arrival time.

$1/\mu$ is my service time.

That is the predicted performance of the workload, increases by 50% more transaction per second

$$\lambda = 30 \quad 50\% = 45$$

$$\mu = 1/20 = 50\text{tps.}$$

$$\text{Utilization} = \lambda/\mu = 45/50 = 90\%$$

$$\text{Throughput} = 45\text{tps}$$

$$\text{Queue length} = \lambda/\mu-\lambda = 45/50-45 = 9\text{customers}$$

$$\text{Response time} = 1/\mu-\lambda = 1/50-45 = 1/5\text{sec} = 0.2\text{sec.}$$

By increasing the workload by 50%

$$\text{Utilization} = 60\% \text{ now } 90\%$$

$$\text{Throughput} = 30\text{tps} \text{ now } 45\text{tps}$$

NB: THE ABOVE INCREASED BY 50%

$$\text{Queue length} = 1.5\text{customers} \text{ now } 9\text{customers} = 500\% \text{ increase}$$

$$\text{Response time} = 1/20\text{sec} \text{ now } 1/5\text{sec}$$

$$0.05 \text{ Now } 0.2 = 300\% \text{ increase}$$

What is the predicted performance if the workload increase by 50% but the processor is ungraded by 50%?

Workload by 50% = increase the transaction arrival rate λ

$$\lambda = 30 = 45$$

$\mu = 50\text{tps}$ = the new processor will be able to handle 75tps instead of the old rate of 50 transaction per second.

$$\mu = 75$$

$$\text{Utilization} = \lambda/\mu = 45/75 = 60\%$$

$$\text{Throughput} = \lambda = 45\text{tps}$$

$$\text{Queue length} = \lambda/\mu - \lambda = 45/75 - 45 = 45/30 = 1.5\text{customers}$$

$$\text{Response time} = 1/\mu - \lambda = 1/75 - 45 = 1/30\text{sec}$$

Utilization = 60% now 60% = no change

Throughput = 30 now 45 = increase by 50%

Queue time = 1.5 now 1.5 = no change

Response time = 1/20 now 1/30 = drop from 1/20 to 1/30

= a 33% improvement.

General –purpose simulation

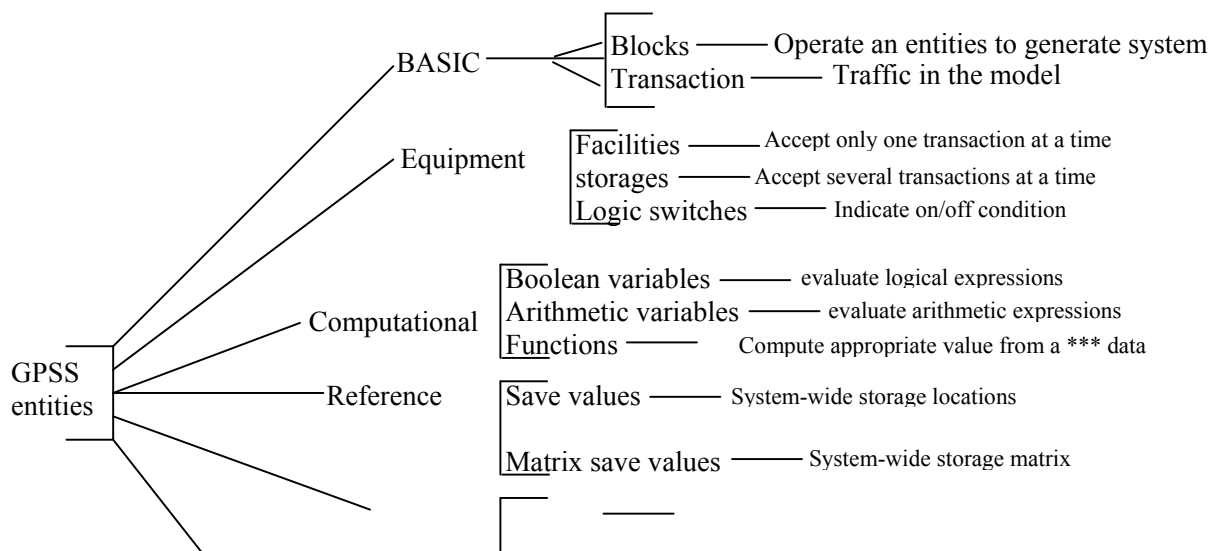
GPSS was originally developed by IBM Gordon in 1962.

- It was invented for use by non- computer specialists
- It was called General purpose system simulator to General purpose simulation system.
- It has interface with Fortran and PL/1.

The GPSS Language is structured as a block- oriented language as the design was influenced by block diagram and flowchart concepts. Several versions of GPSS are currently available on a variety of computer including microprocessors.

GPSS simulates the system by updating the simulation clock (absolute clock) to the time at which the most imminent event is to occur and not by evaluating the model at each successive interval of time.

There are fourteen types of entities available in GPSS for model representation



Statements in a GPSS program are called Blocks. The blocks are subroutines or macro-instructions which perform a series of operations

Block		Source program	
		Simulate	
6,2	Generate	GENERATE 6,2	micros arrive every 4 to 8 min
	Queue	Queue 1,1	wait in queue 1
	Seize	TEST T	Engage test station if free
	Depart	1,1	remove one micro from queue
	Advance	5,3	Process for 2 to 8 minutes representing the processing time
	Release	Test R	free test station
	Terminate	1	remove rested micro from system
	START	1000	Stop after simulation micros

Microprocessor

Manufacturing plant every 6 ± 2 minutes and the testing time per micro was found to be 5 ± 3 minutes. The service is on a first-come-first-served basis and the units placed in a queue whenever the test facility is busy. Assuming that the inter-arrival times and service times are uniformly distributed, simulate the processing of 1000 micros at the test station.

START & END are control statements.

SIMULATE= statements must be present, if it is absent, the GPSS program will be compiled but not executed.

Output average queue length

= average weighty time =1.62

= Utilization of facilities =0.8257

= average time per transaction =4.92 minutes

- The interarrival times are uniformly distributed in the range of to 8 minutes.
- The transactions wait in queue 1 defined by QUEUE block until the test station is free.
- A transaction gets control of the test facility on entry to the SEIZE block and the moves into the DEPART block when one transactions is removed from the top of queue 1
- The queue statistics are automatically collected by the GPSS system.
 - The transaction then moves into the ADVANCE block where it is delayed by 2 to 8 minutes (5 ± 3) representing the processing time in the test facility.
 - On completion of testing a micro, the transaction moves out of the ADVANCE block and enters the RELEASE block, where it gives up the test station which is now made available to the next transaction waiting in the QUEUE block.
 - The next block in the TERMINATE block where the transaction are destroyed and removed from the simulation model.
 - Whenever a transaction enters the TERMINATE block, the value of the first operand (operand A) of the START statement is decremented by the value of the first operand of the TERMINATE block and the simulation stops when the value of the operand A of the START statement becomes zero
- For example, if the test facility is free at the time of the creation of a transaction, then that transaction will move through GENERATE, QUEUE, SEIZE, DEPART blocks in zero time and straight away to the ADVANCE block, where of course will be delayed for specified passengers arrive at the immigration control section of an airport every 7 seconds and join one of the two queues-
 - Queue 1 for aliens
 - Queue 2 local citizens

Sixty percent of the arrivals were found to be local citizens and they have priority over aliens service. The immigration control desk is manned by a single officer who takes 3 minutes to process local passports and 7 ± 3 minutes (uniformly distributed) to process alien passports. Simulate the operation of the immigration control section, with a view to studying the queues of the passengers and the utilization of the immigration officer.

SIMULATE

GENERATE 7 passengers arrive every 7 minutes
 TRANSFER 600 40% Aliens are separate
 QUEUE 1 aliens stand in Queue 1
 SEIZE OFFCR engage immigration officer if free
 DEPART 1 remove one alien passenger from queue 1
 ADVANCE 7,3 immigration officer takes 4 to 10 minutes
 RELEASE OFFCR make immigration officer free
 TERMINATE 1 remove 1 alien passenger from system
 Local Priority 1 give priority 1 to local passenger
 QUEUE 2 local passenger stand in queue 2
 SEIZE OFFCR engage immigration officer, if free
 DEPART 2 remove one local passengers from queue 2
 ADVANCE 3 immigration officer takes 3 minutes
 RELEASE OFFCR make immigration officer free
 TERMINATE 1 remove 1 local passenger from system
 START 1000 stop after simulating 1000 passengers
 STOP

Service time = 16
 minutes

interarrival time can be as short as 12 minutes or as long as 24

A hairat time could be as short as 12 minutes or as long as 20 minutes

Means for interarrival = 18 interarrival = 18 ± 6

Shortest minutes = 12

Longest minutes = 24

Difference = $12/2 = 6$

Means for service time = 16 service time = 16 ± 4

Shortest minutes = 12

Longest minutes = 20

Difference = $8/2 = 4$

Generate 18,6 = customers arrive every 12 and 24

Queue line 1 = wait in queue

SEIZE JOE = engage test station from queue 1

DEPART LINE = remove one customer from queue 1

ADVANCE 16,4 = process for 12 to 20 minutes representing the service time

RELEASE JOE ——— server is free

TERMINATE 1 ——— remove the service

START 25 ——— stop after similarity 25 customers

STOP.

MAJOR SOURCES OF ERROR

- Errors in data collection or data used
- Errors associated with the method of collection
- Analysis of the output
- Interpretation of the output

- Modelling errors – invalid models may result in serious simulation errors
- Computer programming error - arithmetic errors to logical errors
 - omission of control statement such as simulate, start,stop
- Sampling errors – poor random number generators may cause serious errors in simulation
- Errors in parameter estimation