CDURSE CDIDE:
CDURSE TITLE:
NUMBER (DF UNITS:
CDURSE DURATIDN:

MCE 506
Fluid Machinery
3 Units
Three hours per week

## CDURSE DETAILS:

Course Coordinator: Email: Office Location: Other Lecturers:

Engr. Dr. Bukola Olalekan Bolaji B.Eng., M.Eng., $P h D$
bolajibo@unaab.edu.ng, bobbolaji2007@yahoo.com
HOD MCE's Office, COLENG
None

## CDURSE CDNTENT:

Types of fluid machines, classifications. Positive displacement machines: Reciprocating and rotary pumps. Compressors. Hydraulic motors. Rotodynamic machines: Basic theory, performance. Axial-flow, centrifugal and mixed-flow pumps, fans, compressors and turbines, selection of appropriate fluid machines.

## COURSE MequIdemidyis:

This is a compulsory course for all 500 level students in the Department of Mechanical Engineering. In view of this, students are expected to participate in all the course activities and have minimum of $75 \%$ attendance to be able to write the final examination.

## BDADING MST:

1. Bolaji, B.O. Introduction to Fluid Mechanics. Nigeria: Ed., Adeksor Nig. Ent. 2008.
2. Douglas, J.F., Gasiorek and Swaffield, Fluid Mechanics. England: Addison Wesley Longman Ltd., 1985.
3. Fox, R.W. and McDonald, A.T. Introduction to Fluid Mechanics. New York: John Wiley and Sons, 1999.
4. Kreith, F. and Berger, S.A. Mechanical Engineering Handbook. Boca Raton: CRC Press, LLC, 1999.
5. Shepherd, D.G. Principles of Turbo-machinery, Macmillan, 1969.
6. Turton, R.K. Principles of Turbo-machinery, E and F Spon 1984.
7. Cohen, H., Rogers, G.F.C. and Saravanamuttoo, H.H. Gas Turbine Theory. $3^{\text {rd }}$ Edition, London: Longman, 1987.
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## LECTURE NOTES

### 1.0 CLASSIFICATION OF FLUID MACHINES

### 1.1 INTRODUCTION

The term "fluid machinery" is used to describe machines which cause a change of total head of the fluid flowing through them by virtue of the dynamic effect they have upon the fluid. When power is produced by a turbomachine, it is called a turbine and when power is absorbed to raise pressure, it is called a pump.

### 1.2 TYPES OF FLUID MACHINES

### 1.2.1 Turbines

(a) Impulse turbine (Pelton wheel)
(b) Reaction turbine:
(i) Radial flow turbine,
(ii) Mixed flow turbine, and
(iii) Axial flow turbine

### 1.2.2 Pumps

(a) Centrifugal or reaction pumps
(i) Radial flow pump (single or double suction)
(ii) Mixed flow pump (single or double suction)
(iii) Axial flow pump (single or multistage)
(b) Positive displacement pumps
(i) Reciprocating (piston, plunger, and diaphragm)
(ii) Rotary:
(1) Single rotor (vane, piston, and screw types)
(2) Multiple rotors (gear, lobe, and screw types)

### 1.2.3 Fans

(i) Radial flow fan
(ii) Mixed flow fan
(iii) Axial flow fan
(iv) Cross flow fan

### 1.3 TURBINES

There are two types of turbines, namely, impulse and reaction turbines. When a liquid passes through a machine, both the kinetic and pressure energy of the liquid may change. The distribution of the change between kinetic and pressure energy defines the degree of reaction. When there is no change in static pressure across the runner, the degree of reaction is zero and the machine is an impulse design. There are no impulse pumps but impulse turbines, known as Pelton wheels.

## Reaction Turbines

The reaction turbine is employed for powering generators. Its runner is enclosed in a chamber which is completely filled with water under high hydrostatic pressure. The arrangement of a turbine plant is shown Fig. 1.1. Water is supplied to the turbine from a reservoir and thence,
via a draft tube, to a tail race. The pressure of the water drops from inlet to exit as it flows through the runner. Guide vanes are installed on the periphery of the runner to ensure that the flow enters the runner vanes radially; thereafter the flow through the runner remains radial. Reaction turbines include Francis and Kaplan (or Propeller) types which offer a range of specific speeds.

The objective of the draft tube is to keep the turbine full of water and it is divergent to reduce the final velocity of the water, this keeping the loss of kinetic energy of exit, $\mathrm{V}^{2}{ }_{\text {exit }} / 2 \mathrm{~g}$ to a minimum. The height of the draft tube, $z$, is limited by the need to keep the outlet pressure, $\mathrm{p}_{2}$, at the runner above about 2.5 m of water absolute, otherwise bubbles of vapour will be released which will damage the turbine. This phenomenon is known as cavitation. Some of the gross head is lost to pipe friction, entry loss, bends, etc., and some is lost in kinetic energy at exit; the remaining head is available to produce power. Thus net head,


Fig. 1.1: Schematic diagram of a turbine plant.
H = gross head - losses - exit K.E.

### 1.4 TYPES OF REACTION TURBINE

### 1.4.1 Radial Flow Turbine

In the Francis radial flow turbine, water enters a spiral volume chamber and then flows radially through stationary pivoted guide vanes; these direct the water, ideally without shock, on to the moving vanes attached to the runner. The water flows radially inwards through the vanes but leaves the runner axially to enter the draft tube. Francis turbines have medium specific speeds $\left(\mathrm{N}_{\mathrm{ST}}\right)$ between 40 and $350 \mathrm{rev} . / \mathrm{min}$, and are suited to heads between 20 and 200 m .

### 1.4.2 Mixed Flow Turbines

The runner is generally similar to that of the radial flow turbine but is designed to give partly radial and partly axial flow. Fig. 1.2 shows variations in the flow direction for different runners, together with typical specific speeds. The specific speed increases as the flow becomes more axial and less radial.


Fig. 1.2: Mixed flow turbine.

### 1.4.3 Axial Flow Turbine

In a Kaplan axial flow or propeller turbine, the water enters a spiral volute chamber and then flows radially through stationary pivoted guide vanes. It is then turned into the axial direction before passing through the runner, which is similar to a propeller. Kaplan turbines have a high specific speed $\left(\mathrm{N}_{\mathrm{ST}}=430\right.$ to 750$)$ and are suited to low net heads, i.e. between 3 and 25 m .

### 1.5 PUMPS

Pumps are devices that impart a pressure increase to a liquid. The pressure rise found in pumps can vary tremendously, and this is a very important design parameter along with the liquid flow rate. This pressure rise can range from simply increasing the elevation of the liquid to increasing the pressure hundreds of atmospheres. Lifting of water from wells and cisterns is the earliest form of pumping. Modern applications are much broader, and these find a wide variety of pumps in use. There are two categories of pumps, viz: centrifugal or reaction pumps, and positive displacement pumps.

### 1.5.1 Centrifugal and Other Reaction Pumps

Centrifugal pumps are used in more industrial applications than any other kind of pump. This is primarily because these pumps offer low initial and upkeep costs. Traditionally, pumps of this type have been limited to low-pressure-head applications, but modern pump designs have overcome this problem unless very high pressures are required. Some of the other good characteristics of these types of devices include smooth (non-pulsating) flow and the ability to tolerate non-flow conditions.

The most important parts of the centrifugal pump are the impeller and volute. An impeller can take on many forms, ranging from essentially a spinning disk to designs with elaborate vanes. Impeller design tends to be somewhat unique to each manufacturer, as well as finding a variety of designs for a variety of applications. This device imparts a radial velocity to the fluid that has entered the pump perpendicular to the impeller. The volute (there may be one or more) performs the function of slowing the fluid and increasing the pressure.

### 1.5.2 Positive Displacement Pumps

Positive-displacement pumps demonstrate high discharge pressures and low flow rates. Usually, this is accomplished by some type of pulsating device. A piston pump is a classic example of positive displacement machines. Rotary pumps are one type of positive displacement device that do not impart pulsations to the existing flow. Several techniques are available for dealing with pulsating flows, including use of double-acting pumps (usually of the reciprocating type) and installation of pulsation dampeners. Positive-displacement pumps usually require special seals to contain the fluid. Costs are higher both initially and for maintenance compared with other pumps.

### 1.6 TYPES OF REACTION PUMPS

### 1.6.1 Radial Flow Pump

In the radial flow or centrifugal pump, water enters the impeller (or rotor) eye axially and then flows radially outwards through the blade passages. The specific speed of a centrifugal pump may be up to 100 but when $\mathrm{N}_{\mathrm{SP}}<30$, a set of diffuser blades is fixed round the rotor to improve efficiency. Centrifugal pumps give relatively high heads with a low flow rate.

In a mixed flow pump, the water enters axially and the passes through an impeller which gives it a partly radial flow at exit. Also, as with turbine, the transition from axial to radial flow is gradual and the specific speed and general performance lie between those for centrifugal and axial flow pumps. Specific speeds range from 100 to 200.

### 1.6.3 Axial Flow or Propeller Pump

In the axial flow, water flows axially through a propeller type runner and then through stationary guide vanes. The specific speed of a propeller pump is high $\left(\mathrm{N}_{\mathrm{SP}}>200\right)$ and this design is suitable for cases where a high flow rate with low head is required.

### 1.7 FANS

Fans are devices that impart air movement due to rotation of an impeller inside a fixed casing. Fans find application in many engineering systems. Along with the chillers and boilers, they are the heart of heating, ventilating, and air conditioning (HVAC) systems. Many types of fans are found in power plants. Very large fans are used to furnish air to the boiler, as well as to draw or force air through cooling towers and pollution-control equipment. Electronic cooling finds applications for small units. Even automobiles have several fans in them. Generally fans are classified according to how the air flows through the impeller. These flows may be axial (essentially a propeller in a duct), radial (conceptually much like the centrifugal pumps), mixed, and cross. Mixed-flow fans are so named because both axial and radial flow occurs on the vanes.

### 2.0 SPECIFIC SPEED OF FLUID MACHINES

### 2.1 PERFORMANCE PARAMETERS

The specific speed of a turbine or a pump is the speed at which a geometrically similar machine would need to run to produce unit output from unit input at maximum efficiency. This speed has a typical value for each different design of machine and is a useful parameter for selecting the optimum type for given performance requirements.

$$
\begin{array}{ll} 
& Q \propto N D^{3} \\
\text { or } \quad \frac{Q}{N D^{3}}=C
\end{array}
$$

The discharge $Q$ is given as:

$$
\begin{equation*}
Q=C_{d} A \sqrt{2 g H} \tag{2.2}
\end{equation*}
$$

Since A is proportional to $D^{2}$, the discharge equation may be:

$$
\begin{equation*}
\frac{Q}{D^{2} \sqrt{H}}=C \tag{2.3}
\end{equation*}
$$

Eliminating $Q$ between Eqs. (2.1) and (2.3) gives

$$
\begin{equation*}
\frac{H}{N^{2} D^{2}}=C \tag{2.4}
\end{equation*}
$$

Equations (2.1 and (2.4) are most useful in determining performance characteristics for a range of geometrically similar turbines or pumps. These parameters are found by experiment for a particular design of machine at the point of maximum efficiency; these will be the same for all similar machines and are given in the following names:

Flow coefficient, or discharge number, $C_{Q}=\frac{Q}{N D^{3}}$
Head Coefficient, $C_{H}=\frac{g H}{N^{2} D^{2}}$
Power Coefficient, $C_{P}=\frac{P}{\rho N^{3} D^{5}}$

The specific speed $N_{S}$ is usually defined differently for a turbine and a pump.

### 2.2 THE SPECIFIC SPEED OF TURBINE

The specific speed of a turbine $\left(N_{S T}\right)$ is defined as the speed at which a similar turbine would generate an output of a unit power under a unit head. Since power is proportional to $Q H$,

$$
\begin{equation*}
\text { therefore, } \quad \frac{P}{Q H}=C \tag{2.8}
\end{equation*}
$$

The terms $D$ and $Q$ may be eliminated from Eqs. (2.4), (2.7) and (2.11) to produce:

$$
\begin{equation*}
\frac{N \sqrt{P}}{H^{5 / 4}}=C \tag{2.9}
\end{equation*}
$$

For unit power and unit head the constant of Eq. (2.9) becomes the speed, or the specific speed of a turbine $\left(\mathrm{N}_{\mathrm{ST}}\right)$, so that,

$$
\begin{equation*}
N_{S T}=\frac{N \sqrt{P}}{H^{5 / 4}} \tag{2.10}
\end{equation*}
$$

where P is in $\mathrm{kW}, N$ is in rev/min, and $H$ in m .

### 2.3 THE SPECIFIC SPEED OF PUMP

The specific speed of a pump $\left(N_{S P}\right)$ is defined as the speed at which a similar pump would deliver an output of a unit discharge at a unit head. This is determined by eliminating $D$ in Eqs. (2.1) and (2.4) to give:

$$
\begin{equation*}
\frac{N \sqrt{Q}}{H^{3 / 4}}=C \tag{2.11}
\end{equation*}
$$

By definition of specific speed, the constant is $N_{S P}$, the speed of a unit for $Q=1 \mathrm{~m}^{3} / \mathrm{s}$ and $H=1$ m is:

$$
\begin{equation*}
N_{S P}=\frac{N \sqrt{Q}}{H^{3 / 4}} \tag{2.12}
\end{equation*}
$$

where, $Q$ in $\mathrm{m}^{3} / \mathrm{s}, N$ is in rev/min, and $H$ in m . The specific speed of a unit required for a given discharge and head can be estimated from Eqs (2.10) and (2.12).

## QUESTION

Francis turbine is required to develop about 4.5 MW at maximum efficiency when running at $110 \mathrm{rev} / \mathrm{min}$ with a head of 14 m . A scale model is tested with a head of 3.7 m and at maximum efficiency, the speed was $210 \mathrm{rev} / \mathrm{min}$, the flow rate was $1.45 \mathrm{~m}^{3} / \mathrm{s}$ and the power output was 44.25 kW . Determine: (i) the ratio of sizes of turbine and model, (ii) the specific speed, (iii) the flow rate required through the turbine, (iv) the turbine power output, and (v) the turbine efficiency.

### 3.0 THEORY OF FLUID MACHINES

### 3.1 INTRODUCTION

Turbines extract useful work from fluid energy; and pumps, blowers, and turbocompressors add energy to fluids by means of a runner consisting of vanes rigidly attached to a shaft. Since the only displacement of the vanes is in the tangential direction work is done by the displacement of the tangential components of force on the runner. The radial components of force on the runner have no displacement in a radial direction and hence can done work.

### 3.2 HEAD AND ENERGY RELATIONS

For no losses the power available from a turbine is Power $=Q \Delta P=\rho g Q H$, in which $H$ is the head on the runner, since $\rho g Q$ is the weight per unit time and $H$ the potential energy for unit weight. Similarly, a pump runner produces work $\rho g Q H$, in which $H$ is the pump head. The power exchange is:

$$
\begin{equation*}
T \omega=\rho g Q H \tag{3.1}
\end{equation*}
$$

The pump head is given as:

$$
\begin{equation*}
H=\frac{u_{2} V_{u 2}-u_{1} V_{u 1}}{g} \tag{3.2}
\end{equation*}
$$

For turbines the sign is reversed in Eq. (3.4). For pumps the actual head $H_{a p}$ produced is:

$$
\begin{equation*}
H_{a p}=\eta_{h} H=H-H_{L} \tag{3.3}
\end{equation*}
$$

and for turbines the actual head $H_{a T}$ is:

$$
\begin{equation*}
H_{a p}=H / \eta_{h}=H+H_{L} \tag{3.4}
\end{equation*}
$$

where $\eta_{h}$ is the hydraulic efficiency of the machine and $H_{L}$ represents all the internal fluid losses in the machine. The overall efficiency of the machines is further reduced by:
(i) bearing friction,
(ii) friction caused by fluid between runner and housing, and
(iii) by leakage or flow that passes round the runner without going through it. These losses do not affect the head relations.
Pumps are generally so designed that the angular momentum of fluid entering the runner (impeller) is zero. Then

$$
\begin{equation*}
H=\frac{u_{2} V_{2} \cos \alpha_{2}}{g} \tag{3.5}
\end{equation*}
$$

Turbines are so designed that the angular momentum is zero at the exit section of the runner for conditions at best efficiency; hence,

$$
\begin{equation*}
H=\frac{u_{1} V_{1} \cos \alpha_{1}}{g} \tag{3.6}
\end{equation*}
$$

## QUESTION

A centrifugal pump with a 700 mm diameter impeller runs at $1800 \mathrm{rev} / \mathrm{min}$. The water enters without whirl, and $\alpha_{2}=60^{\circ}$. The actual head produced by the pump is 17 m . Calculate the theoretical head and the hydraulic efficiency when $V_{2}=6 \mathrm{~m} / \mathrm{s}$.

### 3.3 REACTION TURBINES

In the reaction turbine a portion of the energy of the fluid is converted into kinetic energy by the fluids passing through adjustable gates before entering the runner, and the remainder of the conversion takes place through the runner. All passages are filled with liquid, including the passage (draft tube) from the runner to the downstream liquid surface. The static fluid pressure occurs on both sides of the vanes and hence does no work. The work done is entirely due to the conversion to kinetic energy.

The reaction turbine is quite different from the impulse turbine. In an impulse turbine all the available energy of the fluid is converted into kinetic energy by a nozzle that forms a free jet. The energy is then taken from the jet by suitable flow through moving vanes. The vanes are partly filled, with the jet open to the atmosphere throughout its travel through the runner.

In contrast, in the reaction turbine the kinetic energy is appreciable as the fluid leaves the runner and enters the draft tube. The function of the draft tube is to reconvert the kinetic energy to flow energy by a gradual expansion of the flow cross-section. Application of the energy equation between the two ends of the draft tube shows that the action of the tube is to reduce the pressure at its upstream end to less than atmospheric pressure, therefore, increasing the effective
head across the runner to the difference in elevation between head water and tail water, which reduce losses.

By referring to Fig. 3.2, applying the Bernoulli's equation;

$$
\begin{equation*}
z_{1}+\frac{v_{1}^{2}}{2 g}+\frac{p_{1}}{\rho g}=z_{2}+\frac{v_{2}^{2}}{2 g}+\frac{p_{2}}{\rho g}+\text { losses } \tag{3.7}
\end{equation*}
$$

Therefore, the energy equation from (1) to (2) yields:

$$
z+\frac{v_{1}^{2}}{2 g}+\frac{p_{1}}{\rho g}=0+0+0+\text { losses }
$$

The losses include the expansion loss, friction, and velocity head loss at the exit from the draft tube, all of which are quite small;

$$
\begin{equation*}
\text { hence, } \quad \frac{p_{1}}{\rho g}=-z-\frac{v_{1}^{2}}{2 g}+\text { losses } \tag{3.8}
\end{equation*}
$$

Equation (3.8) shows that considerable vacuum is produced at section (1), which effectively increase the head across the turbine runner. If power input to the turbine $\left(P_{i n}\right)=\rho g H Q$ and efficiency $(\eta)$ is the ratio of power output to power input, therefore, power output

$$
\begin{equation*}
\left(P_{\text {out }}\right)=\eta(\rho g H Q) . \tag{3.9}
\end{equation*}
$$

## QUESTION

A turbine has a velocity of $6 \mathrm{~m} / \mathrm{s}$ at the entrance to the draft tube and a velocity of $1.2 \mathrm{~m} / \mathrm{s}$ at the exit. For friction losses of 0.1 m and tail water 5 m below the entrance to the draft tube, find the pressure head at the entrance.

### 3.4 IMPULSE TURBINE.

In the impulse turbine, such as the Pelton wheel (Fig. 3.3) the energy of the fluid supplied to the machine is converted by one or more nozzles into kinetic energy. The jet strikes a series of buckets on the circumference of the wheel and is turned through an angle $\theta$ (usually $165^{\circ}$ ) thus producing a force on the bucket and a torque on the wheel. The interior of the casing of the Pelton wheel is at atmospheric pressure and is not filled with water. The wheel must be placed above tail water level so that the water leaving the buckets falls clear of the wheel.


Fig. 3.3: Pelton wheel turbine.

## The Power Developed and Hydraulic efficiency of Pelton Wheel.

Fig. 3.4 shows the inlet and outlet velocity triangles.


Fig. 3.4: Inlet and outlet velocity triangles.
If $H=$ head at the nozzle
$v_{1} \quad=\quad$ absolute velocity of jet and entry to the bucket
$u \quad=\quad$ mean bucket speed
$v_{r 1} \quad=\quad$ velocity of jet relative to bucket at entry
$=\quad v_{1}-u$ from the inlet triangle
$v_{2} \quad=\quad$ absolute velocity of the water leaving the bucket
$v_{r 2} \quad=\quad$ relative velocity of water leaving bucket
$Q \quad=\quad$ volume of water deflected per second.
Force exerted on bucket, $F$, is the rate of change of momentum of water in the plane of the wheel, therefore,

$$
\begin{equation*}
F \quad=\quad \dot{m} \Delta v \tag{3.14}
\end{equation*}
$$

where, $\dot{m} \quad=\quad$ mass of water deflected per second $=\rho Q(\mathrm{~kg} / \mathrm{s})$
$\Delta v \quad=\quad$ change of velocity in direction of motion of bucket.
Initial absolute velocity of water indirection of bucket is $v_{1}$ and the component of final absolute velocity in this direction is $v_{2} \cos \beta$. Change of absolute velocity in this direction, $\Delta v=v_{1}-$ $v_{2} \cos \beta$. Therefore,

$$
\begin{equation*}
F=\rho Q\left(v_{1}-v_{2} \cos \beta\right) \tag{3.15}
\end{equation*}
$$

From the outlet velocity triangle

$$
v_{2} \cos \beta=u-v_{r 2} \cos \alpha=u-v_{r 2} \cos (180-\theta)
$$

where $\theta$ is the deflection angle. If there is no friction on the surface of the bucket the water enters and leaves with the same relative velocity so that $v_{r 2}=v_{r 1}=v_{1}-u$ and

$$
\begin{equation*}
v_{2} \cos \beta=u-\left(v_{1}-u\right) \cos (180-\theta) \tag{3.16}
\end{equation*}
$$

Force exerted on bucket, Eq. (3.15) becomes

$$
\begin{align*}
& \quad F=\rho Q\left\{v_{1}-\left[u-\left(v_{1}-u\right) \cos (180-\theta)\right]\right\} \\
&  \tag{3.17}\\
& F=\rho Q\left(v_{1}-u\right)[1+\cos (180-\theta)] \\
& \\
& \text { Power developed }  \tag{3.18}\\
& \quad \begin{aligned}
& =\text { work done per second } \\
& =\text { (Force on bucket) } \times \text { (bucket speed) }
\end{aligned} \\
& \therefore \quad P_{\text {dev }} \quad=\rho Q u\left(v_{1}-u\right)[1+\cos (180-\theta)]
\end{align*}
$$

Power supplied to the nozzle = (weight per sec.) x (head at nozzle)

$$
\begin{equation*}
P_{\text {sup }}=\rho g Q H \tag{3.19}
\end{equation*}
$$

Hydraulic efficiency, $\eta_{h}=\frac{\text { power output }}{\text { power supplied }}$

$$
\begin{align*}
& \eta_{h}=\frac{\rho Q u\left(v_{1}-u\right)[1+\cos (180-\theta)]}{\rho g Q H} \\
& \eta_{h}=(u / g H)\left(v_{1}-u\right)[1+\cos (180-\theta)] \tag{3.20}
\end{align*}
$$

## QUESTION

A Pelton wheel is supplied with water under a head 30 m at a rate of $41 \mathrm{~m}^{3} / \mathrm{min}$. The buckets deflect the jet through an angle of $160^{\circ}$ and the mean bucket speed is $12 \mathrm{~m} / \mathrm{s}$. Calculate the power and the hydraulic efficiency of the machine.

### 4.0 CENTRIFUGAL PUMPS

### 4.1 INTRODUCTION

The centrifugal pump is one of the most common types of fluid machine and it consists essentially of a runner or impeller which carries a number of backward curved vanes and rotates in a casing (Fig. 4.1). Liquid enters the pump at the centre and work is done on it as it passes centrifugally outwards so that it leaves the impeller with high velocity and increased pressure. In the casing, part of the kinetic energy of the fluid is converted into pressure energy as the flow passes to the delivery pipe. Fig. 4.1 shows a volute casing which increases in area towards the delivery thus reducing the velocity of the liquid and increasing the pressure to overcome the delivery head. This type of casing has a low efficiency as there is a large loss of energy in eddies.


Fig. 4.1: Centrifugal pump

### 4.2 THEORETICAL HEAD DISCHARGE

The momentum equation is used to determine the force on a curved blade and this forms the basis of the action of a turbomachine. It is common to resolve the absolute velocity of the water into components in the radial, tangential and axial direction; the force in any direction then derives from the change in momentum in that direction. The velocity component tangential to the rotor circumference is that which gives rise to the torque and power and is therefore the most important force considered in turbo machine analysis. In this analysis, the following notation is used:
$V_{1}$ and $V_{2} \quad=\quad$ absolute velocity at inlet and outlet, respectively
$u_{1}$ and $u_{2} \quad=\quad$ relative velocity at inlet and outlet, respectively $v_{1}$ and $v_{2} \quad=\quad$ tangential velocity at inlet and outlet, respectively $v_{w 1}$ and $v_{w 2}=\quad$ velocity of whirl at inlet and outlet, respectively $v_{r 1}$ and $v_{r 2} \quad=\quad$ radial velocity at inlet and outlet, respectively where $v_{1}=\omega r_{1}$ and $v_{2}=\omega r_{2}$.

Figure 4.2 shows the relative velocity triangles at inlet and outlet of a typical blade of a radial flow (outward) pump impeller. The directions of $u_{1}$ and $u_{2}$ are tangential to the blade tips for smooth flow; the directions of $v_{1}$ and $v_{2}$ are tangential to the impeller circumference at the blade tips. The torque about the shaft axis, exerted on the water, is given by the rate of change of the angular momentum of the water about that axis. Momentum of water at inlet per second is equal to the product of the mass of water per second and the tangential velocity $\left(v_{w 1}\right)$.
Therefore, Momentum of water at inlet $/ \mathrm{sec}=\rho Q \times v_{w 1}$
Moment of momentum (or angular momentum) of water/sec $=\rho Q v_{w 1} r_{1}$
similarly angular momentum of water/sec at outlet $=\rho Q v_{w 2} \cdot r_{2}$
$\therefore \quad$ torque $=$ change of angular momentum $/$ sec.
i.e. $\quad T=\rho Q\left(v_{w 2} r_{2}-\quad v_{w 1} r_{1}\right)$


Fig. 4.2: Relative velocity triangles at inlet and outlet.
The power input, $P=T \omega=\rho Q\left(v_{w 2} r_{2}-v_{w 1} r_{1}\right) \omega$
But $\omega r_{1}=v_{1}$ and $\omega r_{2}=v_{2}$

$$
\begin{equation*}
\therefore \quad P=\rho Q\left(v_{w 2} v_{2}-\quad v_{w 1} v_{1}\right) \tag{4.5}
\end{equation*}
$$

Equation (4.5) represents the energy input per second but in pump analysis, it is often more convenient to work in terms of the head produced, rather than the power. The head $H$ represents the energy per unit weight.

Power, $P=\rho g Q H \quad$ or $\quad H=P /(\rho g Q)$
Substituting Eq.(4.5) in Eq. (4.6) we have:

$$
\begin{equation*}
H=\frac{\rho Q\left(v_{w 2} v_{2}-v_{w 1} v_{1}\right)}{\rho g Q}=\frac{v_{w 2} v_{2}-v_{w 1} v_{1}}{g} \tag{4.7}
\end{equation*}
$$

Equation (4.7) represents the Euler head.

### 4.2 APPLICATIONS OF HEAD DISCHARGE

In centrifugal pumps, the water velocity has no component in the tangential direction at inlet to the impeller, that is, the velocity of whirl $v_{w 1}$ is zero and the velocity triangle is right-angled. Hence equations (4.5) and (4.7) reduce to

$$
\begin{equation*}
P=\rho Q v_{w 2} v_{2} \tag{4.8}
\end{equation*}
$$

and

$$
\begin{equation*}
H=v_{w 2} v_{2} / g \tag{4.9}
\end{equation*}
$$

The Euler head is greater than the actual head produced, due to losses. The actual head produced comprises the suction head $\left(H_{s}\right)$, the discharge head $\left(H_{d}\right)$, the friction losses in the suction and delivery pipes, $H_{f s}$ and $H_{f d}$, respectively, and the discharge velocity head ( $v_{d}{ }^{2} / 2 g$ ). This head is known as the manometric head, $H_{m}$, since it is the head which would be registered by a manometer connected across the pump at inlet and outlet if the inlet and outlet pipes were the same diameter. Therefore:

$$
\begin{equation*}
H_{m}=H_{s}+H_{d}+H_{f s}+H_{f d}+v_{d}^{2} / 2 g \tag{4.10}
\end{equation*}
$$



Fig. 4.3: Distribution of the components of manometric head of a pump
The distribution of these heads is shown in Fig. 4.3. The ratio of the manometric head to the Euler head is called the manometric efficiency ( $\eta_{m}$ ).
i.e., $\quad \eta_{m}=\frac{H_{m}}{v_{w 2} v_{2} / g}=\frac{g H_{m}}{v_{w 2} v_{2}}$

The difference between $v_{w 2} v_{2} / g$ and $H_{m}$ is made up of hydraulic losses such as friction, bends eddies, etc., and the effectiveness of the volute casing or diffuser in converting the impeller exit velocity to pressure energy. In addition to these hydraulic losses, there are also mechanical losses such as bearing friction; therefore, the overall efficiency of the pump will be less that the manometric efficiency.

## Example 4.1

The diameter of the impeller of a centrifugal pump is 1.2 m and the velocity of the whirl at outlet is $6.4 \mathrm{~m} / \mathrm{s}$. If the pump discharges $3.4 \mathrm{~m}^{3} / \mathrm{min}$, what will be turning moment on the shaft?

## Example 4.2

A centrifugal blower of outer diameter 500 mm delivers air weighing $1.25 \mathrm{~kg} / \mathrm{m}^{3}$ at a rate of $3.1 \mathrm{~m}^{3} / \mathrm{s}$ and speed of $900 \mathrm{rev} / \mathrm{min}$. The manometric head is 26.4 m of air, and the power supplied to the blower shaft is 1.65 kW and the mechanical efficiency is $93 \%$. If the velocity of the whirl at outlet is $14 \mathrm{~m} / \mathrm{s}$. Calculate (a) the Euler head (b) power supplied to the fluid by the impeller (c) the power output of blower (d) manometric and overall efficiencies (e) power lost in (i) bearing friction and windage, (ii) the diffuser (iii) the impeller.

### 5.0 HYDRAULIC MACHINES

### 5.1 RECIPROCATING PUMPS

A reciprocating pump for incompressible fluids uses a piston and cylinder mechanism. Liquid is drawn into the pump cylinder through an inlet valve as the piston moves back, then, as the piston moves forward the inlet valve closes and the liquid is raised in pressure until the delivery valve opens when the required pressure is reached, (Fig. 5.1).


Fig. 5.1: Piston and cylinder mechanism of reciprocating pump.

### 5.2 HYDRAULIC PUMP WITH CUSHION CHAMBERS

Air cushion chamber is usually filled on the suction side of hydraulic pumps to smooth fluctuations and reduce the risk of cavitation. Similarly; on the delivery side, an air cushion chamber will smooth pulsating flow. Fig. 4.3 shows the arrangement of air cushion chambers. The effect of the cushion chamber is to reduce the amount of liquid being accelerated to that between each chamber and the pump.


Fig. 5.3: Reciprocating pump fitted with cushion chambers

### 5.3 RECIPROCATION PUMP PERFORMANCE

Pump delivery increases with speed as would be expected but is reduced by slip which is the difference between the swept volume and delivered volume. Therefore, volumetric efficiency is defined by:

$$
\begin{align*}
& \eta_{\text {vol }}=\frac{\text { delivered volume }}{\text { swept volume }} \\
\therefore & \eta_{\text {vol }}=\frac{Q}{Q_{o}}=\frac{Q_{o}-Q_{L}}{Q_{o}}=1-\frac{Q_{L}}{Q_{o}} \tag{5.6}
\end{align*}
$$

Where $Q=$ delivered volume ( $\mathrm{m}^{3} / \mathrm{s}$ ); $Q_{L}=$ loss due to slip; and $Q_{o}=$ (swept volume) x (speed). The overall efficiency of the pump is given by:

$$
\begin{equation*}
\eta_{o}=\frac{\rho Q g H}{P} \tag{5.7}
\end{equation*}
$$

Where $P$ is the power input to the pump, $H$ is the head and $\rho$ is the fluid density.

### 5.4 ROTARY POSITIVE DISPLACEMENT PUMPS

These comprise a group of pumps which use close meshing or sliding of rotating parts to move liquid from the low pressure supply side to the high pressure delivery side. Some common designs include gear pumps, lobe pumps and sliding vane pumps. The gear pump type is shown in Fig. 5.4. The performance characteristics of these pumps are similar to those of reciprocating pumps.


Fig. 5.4: Gear Pump

### 5.5 HYDRAULIC ACCUMULATORS

The hydraulic accumulator is used for temporary storage of high pressure water (Fig. 5.5). It consists of a vertical cylinder with a sliding ram around which are attached circumferential containers filled with heavy material. Water is pumped into the cylinder and lifts the ram and heavy material until the cylinder is full.

When the machine served by the accumulator requires the high pressure water it is passed to the working cylinder of the machine. An accumulator is also useful for overcoming the fluctuating nature of reciprocating pump flow to allow a steady power supply at constant pressure. The maximum energy or capacity stored by an accumulator is the product of the accumulator pressure $P$ and the accumulator volume v.


Fig. 5.5: Hydraulic accumulator

### 5.6 HYDRAULIC INTENSIFIERS

The hydraulic intensifier is a device for increasing the pressure of a quantity of water. In order to achieve this, a large amount of low pressure water is required. The intensifier has a fixed ram through which high pressure water is supplied to a machine. A sliding ram is fitted over the fixed ram and the outer wall of this sliding ram moves inside a fixed cylinder into which low pressure water is passed. Due to the area difference, the sliding ram is forced down over the lower fixed ram, increasing the pressure in the sliding ram. The sliding ram passes this high pressure water to the machine in use (Fig. 5.6). When the sliding ram reaches the bottom of the stroke, valves are operated to allow the sliding ram to move up to the beginning of the stroke and repeat the process.


Fig. 5.6: Hydraulic intensifier

### 5.7 HYDRAULIC BRAKES

The operating of hydraulic vehicle brakes is similar to that of the press (Fig. 5.7). A large movement of the vehicle brake pedal raises the pressure in the "master cylinder'. This pressure is transmitted to the larger area of the wheel cylinders which operate the brake shoes or disc pads forcing them against the brake drums or brake disc, respectively.

In order to avoid inward air leaks, enough fluid must always be in the system. Air is a compressible fluid which would lead to inefficient braking, perhaps causing an accident. For this reason the master cylinder is connected to a brake fluid reservoir and some designs involve retaining a small pressure in the hydraulic system so that fluid leaks are out ward which is safer than air leaking inwards.


Fig. 5.7: Hydraulic Vehicle brakes

## QUESTION 1

A reciprocating pump has a stroke of 300 mm and a bore of 440 mm . Water is to be lifted through a total height of 12 m . The pump is driven by an electric motor at $70 \mathrm{rev} / \mathrm{min}$ and delivers $0.052 \mathrm{~m}^{3} / \mathrm{s}$ of water. Determine: (i) The swept volume of piston and ideal volume flow rate, (ii) the percentage slip, and (iii) the power required to drive the pump if the overall efficiency of the system is $95 \%$.

## QUESTION 2

A single-acting reciprocating pump runs at $28 \mathrm{rev} / \mathrm{min}$. The pump has a piston of diameter 125 mm and a stroke of 300 mm . The suction pipe is 10 m in length and the diameter is 75 mm . Calculate the acceleration head at the beginning of the suction stroke.

## QUESTION 3

A fluid of specific gravity 0.8 is raised through a total height of 20 m by a single acting reciprocating pump. The bore of the pump is 150 mm and the stroke is 350 mm . The pump is driven at $36 \mathrm{rev} / \mathrm{min}$ by an electric motor. When tested, the pump delivered $13 \mathrm{~m}^{3} / \mathrm{h}$. Calculate the percentage slip and the power required from the electric motor if mechanical efficiency of the system is $93 \%$.

### 6.0 THE POWER OF A STREAM OF FLUID

### 6.1 ENERGY EQUATION

$$
\begin{align*}
& z+\frac{P}{\rho g}+\frac{v^{2}}{2 g}=\text { constant }  \tag{6.1}\\
& z_{1}+\frac{P_{1}}{\rho g}+\frac{v_{1}^{2}}{2 g}=z_{2}+\frac{P_{2}}{\rho g}+\frac{v_{2}^{2}}{2 g} \tag{6.2}
\end{align*}
$$

where z = elevation; $\mathrm{P}=$ Pressure; and $\mathrm{v}=$ average (uniform) velocity of the fluid at a point in the flow under consideration. Equation (6.2), known as Bernoulli's equation, is sometimes called the energy equation for steady ideal fluid flow along a streamline between two sections 1 and 2 .

Bermnoulli's theorem states that the total energy of all points along a steady continuous stream line of an ideal incompressible fluid flow is constant although its division between the three forms of energy may vary and it is written as Equation (6.1). The three terms on the lefthand side of Equation (6.1) have the dimension of length or head and the sum can be interpreted as the total energy of a fluid element of unit weight.

The first term z , is referred to as the potential head of the liquid. The second term $\mathrm{P} / \rho \mathrm{g}$, is referred to as the pressure head and the third term $\mathrm{v}^{2} / 2 \mathrm{~g}$, is referred to as the velocity head. The addition of the three heads is constant and it is referred to as total head H .
Total head $=$ potential head + Pressure head + Velocity head

$$
\begin{equation*}
H=z+\frac{P}{\rho g}+\frac{v^{2}}{2 g} \tag{6.3}
\end{equation*}
$$

where H is the total energy per unit weight.
Potential Head (z): Potential head is the potential energy per unit weight of fluid with respect to an arbitrary datum of the fluid. z is in $\mathrm{J} / \mathrm{N}$ or m

Pressure Head ( $\mathbf{P} / \mathbf{p g}$ ): Pressure head is the pressure energy per unit weight of fluid. It represents the work done in pushing a body of fluid by fluid pressure.
$\mathrm{P} / \mathrm{\rho g}$ is in $\mathrm{J} / \mathrm{N}$ or m .
Velocity Head ( $\mathbf{v}^{2} / \mathbf{2 g}$ ): Velocity head is the kinetic energy per unit weight of fluid in $\mathrm{J} / \mathrm{N}$ or m .
In formulating Bernoulli's equation (Equation 6.2), it has been assumed that no energy has been supplied to or taken from the fluid between points 1 and 2. Energy could have been supplied by introducing a pump; equally, energy could have been lost by doing work against friction or in a machine such as a turbine. Bernoulli's equation can be expanded to include these conditions, giving

$$
\begin{equation*}
z_{1}+\frac{P_{1}}{\rho g}+\frac{v_{1}^{2}}{2 g}=z_{2}+\frac{P_{2}}{\rho g}+\frac{v_{2}^{2}}{2 g}+h+w-q \tag{6.4}
\end{equation*}
$$

where h is the loss per unit weight; wis the work done per unit weight; q is the energy supplied per unit weight

### 6.2 TOTAL ENERGY AND POWER OF FLOWING FLUID

The total energy per unit weight H of the fluid is given by (Equation 6.3). If the volume rate of flow $(\mathrm{Q})$ is known and the density of the fluid is $\rho$, therefore weight per unit time of fluid flowing can be calculated using Equation (6.5).

Weight per unit time $=\rho g Q(\mathrm{~N} / \mathrm{s})$
Therefore, power of fluid flowing can be calculated as the product of energy per unit weight H (in m or $\mathrm{J} / \mathrm{N}$ ) and weight per unit time in $\mathrm{N} / \mathrm{s}$.

$$
\begin{align*}
\text { Power } & = & & \text { Energy per unit time } \\
& = & & (\text { weight } / \text { unit time }) \times(\text { energy/unit weight }) \\
\text { Power } & = & & \rho g Q H(\mathrm{~W} \text { or } \mathrm{kW}) \tag{6.6}
\end{align*}
$$

## QUESTION 1

Water is flowing along a pipe with a velocity of $7.2 \mathrm{~m} / \mathrm{s}$. Express this as a velocity head in metres of water. What is the corresponding pressure in $\mathrm{kN} / \mathrm{m}^{2}$ ?

## QUESTION 2

Water in a pipeline 36 m above sea level is under a pressure of $410 \mathrm{kN} / \mathrm{m}^{2}$ and the velocity of flow is $4.8 \mathrm{~m} / \mathrm{s}$. Calculate the total energy pr unit weight reckoned above sea level.

## QUESTION 3

Water flow from a reservoir into a closed tank in which the pressure is $70 \mathrm{kN} / \mathrm{m}^{2}$ below atmospheric. If the water level in the reservoir is 6 m above that in the tank, find the velocity of water entering the tank, neglecting friction.

## QUESTION 4

A pipe carrying water tapers from 160 mm diameter at A to 80 mm diameter at B . Point A is 3 m above B. The pressure in the pipe is $100 \mathrm{kN} / \mathrm{m}^{2}$ at A and $20 \mathrm{kN} / \mathrm{m}^{2}$ at B, both measured above atmosphere. The flow is $4 \mathrm{~m}^{3} / \mathrm{min}$ and is in direction A to B. Find the loss of energy, expressed as a head of water, between points $A$ and $B$.

