COURSE CODE: *MCE 205*

COURSE TITLE: *Fluid Mechanics I*

NUMBER OF UNITS: *2 Units*

COURSE DURATION: *Two hours per week*

COURSE DETAILS:

Course Coordinator: **Engr. Dr. Bukola Olalekan Bolaji** *B.Eng., M.Eng., PhD*
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Office Location: **HOD's Office, MCE Department, COLENG**
Other Lecturers: **None**

COURSE CONTENT:

Elements of fluid statics, density, pressure, surface tension, viscosity, compressibility etc. hydrostatic forces on submerged surfaces due to incompressible fluid. Static forces on surface stability of floating bodies. Introduction to fluid dynamics – conservation laws. Introduction to viscous flows. Fluid friction, friction factor and its relation to pipe losses; pipes in parallel and series. Fluid flow measurements, venturi meter.

COURSE REQUIREMENTS:

This is a compulsory course for all 200 level students in the College of Engineering, 200 level students in the Department of Water Resources Management and Agricultural Meteorology, and 300 level students in the Department of Food Science and Technology. In view of this, students are expected to participate in all the course activities and have minimum of 75% attendance to be able to write the final examination.

READING LIST:

1. Bolaji, B.O. *Introduction to Fluid Mechanics*. Nigeria: Ed., Adeksor Nig. Ent. 2008.
2. Douglas, J.F., Gasiorek and Swaffield, *Fluid Mechanics*. England: Addison Wesley Longman Ltd., 1985.
3. Fox, R.W. and McDonald, A.T. *Introduction to Fluid Mechanics*. New York: John Wiley and Sons, 1999.
4. Kreith, F. and Berger, S.A. *Mechanical Engineering Handbook*. Boca Raton: CRC Press, LLC, 1999.
5. Trefethen, L. *Surface Tension in Fluid Mechanics*: In Illustrated Experiments in Fluid Mechanics. Cambridge: The MIT Press, 1972.
6. Yaws, C.L. *Handbook of Viscosity*. Houston: Gulf, 1994.

LECTURE NOTES

1.0 INTRODUCTION

Fluid Mechanics is a branch of applied mechanics concerned mainly with the study of the behaviour of fluids either at rest or in motion.

Fluid: *A fluid is a material substance, which cannot sustain shear stress when it is at rest. In other words, a fluid is a substance, which deforms continuously under the action of shearing forces, however small they may be.*
The major differences between liquids and gases are:
Liquids are practically incompressible whereas gases are compressible
Liquids occupy definite volumes and have free surfaces whereas a given mass of gas expands until it occupies all portions of any containing vessel.

PROPERTIES OF FLUIDS

DENSITY

The density or mass density of the fluid (ρ) is defined as the *mass per unit volume*. Its unit of measurement is kg/m^3 i.e.

$$\rho = m/V. \qquad (1.1)$$

SPECIFIC VOLUME

Specific volume is defined as *volume per unit mass*. Its unit of measurement is (m^{3}kg^{-1})

$$V_s = \frac{V}{m} = \frac{1}{\rho} = \frac{RT}{P} \qquad (1.2)$$

SPECIFIC WEIGHT

The specific weight 'Y', of a fluid is its *weight per unit volume*. Unit is N/m^3.

$$Y = mg/V = \rho g \qquad (1.3)$$

RELATIVE DENSITY

The relative density RD or specific gravity of a substance is *mass of the substance to the mass of equal volume of water* at specified temperature and pressure.

$$RD = \frac{\Gamma_s}{\Gamma_w} = \frac{\rho_s}{\rho_w} = \frac{m_s}{m_w} \qquad (1.4)$$

COMPRESSIBILITY OF FLUIDS

The compressibility of any substance is measure in terms of *bulk modulus of elasticity*, K.

BULK MODULUS OF ELASTICITY

Also known as Modulus of volume expansion is defined as *the ratio of the change in pressure to the corresponding volumetric strain.*

$$K = \frac{-dP}{\frac{dV}{V}} \qquad (1.5)$$

or

$$K = \rho\left(\frac{dP}{d\rho}\right) \qquad (1.6)$$

VISCOSITY OF FLUIDS

The viscosity of a fluid is that property which determines its ability to resist shearing stress or angular deformation.

Shear stress, τ, varies with velocity gradient, du/dy.

$$\tau = \mu\left(\frac{du}{dy}\right) \qquad (1.7)$$

The Dynamic viscosity, μ is defined as the shear force per unit area required to draw one layer of fluid with unity velocity past another layer unit distance away from it in the fluid. Unit is Ns/m^2.

KINEMATIC VISCOSITY

Kinematic viscosity, ν is defined as the ratio of dynamic viscosity to mass density. Unit is m^2s^{-1}

$$\nu = \mu/\rho \qquad (1.8)$$

NEWTONIAN AND NON-NEWTONIAN FLUIDS

Ideal Fluid: For the ideal fluid, the resistance to shearing deformation is zero, and hence the plotting coincides with the x-axis.

Ideal or Elastic Solid: For the ideal or elastic solid, no deformation will occur under any loading condition, and the plotting coincides with y-axis.

Newtonian Fluids: Fluids obeying Newton's law of viscosity and for which μ has a constant value.

Non-Newtonian Fluids: These are fluids which do not obey Newton's law of viscosity.

SURFACE TENSION

The surface tension, σ, is defined as ***the force in the liquid normal to a line of unit length drawn in the surface***. Its unit of measurement is N/m.

CAPILLARITY

Another interesting consequence of surface tension is the *capillary effect*, which is the rise and fall of a liquid in a small-diameter tube inserted into the liquid.

The height of liquid rise (h) is obtained as:

$$h = \frac{4\sigma\cos\theta}{\rho g d} \qquad (1.9)$$

FLUID PRESSURE

Pressure is express as the *force per unit area.*

$$P = F/A. \quad (Nm^{-2}) \qquad (1.10)$$

Atmospheric Pressure: This is the pressure due to the atmosphere at the earth surface as measured by a barometer. Pressure decreases with altitude

Gauge Pressure: This is the intensity of pressure measured above or below the atmospheric pressure.

Absolute Pressure: This is the summation of Gauge and atmospheric pressure.

Vacuum: A perfect vacuum is a completely empty space; therefore, the pressure is zero.

2.0 FLUID STATICS

Fluid statics or hydrostatics is the study of force and pressure in a fluid at rest with no relative motion between fluid layers.
From the definition of a fluid, there will be no shearing forces acting and therefore, all forces exerted between the fluid and a solid boundary must act at *right angles* to the boundary.
If the boundary is curved, it can be considered to be composed of a series of chords on which a force acts perpendicular to the surface concerned.

TRANSMISSION OF FLUID PRESSURE

The principle of transmission of fluid pressure states that *the pressure intensity at any point in a fluid at rest is transmitted without loss to all other points in the fluid.*

PRESSURE DUE TO FLUID'S WEIGHT

Fluids of Uniform Density

Total weight of fluid (W) = mg

$$W = \rho g A h \tag{2.1}$$

Pressure (P) = Weight of fluid/Area

$$P = \rho g h \tag{2.2}$$

STRATIFIED FLUIDS

Stratified fluids are two or more fluids of different densities, which float on the top of one another without mixing together.

$P_1 = \rho_1 g h_1$ and $W_1 = \rho_1 g h_1 A$.

$P_2 = \rho_2 g h_2$ and $W_2 = \rho_2 g h_2 A$

Total pressure,

$$P_T = \rho_1 g h_1 + \rho_2 g h_2$$

Total weight,

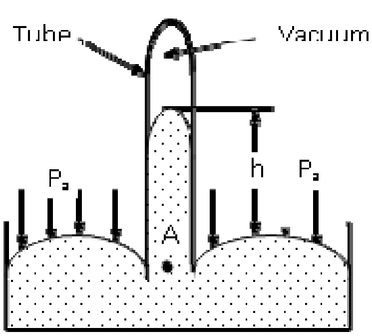
$$W_T = (\rho_1 g h_1 + \rho_2 g h_2)A$$

$$W_T = P_T A \tag{2.4}$$

PRESSURE MEASUREMENT BY MANOMETER

Measurement of Absolute Pressure

The absolute pressure of a liquid is measured by a *barometer*.



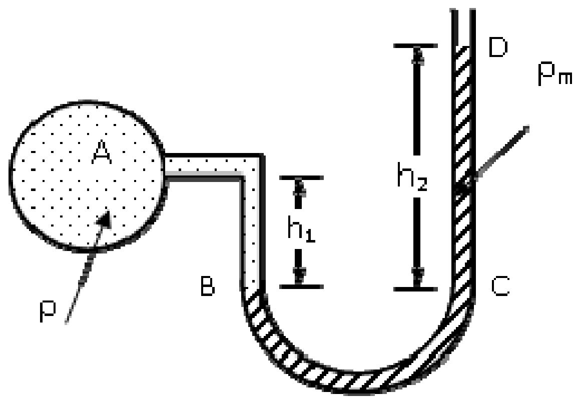
$P = \rho gh$ (2.5)

Piezometer Tube

Piezometer consists of a single vertical tube, inserted into a pipe or vessel containing liquid under pressure which rises in the tube to a height depending on the pressure. The pressure due to column of liquid of height h is:

$$P = \rho gh \tag{2.6}$$

OPEN–END U-TUBE MANOMETER



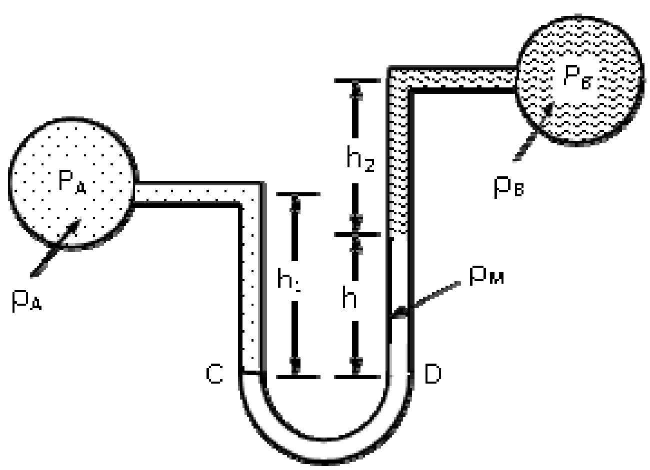
Pressure $P_B = P_A + \rho gh_1$

Pressure $P_C = 0 + \rho_m gh_2$

$P_A + \rho gh_1 = \rho_m gh_2$ (Since $P_B = P_C$)

$$P_A = \rho_m gh_2 - \rho gh_1 \tag{2.7}$$

CLOSE-END U-TUBE MANOMETER



$P_C = P_A + \rho_A gh_1$

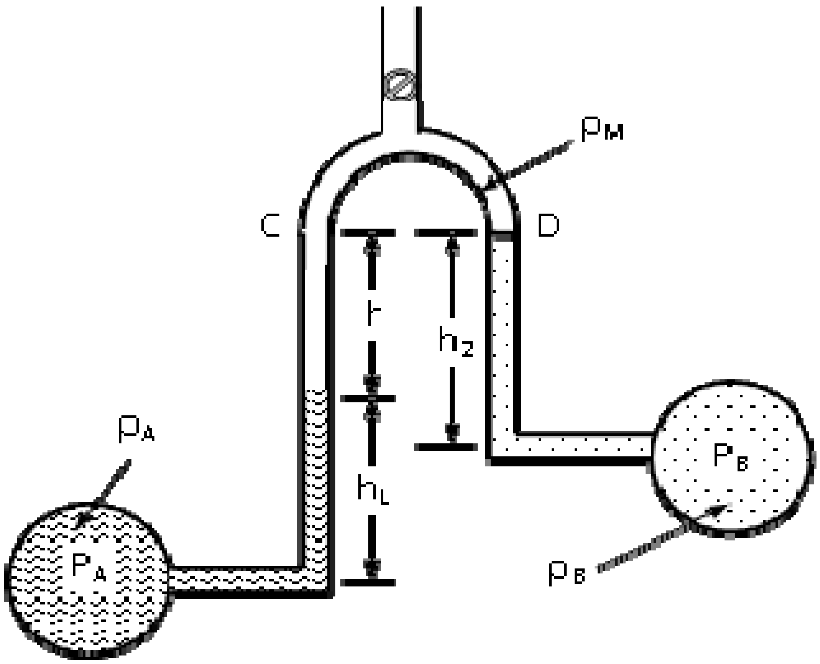
$P_D = P_B + \rho_B gh_2 + \rho_m gh$

But $P_C = P_D$, hence,

$$P_A + \rho_A gh_1 = P_B + \rho_B gh_2 + \rho_m gh$$

$P_A – P_B = P_B gh_2 + \rho_m gh – \rho_A gh_1$ (2.8)

INVERTED U-TUBE MANOMETER



$P_A = \rho_A gh_1 + \rho_m gh + P_C$
$P_B = \rho_B gh_2 + P_D$
Since $P_C = P_D$
$P_A – P_B = \rho_A gh_1 + \rho_m gh – \rho_B gh_2$ (2.9)

If the top of the tube is filled with air
$P_A – P_B = \rho_A gh_1 – \rho_B gh_2$ (2.10)

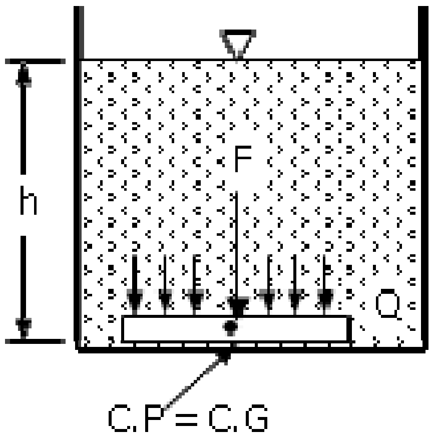
If fluids in A and B are the same
$P_A – P_B = pg(h_1 – h_2) + \rho_m gh$ (2.11)

Combining conditions for Eqs. (2.10) and (2.11):
$P_A – P_B = pg(h_1 – h_2)$ (2.12)

3.0 FORCES ON SUBMERGED SURFACES

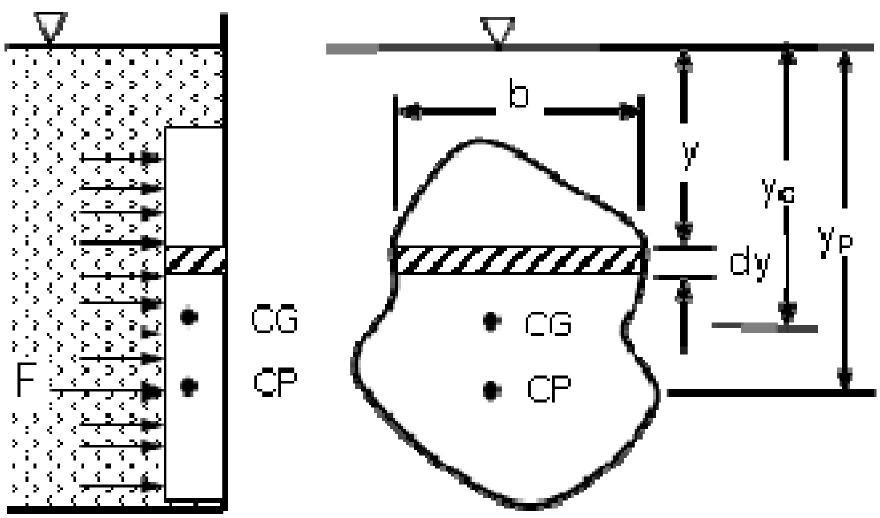
A submerged surface can be defined as a surface of a body below the liquid surface. There are two types of surfaces, namely:
Plane surface
Curved surface

SUBMERGED HORIZONTAL PLANE SURFACE



$$P = \rho gh \tag{3.1}$$
$$F = \rho ghA \tag{3.2}$$

SUBMERGED VERTICAL PLANE SURFACE



Elemental force,

$$dF = PdA$$

$\int dF = \rho g \int y dA$

But $\int y dA$ is the first moment of area about the liquid surface, hence

$F = \rho g A y_G$ (3.3)

DETERMINATION OF CENTRE OF PRESSURE (y_p)

$dF = \rho g y dA$

Taking moment about the liquid surface

$dF.y = \rho g y^2 dA$

$\int dF.y = \rho g \int y^2 dA$

But the $\int y^2 dA$ is the second moment of area I, about the surface level

$F y_p = \rho g \int y^2 dA = \rho g I$ (3.4)

$y_p = I/Ay_G$ = Ratio of Second moment of Area to First moment of Area

Using parallel axis theorem,

$I_X = I_G + Ay^2$

$I = I_G + Ay^2_G$ (3.5)

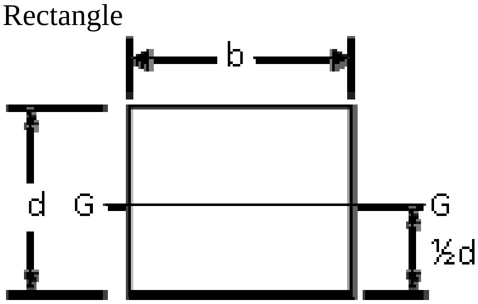
I_G is the second moment of Area about the centroid. Substituting for I, we have

$$y_p = \frac{I_G + Ay_G^2}{Ay_G}$$

$$y_p = \frac{I_G}{Ay_G} + y_G$$ (3.6)

GEOMETRIC PROPERTIES OF SOME SHAPES

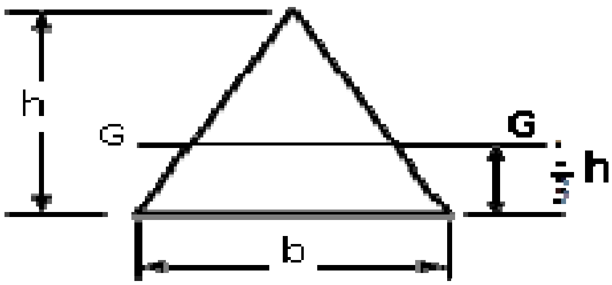
Rectangle



$A = bd$

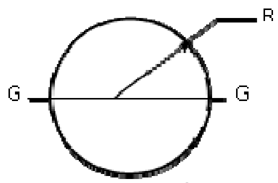
$I_G = bd^3/12$

Triangle



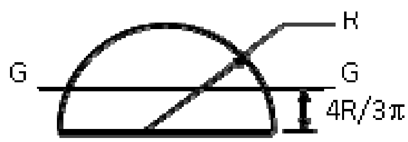
$A = \frac{1}{2} bh$
$I_G = bh^3/36$

Circle



$A = \pi R^2$ and $I_G = \pi R^4/4$

Semicircle



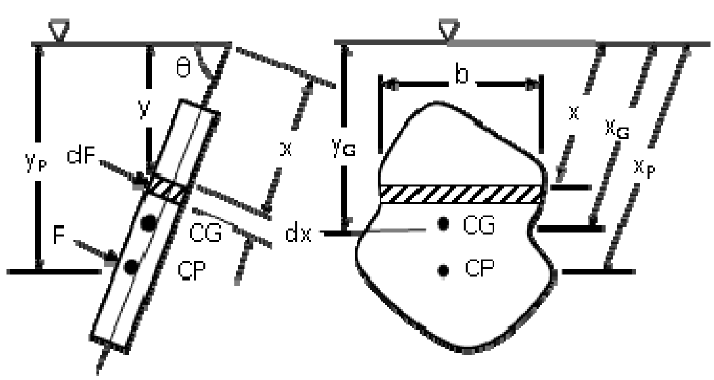
$A = \frac{1}{2} \pi R^2$
$I_G = 0.1102R^4$

QUESTION

A fuel tank 10 m wide by 5 m deep contains oil of relative density 0.7. In one vertical side a circular opening 1.8 m in diameter was made and closed by a trap door hinged at the lower end B held by a bolt at the upper end A. If the fuel level is 1.8 m above the top edge of the opening, calculate the:

- total force on the door
- force on the bolt
- force on the hinge.

SUBMERGED INCLINED PLANE SURFACE



$dF = PdA$

$P = \rho gy$ and $y = x.\sin\theta$

$P = \rho gx.\sin\theta$

$dF = \rho gx\sin\theta.dA$ (3.7)

$\int dF = \rho g.\sin\theta \int x.dA$

where $\int x.dA = Ax_G$ first moment of area.

$F = \rho g \sin\theta\, Ax_G$

$\therefore$ $F = \rho gy_G A$ (3.8)

DETERMINATION OF CENTRE OF PRESSURE

Taking moment about the fluid surface,

$dM = xdF$

$dM = \rho gx^2\sin\theta dA$

$\int dM = \rho g.\sin\theta \int x^2 dA$

$I = \int x^2 dA$ (second moment of area), hence

$M = \rho g.\sin\theta\, I.$

Also the total moment $M = Fx_P$,

therefore,

$Fx_P = \rho g.\sin\theta\, I.$

$$x_P = \frac{\rho g \sin\theta I}{F}$$

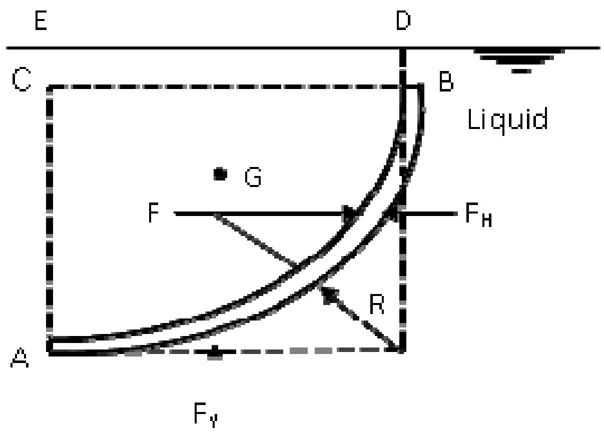
But $F = \rho gy_G A = \rho gx_G.\sin\theta A$

$$x_P = \frac{\rho g \sin\theta I}{\rho g x_G \sin\theta A} = \frac{I}{Ax_G} \quad (3.9)$$

But $I = I_G + Ax_G^2$

$$\therefore \quad x_P = \frac{I_G}{Ax_G} + x_G \quad (3.10)$$

FORCES ON A SUBMERGED CURVED SURFACE



Determine the forces acting on horizontal (F_H) and vertical (F_V) planes. These components are combined into a resultant force (R)

F_H = ρg x Area of EA x depth to centroid of EA

$$F_H = \rho g A y_G \qquad (3.11)$$

Vertical component F_V is equal to the weight of fluid which would occupy ECABD

$$F_V = \rho G v \qquad (3.12)$$

4.0 BUOYANCY AND STABILITY OF FLOATING BODIES

BUOYANCY

The **Upthrust** (upward vertical force due to the fluid) or buoyancy of an immersed body is equal to the weight of liquid displaced

The centre of gravity of the displaced liquid is called the ***centre of buoyancy.***

$R = W$

i.e. $\rho g V = mg$

and $V = mg/\rho g = m/\rho$

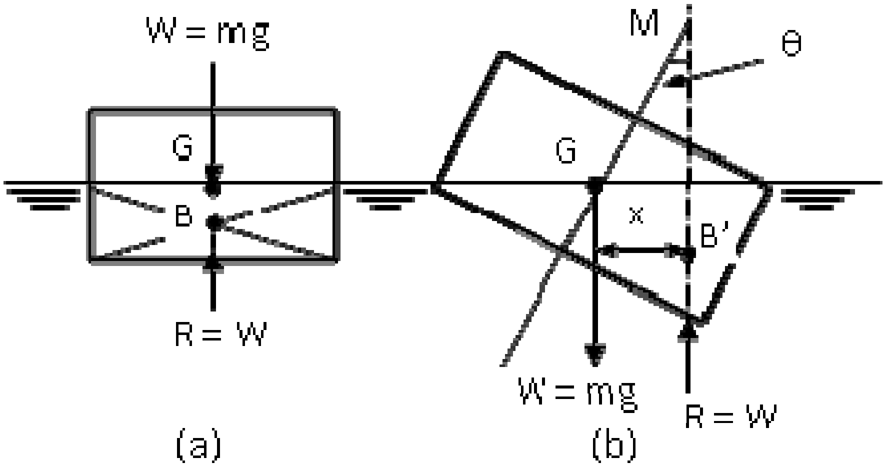
$$\text{Volume of fluid displaced} = \frac{\text{mass of the floating body}}{\text{density of the fluid}} \qquad (4.1)$$

STABILITY OF A SUBMERGED BODY

For stable equilibrium the centre of gravity of the body must lie directly below the centre of buoyancy of the displaced liquid.

If the two points coincide, the submerged body is in neutral equilibrium for all positions.

STABILITY OF FLOATING BODIES

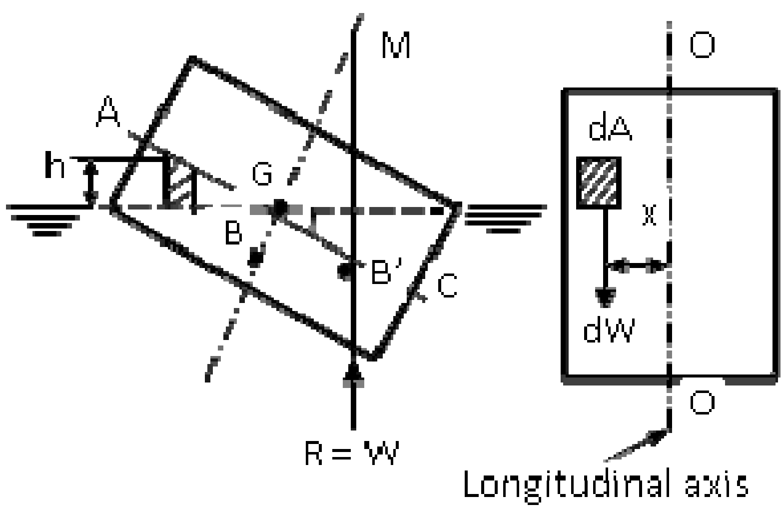


The point M is called the metacentre
Equilibrium is stable if M lies above G
Equilibrium is unstable if M lies below G
If M coincides with G, the body is in neutral equilibrium.

Metacentre: The metacentre is the point at which the line of action of upthrust (or buoyant force) for the displaced position intercept the original Vertical axis through the centre of gravity of the body.

Metacentric Height: The distance of metacentre from the centre of gravity of the body is called metacentric height.

DETERMINATION OF POSITION OF METACENTRE



Consider an elemental horizontal area dA

$$h = x.\tan\theta$$
$$dW = \rho gh.dA$$
$$dW = \rho gx \tan\theta.dA$$

Taking moment about axis OO

$$dM = x.dW$$
$$dM = \rho g.x^2\tan\theta.dA$$

Total moment,

$$M = \int dM$$
$$M = \rho g\tan\theta \int x^2 dA$$

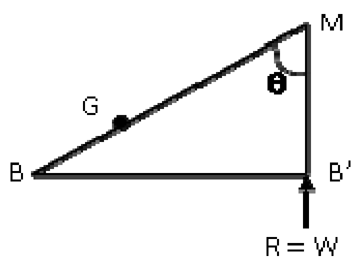
Where

$$\int x^2 dA = I = \text{second moment of area}$$

Therefore,

$$M = \rho g\tan\theta.I \qquad (4.2)$$

The Buoyance Moment



Buoyance Moment,

$$M_B = R.BB'$$

Buoyant force

$$R = \rho gV$$

but $BB' = BM.\sin\theta$,

therefore,

$$M_B = \rho gV.BM\sin\theta \qquad (4.3)$$

Equating Eqs. 4.2 and 4.3, we have

$$\rho g\tan\theta.I = \rho gV.BM\sin\theta$$

$$BM = \frac{I\tan\theta}{V\sin\theta}$$

If the angle of tilt is very small, $\sin\theta = \tan\theta$

therefore, $$BM = \frac{I}{V} \qquad (4.4)$$

The distance BM is known as the metacentric radius

But $GM = BM - BG = (I/V) - BG$ (4.5)

QUESTION 4.1

A stone weighs 400 N in air, and when immersed in water it weighs 222 N. Compute the volume of the stone and its relative density.

Hints

(i) $V = R/\rho g$

(ii) $RD = W/R$

QUESTION 4.2

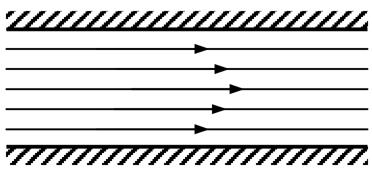
A pontoon is 6 m long, 3 m wide 3 m deep, and the total weight is 260 kN. Find the position of the metacentre for rolling in sea water. How high may the centre of gravity be raised so that the pontoon is in neutral equilibrium? (Take density of sea water to be 1025 kgm^{-3})

5.0 FLUID FLOW AND EQUATION

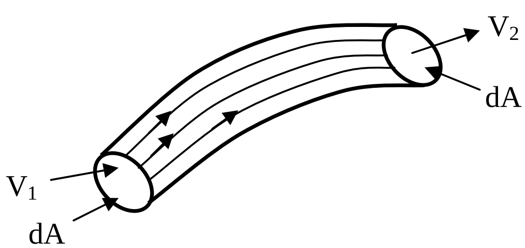
Boundary Layer: The layer of fluid in the immediate neighbourhood of an actual flow boundary that has had its velocity relative to the boundary affected by viscous shear is called the *boundary layer*.

Adiabatic Flow: Adiabatic flow is that flow of a fluid in which no heat is transferred to or from the fluid. *Reversible adiabatic* (frictionless adiabatic) flow is called isentropic flow.

Streamline: A streamline is a continuous line drawn through the fluid so that it has the direction of the velocity vector at every point.



Stream Tube: A stream tube is the tube made by all the streamlines passing through a small, closed curve.



DISCHARGE AND MEAN VELOCITY

The total quantity of fluid flowing in a unit time past any particular cross-section of a stream is called the '***discharge'*** or flow at that section. It can be measured either in terms of mass, in which case it is referred to as the ***mass flow rate*** (m) or in terms of volume, when it is referred to as the volumetric flow rate or discharge (Q).

Volumetric Flow rate or Discharge **(Q):** It is defined as the volume of fluid passing a given cross-section in unit time. It is measured in cubic metres per second, (m^3s^{-1}).

Mass Flow Rate **(m):** It is defined as the mass of fluid passing a given cross-section in unit time. It is measured in kilogrammes per second (kgs^{-1}).

Mean Velocity**:** The mean velocity at any cross-section area is the ratio of volumetric flow rate to the cross-sectional area, i.e. V = Q/A.

THE CONCEPTS OF SYSTEM AND CONTROL VOLUME

System**:** A system refers to a definite mass of material and distinguishes it from all other matter called surroundings. Therefore, *a system can be defined as a collection of matter which is separated from the surrounding matter by a boundary and can interact with the surrounding matter via the boundary*. The boundaries of a system form a closed surface. This surface may vary with time, so that it contains the same mass during changes in its condition. The system may contain an infinitesimal mass or a large finite mass and solids at the will of the investigator.

The Law of Conservation of Mass

The law of conservation of mass states that the mass within a system remains constant with time disregarding relativity effects. Therefore,

$$\frac{dm}{dt} = 0 \qquad (5.1)$$

where m is the total mass.

Conservation of Momentum

The conservation of (linear) momentum is expressed through Newton's second law of motion as:

$$\sum F = \frac{d}{dt}(mv) \qquad (5.2)$$

where m is the constant mass of the system. ΣF refers to the resultant of all external forces acting on the system, including body forces such as gravity, and 'v' is the velocity of the centre of mass of the system.

Control Volume

A control volume refers to a region in space and is useful in the analysis of situations where flow occurs into and out of the space. The boundary of a control volume is its control surface. The content of the control volume is called the system.

CONTINUITY EQUATION

The continuity equation is developed from the general principle of conservation of mass, Equation (5.1). *The continuity equation for a control volume states that the time rate of* *increase of mass within a control volume is just equal to the net rate of mass inflow to the control volume.*

Consider the flow through an infinitesimal stream tube represented in Fig. 5.2. Since the cross sectional areas at inlet and outlet are very small, it can be assumed that the flow velocity is uniform over each cross section. The density and the velocity are ρ_1 and v_1 for the inlet section and ρ_2 and v_2 for the outlet section. Consequently, the corresponding mass flow rates are $\rho_1 v_1 dA_1$ and $\rho_2 v_2 dA_2$ respectively. For conservation of mass, the rate of increase of mass in the system must be equal to the net inflow of mass into the control volume. Therefore,

$$\frac{dm}{dt} = \rho_1 v_1 dA_1 - \rho_2 v_2 dA_2 \qquad (5.3)$$

where dm/dt is the rate of increase of mass of the system due to net inflow. Since a system is one whose mass is always constant with respect to time, therefore, dm/dt is equal to zero, Equation (5.3) reduces to:

$$\rho_1 v_1 dA_1 = \rho_2 v_2 dA_2 \qquad (5.4)$$

Equation (5.4) can be extended to large cross sections by integrating the mass flow rate across each cross section:

$$\int_{A_1} \rho_1 v_1 dA_1 = \int_{A_2} \rho_2 v_2 dA_2 \qquad (5.5)$$

If ρ and v are constant over each section, the equation reduces to:

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2 \qquad (5.6)$$

For an incompressible fluid, the density is constant, ($\rho_1 = \rho_2$) and Equation (5.6) becomes

$$v_1 A_1 = v_2 A_2 = Q \qquad (5.7)$$

Therefore, the volumetric flow rate or discharge (Q) is constant from section to section as long as the same flow goes through all the sections. Equation (5.7) shows that the flow velocity (v) is inversely proportional to the flow area (A).

The control volume concept can be applied to continuity equation (Equation 5.7). For a control volume with many outlets and inlets ports, the sum of the flows through the inlet must be equal to the sum of the flows through the outlet ports.
Therefore,

$$\sum_{a=1}^{p} Q_a = \sum_{b=1}^{q} Q_b \qquad (5.8)$$

where p is the number of inlet ports; and q is the number of outlet ports.

ENERGY EQUATION FOR AN IDEAL FLUID FLOW

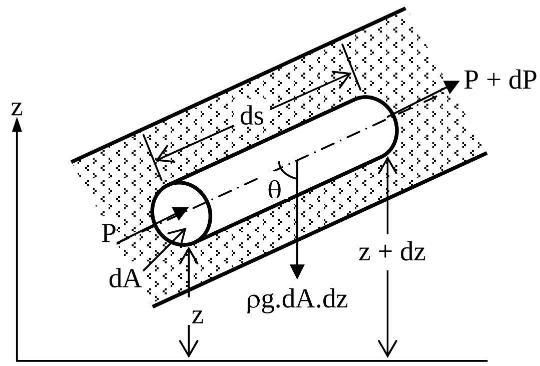
Consider an elemental stream tube in motion along a streamline (Fig. 5.4) of an ideal fluid flow. The forces responsible for its motion are the pressure forces, gravity and accelerating force, due to change in velocity along the streamline. All frictional forces are assumed to be zero and the flow is irrotational i.e. Uniform velocity distribution across streamlines.

Since Force = Mass x Acceleration or Pressure x Area

Therefore, $PdA - (P + dP)dA - \rho g dA ds \cos\theta = \rho dA ds \left(\frac{dv}{dt}\right)$

or $$-dP - \rho g ds \cos\theta = \rho ds\left(\frac{dv}{dt}\right)$$

The tangential acceleration (along streamline) for steady flow,



$$\frac{dv}{dt} = v\left(\frac{dv}{ds}\right) \quad \text{since } \frac{dv}{dt} = \frac{ds}{dt} \times \frac{dv}{ds}$$

and $$\cos\theta = \frac{dz}{ds}$$

Therefore, $-dP - \rho g dz = \rho v dv$

divide through by ρg to obtain Equation (5.9):

$$dz + \frac{dP}{\rho g} + \frac{dv^2}{2g} = 0$$

since ($dv^2 = 2vdv$)

or $$dz + \frac{dP}{\rho g} + \frac{d(v)^2}{2g} = 0 \tag{5.9}$$

Equation (5.9) is the ***Euler equation*** of motion applicable to steady state, irrotational flow of an ideal incompressible fluid.

On integration along the streamline, we get:

$$z + \frac{P}{\rho g} + \frac{v^2}{2g} = constant \tag{5.10}$$

or $$z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g} \tag{5.11}$$

where z = elevation; P = Pressure; and v = average (uniform) velocity of the fluid at a point in the flow under consideration. Equation (5.11), known as *Bernoulli's equation*, is sometimes called the energy equation for steady ideal fluid flow along a streamline between two sections 1 and 2.

Bermnoulli's theorem states that the total energy of all points along a steady continuous stream line of an ideal incompressible fluid flow is constant although its division between the three forms of energy may vary and it is written as Equation (5.10). The three terms on the left-hand side of Equation 5.10 have the dimension of length or head and the sum can be interpreted as the total energy of a fluid element of unit weight.

The first term z, is referred to as the *potential head* of the liquid. The second term P/ρg, is referred to as the *pressure head* and the third term $v^2/2g$, is referred to as the *velocity head*. The addition of the three heads is constant and it is referred to as total head H.

Total head = potential head + Pressure head + Velocity head

$$H = z + \frac{P}{\rho g} + \frac{v^2}{2g} \qquad (5.12)$$

where H is the total energy per unit weight.

Potential Head (z): Potential head is the potential energy per unit weight of fluid with respect to an arbitrary datum of the fluid. z is in JN^{-1} or m

Pressure Head (P/ρg): Pressure head is the pressure energy per unit weight of fluid. It represents the work done in pushing a body of fluid by fluid pressure.
$P/\rho g$ is in JN^{-1} or m.

Velocity Head (v^2/2g): Velocity head is the kinetic energy per unit weight of fluid in JN^{-1} or m.

In formulating Bernoulli's equation (Equation 5.11), it has been assumed that no energy has been supplied to or taken from the fluid between points 1 and 2. Energy could have been supplied by introducing a pump; equally, energy could have been lost by doing work against friction or in a machine such as a turbine. Bernoulli's equation can be expanded to include these conditions, giving

$$z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h + w - q \qquad (5.13)$$

where h is the loss per unit weight; w is the work done per unit weight; q is the energy supplied per unit weight

THE POWER OF A STREAM OF FLUID

The total energy per unit weight H of the fluid is given by (Equation 5.12). If the volume rate of flow (Q) is known and the density of the fluid is ρ, therefore weight per unit time of fluid flowing can be calculated using Equation (5.14).

Weight per unit time = ρgQ (Ns^{-1}) (5.14)

Therefore, power of fluid flowing can be calculated as the product of energy per unit weight H (in m or JN^{-1}) and weight per unit time in $N\,s^{-1}$.

Power = Energy per unit time
= (weight/unit time) x (energy/unit weight)
Power = ρgQH (W or kW) (5.15)

QUESTION 5.1

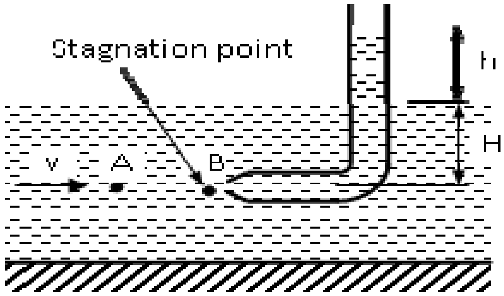
A siphon has a uniform circular bore of 75 mm diameter and consists of a bent pipe with its crest 1.8 m above water level discharging into the atmosphere at a level 3.6 m below water level. Find the velocity of flow, the discharge and the absolute pressure at crest level if the atmospheric pressure is equivalent to 10 m of water. Neglect losses due to friction.

QUESTION 5.2

A pipe carrying water tapers from 160 mm diameter at A to 80 mm diameter at B. Point A is 3 m above B. The pressure in the pipe is 100 kN/ at A and 20 kN/m^2 at B, both measured above atmosphere. The flow is 4 m^3/min and is in direction A to B. Find the loss of energy, expressed as a head of water, between points A and B.

6.0 FLOW MEASURING DEVICES

PITOT TUBE



$+ v^2/2g = H + h$

$\therefore \quad v^2/2g = h$

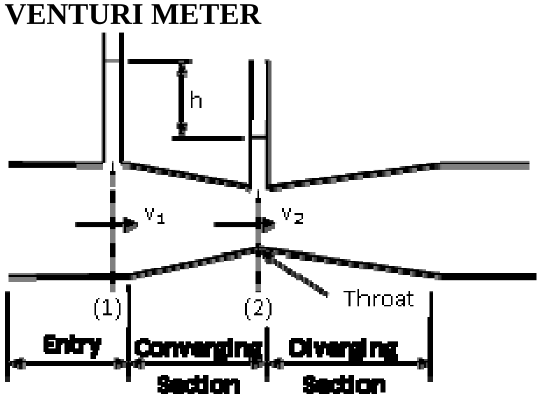
or $\quad v = \sqrt{2gh}$

PITOT-STATIC TUBE

Pitot tubes may be used in the following area:

- they can be used to measure the velocity of liquid in an open channel or in a pipe.
- they can be used to measure gas velocity if the velocity is sufficiently low so that the density may be considered constant.
- they can also be used to determine the velocities of aircraft and ships.

VENTURI METER



$A_1 v_1 = A_2 v_2$

or $\quad v_2 = (A_1/A_2) v_1$

$$z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

$z_1 = z_2$ (Horizontal)

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

Let Pressure difference

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = h$$

$$h = \frac{v_1^2}{2g}\left(\frac{A_1}{A_2}\right)^2 - \frac{v_1^2}{2g}$$

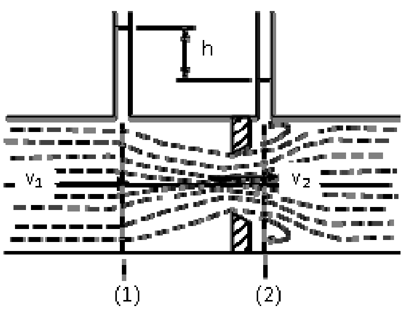
hence

$$v_1 = \left(\frac{A_2}{\sqrt{A_1^2 - A_2^2}}\right)\sqrt{2gh}$$

and

$$Q = A_1 v_1 = \left(\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\right)\sqrt{2gh}$$

ORIFICE METER



In an orifice meter, a pressure differential is created along the flow by providing a sudden constriction in the pipeline.
The principles of operation is the same with that of Venturi meter, except that it has lower coefficient of discharge due the sudden contraction.