MCE 403: HEAT TRANSFER (3 UNITS)

Theory of steady state heat conduction, convection and radiation. Dimensional analysis and similitude in heat transfer theory. Analogy between mass and momentum transfer, Sunday layer flows relations use in convection heat transfer calculations. Materials and design of heat exchange. Introduction to mass transfer, analogy between heat and mass transfer.

Introduction:

Heat transfer (or heat) is energy in transit due to temperature difference.

There are three modes of heat transfer.

When a temperature gradient exists in a stationary medium, which may be a solid or a fluid, we use the term conduct ion to refer to the heat transfer that will occur across the medium.

Conversion refers to heat transfer that will occur between a surface and a moving fluid when they are at different temperatures.

Thermal radiation is energy omitted by matter that is at a fruit temperature.

- To update heat exchange
- Radiation heat transfer
- Conversion heat transfer
- Conduction heat transfer
- Mass transfer

Relevance of Heat Transfer

- Indeed a relevant subject in many industrial and environmental problems.
- In energy production and conversion; i.e. in the generation of electrical power whether through nuclear fission or fusion, the combustion of fossil fuels, magneto hydrodynamic process, or the use of geothermal, energy sources, there are numerous heat transfer problems that must be solved.

- Development of solar energy conversion systems for space heating, a, well as for electric power production.
- In propulsion system such as internal combustion, gas turbine, and rocket engines.
- Designs of convectional spouse and water heading system, in the design of incinerator and cryogenic storage equipment, in the cooking of electronic equipment, in the design of refrigeration and air conditioning systems, and in many manufacturing processes.
- Also relevant to air and water pollution and strongly influences local and global climate.

Theory of heat transfer by conduction.

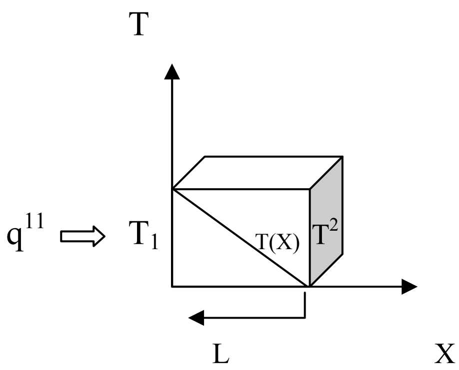
All matters consist of roleenless that are in random translational motion. Higher temperature, are associated with higher molecular energies, and when neighbouring molecules collide, as they are constantly doing, a transit, of energy from the more energetic to the less energetic molecules must occur. Energy transfer by conduction occurs in the direction of decreasing temperature.

The amount of decreasing temperature.

The amount of energy transfer by indirection between two surfaces ban be determined by Fourier's law. For one-dimensional plane will show below, the heat flute $q^{11}\pi\ 1^3$ given as.

$$q_n^1 = -K\frac{dT}{dx}$$

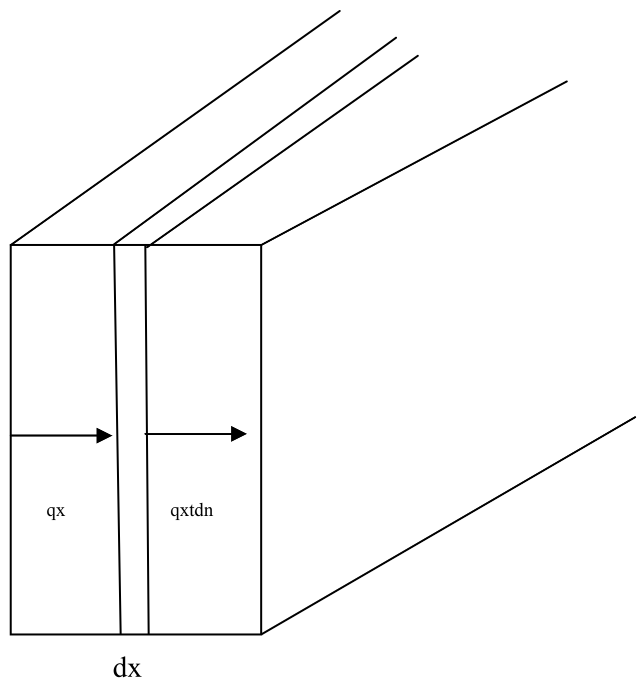
$$q = -KA\frac{dT}{dx}$$



Where q_n^1 is the heat transfer rate in the π direction per unit area perpendicular to the direction of transfer, and is proportional to the temperature gradient $\partial T/\partial\pi$ in this direction. The proportionality constant k (w/m.k) is a transport property known as the thermal conductivity. Thermal conductivity is a characteristic of the wall material.

Example

The wall of an industrial furnace is constructed firm 0.15.m thick fireclay brick hang a thermal conductivity of 1.7w/m.k measurements made during steady-state operation reveal temperatures of 1,400 and 1150k at the inner and outer surfaces respectively. What is the rate of heat loss through a wall treat is 0.5m by 3.0m on a side?



Consider the one-dimensional system shown above, the unsteady energy balance may be written as follows:

Energy conducted in left face + heat generated within element = change in internal energy + energy conducted out right fork.

The energy quantities are given as follows:

Energy in left face = qx $= -KA \frac{\partial T}{\partial x}$

Energy generated within element = $q\ A\ dx$

Change generated in internal energy = $PCA\frac{\partial T}{\partial T}dx$

Energy out right face = $qx + dx = -KA\frac{\partial T}{d\pi}$

$$= -A\left\{N\frac{\partial T}{\partial \pi} + \frac{\partial d}{d\pi}\left(K\frac{\partial y}{\partial x}\right)\partial x\right.$$

where q = energy generated per unit volume

c = specific heat of material

p = density

Combining the relations above gives

$$-KA + \frac{\partial y}{\partial x} + qA\partial\pi = PCA\frac{\partial T}{\partial \pi}\partial x - A\left\{K\frac{\partial T}{\partial \pi} + \frac{\partial T}{\partial \pi}\left(K\frac{\partial T}{\partial x}\right)\partial\right.$$

or $\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + q = PC\frac{\partial T}{\partial x}$

This is the one-dimensional heat conduction equation.

The energy balance in 3 – dimensional heat

Conduction is

$q_x + q_y + q_z + q_{gln} = q_{ntd\pi} + q_{yTdy} + q_{z\,td7} + \frac{\partial E}{\partial t}$

This result into

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial T}\left(k\frac{\partial T}{\partial z}\right) + q = c\frac{\partial T}{\partial T}$$

For constant k

$$\Longrightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{1}{\dot{a}}\frac{\partial T}{\partial x}$$

Where x = $\frac{k}{pc}$ called thermal diffusivity

This equation in cylindrical coordinates:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{dT}{dr} + \frac{1}{\partial^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial t^2} + \frac{q}{k} = \frac{1}{\partial}\frac{\partial T}{\partial t}$$

Spherical coordinate

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2 smt}\frac{\partial}{\partial \theta}\left(smt\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 sm^2 t}\frac{\partial^2 T}{\partial \phi^2} + \frac{q}{k} = \frac{1}{\partial}\frac{\partial T}{\partial T}$$

Steady state one dimensional heat flow in cartesion coordinate (no heat generation)

$$\frac{\partial^2 T}{\partial x^2} = 0$$

With heat same

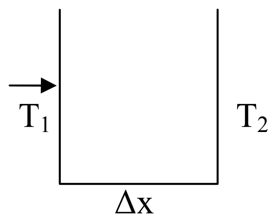
$$\frac{\partial^2 T}{\partial x^2} + \frac{q}{k} = 0$$

Two – D steady state no heat gen

$$\frac{\partial^2 T}{\partial x^2} + \frac{y^2 T}{y^2} = 0$$

Applications of founer's law of heat conduction in $\wedge - D$ systems.

The plane wall



q = - KA $\frac{dT}{dr}$ for constant k

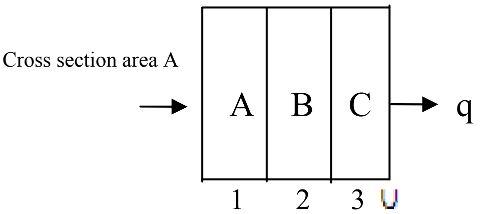
$$= -\frac{KA}{\Delta x}(T_2 - T_1)$$

If $K = K(T) = K_0 (1 + \beta T)$

$$\longrightarrow \quad q = -AK_0(1 + \beta T)\frac{dT}{dx}$$

$$q = \frac{AK_0}{Dx}\left\{ (T_2 - T_1) + \frac{\beta}{2}(T_2^{2} - T_1^{\,2}) \right\}$$

Composite Wall

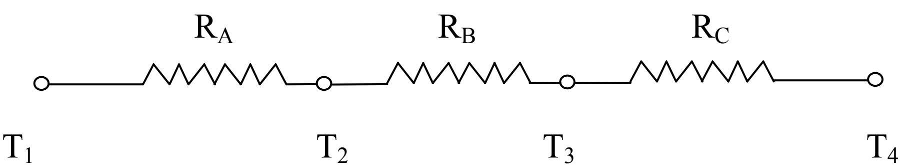


$$q = K_A A\,\frac{T_1 - T_2}{\Delta X_A} = -\,K_B A\,\frac{T_3 - T_2}{\Delta X_B} = -K_C A\,\frac{T_4 - T_3}{\Delta X_C}$$

Heat flow is the same through all sections solving their three equations simultaneously

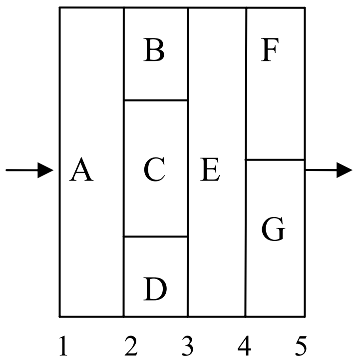
$$q = \frac{T_1 - T_4}{DX_A}$$

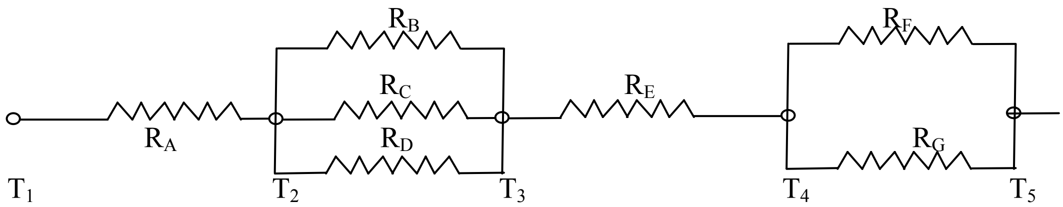
$$\frac{\Delta X_A}{K_A A} \mid \frac{\Delta X_0}{K_B A} \mid \frac{\Delta X_C}{K_C A}$$



$$\frac{\Delta X_A}{K_A A} \qquad \frac{\Delta X_0}{K_B A} \qquad \frac{\Delta X_C}{K_C A}$$

Series and parallel one-dimensions 1 heat transfer through a composite wall



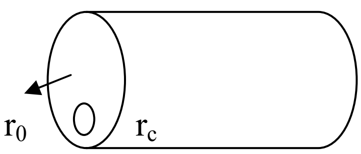


The one-dimensional heat flow equation for this type or problem may be written

$$q = \frac{\Delta T_{overall}}{\Sigma R_{EL}}$$

Where R_{EL} are the thermal resistances of the various materials.

Radial Systems – Cylinders



temperature difference

$= T_t - T_0$

$Ar = 2\pi rl$

Fourier's low : $q_r = - KA_r \frac{dT}{dr}$

$$qr = -\ 2krl\,\frac{dT}{dr}$$

with the boundary conditions

$T = \quad T_0$ at $r = r_0$

$T = \quad T_i$ at $r = r_i$

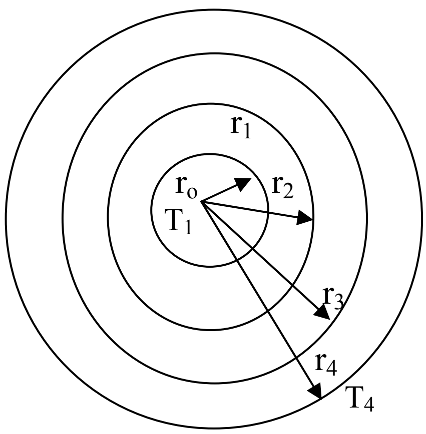
Solving: $q\ \dfrac{2\pi KL(T_1 - T_0)}{\ln(r_0/r_1)}$

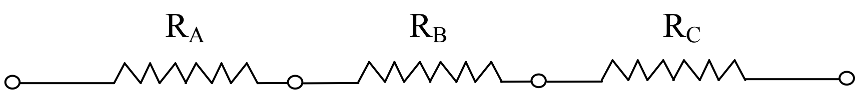
There thermol resistance in this O.K. is

$R_{EL} = \quad \dfrac{\ln(r_0/r_1)}{2\pi KL}$

The thermal resistance for three – layer system is

$$q \ = \ \frac{2\pi L(T_1 - T_4)}{\ln(r_2/r_1)/K_A + \ln(r_3/r_2)/K_B + \ln(r_4/r_3)/K_c}$$





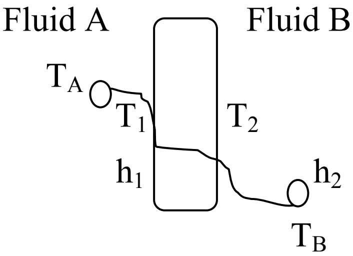
$$\frac{\ln(r_2/r_1)}{2\pi K_A L} \qquad \frac{\ln(r_3/r_2)}{2\pi K_B L} \qquad \frac{\ln(r_4/r_3)}{2\pi K_C L}$$

Convection boundary conditions

$$q_{cmv} = hA\,(T_w - T_w)$$

$$= \frac{(T_w - T_w)}{\gamma hA}$$

Overall heat-transfer coefficient



$$q = h_1A\,(T_A - T_1) = \frac{KA}{\Delta x}(T_1 - T_2) = h_2\,A\,(T_2 - T_B)$$

$$q = \frac{T_A - T_B}{\gamma h_1 A + \frac{\Delta x}{KA} + \gamma h_2 A}$$

V_hA is used to represent the convection resistance.

The overall heat transfer by combined conduction and convection is expressed in terms of an overall heat transfer coefficient U.

- $q = UA\,U\,A\Delta T_{overall}$

where $U = \dfrac{1}{\frac{1}{h_1} + \frac{\Delta x}{K} + \frac{1}{h_2}}$

Plane Wall with heat sources

$$\frac{d^2T}{dx^2} + \frac{q}{K} = 0$$

With b.c. $T = T_w$ at $x = + L$

Solving

T= $\frac{qx^2}{2K} + C_1 + C_2$

Since temperature must be the same on each side of the wall $\Longrightarrow$ $C_1 = 0$

$C_2 = T_0$

$T - T_0 = \frac{-q}{2K} x^2$

n + x = - L

$T_w - T_0 = \frac{-q}{2K} L^2$

$$\frac{T - T_0}{T_w - T_0} = \left(\frac{x}{L}\right)^2$$

Total heat generated = 2 $\left(-KA\frac{dT}{dx} / x = L\right)$

$= \in A2L$

Temperature gradient at the wall

$\frac{dT}{dx} /$ x = L = $(T_w - T_0)$ $\left(\frac{2x}{L^2}\right) / \Longrightarrow$ L

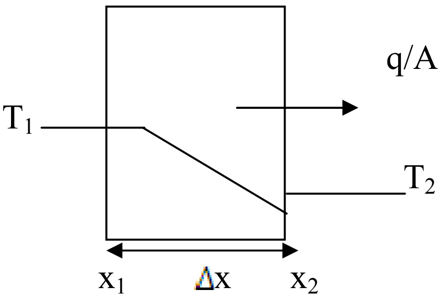
= $(T_w - T_0)$ $\frac{2}{L}$

Then = - K $(T_w - T_0)$ $\frac{2}{L}$ = $\in$L

and T_0 = $\frac{qL^2}{2k} + T_w$

Plane Wall : Fixed surface Temperatures.

Consider a one-dimensional, steady-state heat conduction problem in a plane wall of homogeneous materials having constant thermal conductivity and each face is held at a constant uniform temperature as shown in the figure below:



Starting from the fourier's equation

$$q = -KA\frac{dT}{dx}$$

Separating the variables and integrating the resulting expression given

$$q\int_{x_1}^{x_2} dx = -KA\int_{T_1}^{T_2} dT$$

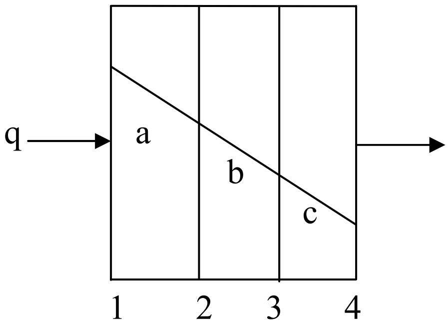
or $$q = -KA\frac{T_2-T_1}{x_2-x_1} = -KA\,\frac{T_2-T_1}{\Delta x}$$

This equation can be rearranged as

$$q = \frac{T_1-T_2}{\Delta x/KA} = \frac{\text{thermal potential difference}}{\text{thermal resistance}}$$

Note that the resistance to the heat flow is directly proportional to the material thickness, inversely proportions to the material thermal conductivity, and inversely proportional to the area normal to the direction of heat transfer.

These principles are readily extended to the case of a composite plane wall as shown in the figure below:



$$\frac{\Delta X_a}{K_a A} \qquad \frac{\Delta X_b}{K_b A} \qquad \frac{\Delta X_c}{K_c A}$$

T_1 T_2 T_3 T_4

In the steady state the heat transfer rate entering the left face is the same as that leaving the right face. Thus:

$$q = \frac{T_1 - T_2}{\Delta x_a / K_a A} \text{ and } q = \frac{T_2 - T_3}{\Delta x_b / K_b A} \qquad q = \frac{T_1 - T_2}{\Delta x_c / K_c A}$$

together these gives

$$q = \frac{T_1 - T_4}{\frac{\Delta x_a}{K_a A} + \frac{\Delta x_b}{K_b A} + \frac{\Delta x_c}{K_c A}}$$

$$\text{i.e. } q = \frac{\text{overall temperature difference}}{\text{summatium of thermal resistances}}$$

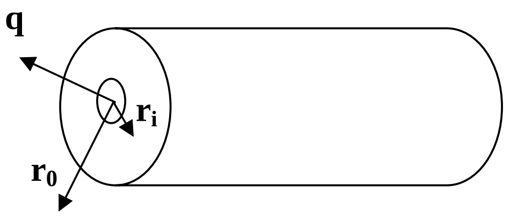
thermal resistence 2k $= \frac{\pi}{2} \; \frac{\Delta X_i}{K_i A}$

$i = 1$

Where n is the number of different wall layers. Note that this is valid only if the effects of connection on the heat transfer on the external walls are neglected.

Radial System – cylinders

Consider a long cylinder of inside radius r_c, outside radius r_0, and length L, such as the one shown below. We expose the cylinder to a temperature differential $T_i - T_0$ and assumed that the heat flows in a radial direction so that the only space coordinate needed to specify the system is r



Fourier's law for system in cylindrical coordinate becomes:

$qr = -KA_r \frac{dT}{dr}$

$Ar = 2\pi rl$

$qr = 2\pi krl \frac{dT}{dr}$

with the boundery conditions

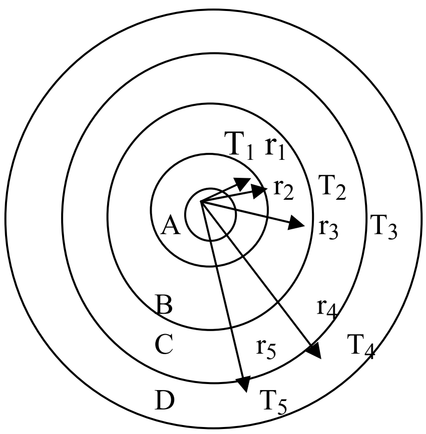
$T = T_i$ at $r = r_i$

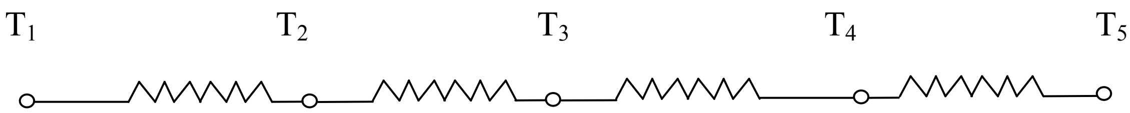
$T = T_0$ at $r = r_0$

$$q = \frac{2\pi KL\,(T_1 - T_0)}{\gamma n\,(r_0 - r_i)} = \frac{(T_1 - T_0)}{\gamma n\,(r_0 - r_i)/2\pi KL}$$

where the thermal resistance

$$R = \frac{\gamma n\,(r_0 - r_i)}{2\pi KL}$$





$$\frac{\gamma n\,(r_2/r_1)}{2\pi K_A L} \qquad \frac{\gamma n\,(r_3/r_2)}{2\pi K_B L} \qquad \frac{\gamma n\,(r_4/r_3)}{2\pi K_C L} \qquad \frac{\gamma n\,(r_5/r_4)}{2\pi K_D L}$$

For multiple-layer cylindrical walls, i.e. four layers as shown above, the heat transfer rate is:

$$q = \frac{T_5 - T_1}{\sum R}$$

$$\overline{Z}\,R = \frac{\gamma n\,(r_2/r_1)}{2\pi K_A L} \qquad \frac{\gamma n\,(r_3/r_2)}{2\pi K_B L} \qquad \frac{\gamma n\,(r_4/r_3)}{2\pi K_C L} \qquad \frac{\gamma n\,(r_5/r_4)}{2\pi K_D L}$$

Spherical systems may be treated as one-dimensional when the temperature is a function of radius only for s3eal system the heat transfer rate q is given as

$$q = \frac{4\pi K\,(T_1 - T_0)}{\frac{1}{r_i} - \frac{1}{r_0}}$$

convection boundery conditions

convection heat transfer can be calculate from

$q = hA(T_w - T_0)$

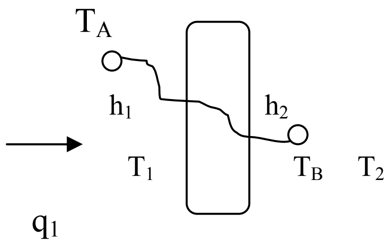
$$= \frac{T_w - T_0}{1/hA}$$

The overall heat-transfer coefficient.

Consider the plane wall shown below exposed to a hot fluid A on one side and a cooker fluid B on the other side.

The heat transfer is expressed by

$$q = h_1A(T_A - T_1) = \frac{KA}{\Delta x}(T_1 - T_2) = h_2A\ (T_1 - T_2)$$



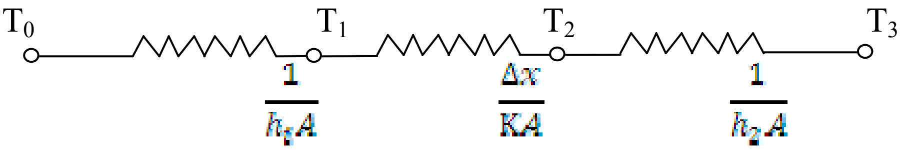
The overall heat transfer is calculated as the ratio of the overall temperature difference to the sum of the thermal resistance:

$$q = \frac{T_A - T_B}{\frac{1}{h_1A} + \frac{\Delta x}{KA} + \frac{1}{h_2A}}$$

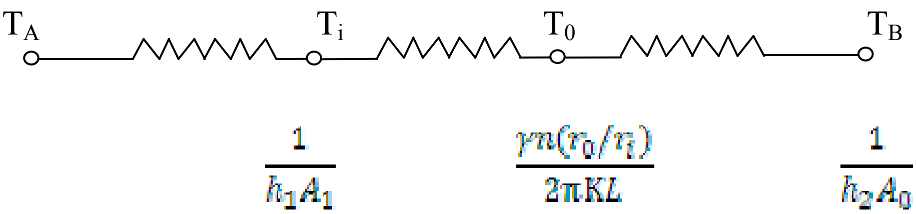
note that 1/hA is used to represent the conversion resistance. The overall heat transfer by combined conduction and convection is frequently expressed in terms of an overall heat-transfer coefficient U, defined by the relation:

$q = UA\Delta T_{overall}$

Where $U = \dfrac{1}{\frac{1}{h_i} + \frac{\Delta x}{K} + \frac{1}{h_2}}$



For a hollow cylinder exposed to a convection environment on its inner and outer surfaces, the electrical resistance analogy would appear as shown in the figure below:



Where T_A and T_B are the two fluid temperatures note that the area of convection is not the same for both fluids in this case. These areas depend on the inside tube diameter and wall thickness. In this case the overall heat transfer would be expressed by:

$$q = \frac{T_A - T_B}{\frac{1}{h_i A_i} + \frac{\ln(r_0/r_i)}{2\pi KL} + \frac{1}{h_0 A_0}}$$

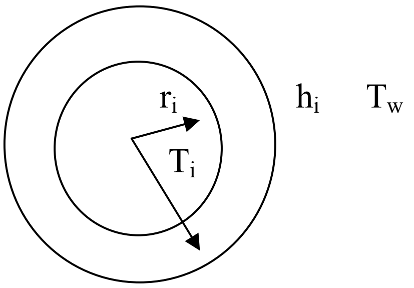
The terms A_i and A_0 represent the inside and outside surface areas of the inner tube. The overall heat transfer co-efficient may be based on either the inside or the outside area of the tube

$$\Longrightarrow U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln(r_0/r_i)}{2\pi KL} + \frac{A_i}{A_0}\frac{1}{h_0}}$$

$$\Longrightarrow U_0 = \frac{1}{\frac{A_0}{A_i}\frac{1}{h_i} + \frac{A_0 \ln(r_0/r_i)}{2\pi KL} + \frac{1}{h_0}}$$

Critical thickness of insulation

Consider a layer of insulation which might be installed around a circular pipe as shown below :



The inner temperature of the insulation is fixed at Ti and the outer surface is exposed to a convection environment at T_w. The heat transfer is

$$q = \frac{2\pi KL\,(T_i - T_w)}{\frac{\ln(r_0/r_i)}{K} + \frac{1}{r_0 h}}$$

The radius of the outer insulation for maximum heat transfer is obtained by interating the above expression with respect to r_0

$$\longrightarrow \quad \frac{dq}{dr} = \frac{2\pi L\,(T_i - T_w)\left(\frac{1}{Kr_0} - \frac{1}{hr_0^2}\right)}{\left(\frac{\ln(r_0/r_i)}{K} + \frac{1}{r_0 h}\right)^2}$$

$$r_0 = \frac{K}{h}$$

These radious the critical radius – of – insulation. If the actual outer radius is less than rentical. Then the heat transfer will be increased by adding more insulation and vis vesa.

Examples:

An exterior wall of a house may be approximated by a 4 – in layer of common brick of thermal conductivity k = 0.7W/M.^{0}C followed by a 1.5 in layer of gypsum plaster of thermal conductivity k = 0.48 W/M.^{0}C. what thickness of loosely packed rock-wool insulation k = 0.065 W/M.^{0}C should be added to reduce the heat loss or gain through the wall by 80 percent?

Solution:

The overall heat loss will be given by

$$q = \frac{\Delta T}{\Sigma\, REL}$$

Because the heat loss with the rock-wool insulation will be only 20% (80% reduction) of that before insulation.

$$\frac{q \text{ with insulation}}{q \text{ without insulation}} = 0.2 = \frac{\overline{Z}\, REL \text{ without insulation}}{\overline{Z}\, REL \text{ with insulation}}$$

for the brick and plaster of unit area:

$$R_b = \frac{\Delta x}{K} = \frac{4(0.0254)}{0.7} = 0.145 m^2\, {}^0C/W$$

$$R_p = \frac{\Delta x}{K} = \frac{1.5(0.0254)}{0.48} = 0.079 m^2\, {}^0C/W$$

The thermal resistance without insulation is

$$R = 0.145 + 0.079$$

$$= 0.224 m^2\, {}^0C/W$$

Then R with insulation $= \frac{0.224}{0.2}$

$$= \quad 1.122\ M^2\ {}^0C/W$$

This represents the sum of our previous value and the resistance for the rock wool.

$$1.122 = 0.224 + R_{rw}$$

$$Rrw = 0.898 = \frac{\Delta x}{K} = \frac{\Delta x}{0.065}$$

$$\Delta x_{rw} = 0.0584\ m$$

Example 2

A thick-walked tube of stainless steel of thermal conductivity K = 19 W/M.^{0}C with 2.cm internal diameter and 4cm outer diameter is covered with a 3-cm layer of asbestos insulation of thermal conductivity K = 0.2 W/M.^{0}C. If the inside wall temperature of the pipe is maintained at 600^{0}C and the outside of the insulation at 100^{0}C, calculate the heat loss per meter of length.

Solution:

Heat flow is given by:

$$\frac{q}{L} = \frac{2\pi(T_1 - T_2)}{\gamma\pi(r_2 - r_1)/kg + \gamma n(r_3 - r_2)/k_a}$$

$$= \frac{2\pi(600 - 100)}{\gamma\pi(2)/19 + \gamma n(\frac{5}{2})/0.2}$$

$$= \quad 680w/m$$

Examples 3

Calculate the critical radius of insulation for asbestos of thermal conductivity k = 0.17 w/m.^{0}C surrounding a pipe and exposed to room air with h = 3.0w/m^2 ^{0}C. Calculate the heat loss from a 200^{0}C. 5.0cm diameter pipe when covered with the critical radius of insulation and without insulation.

Solution:

$$r_{critical} = \frac{k}{h} = \frac{0.17}{3.0}$$

= 0.0567m = 5.67cm

The inside radius of the insulation is 5/2 = 2.5cm

$$\longrightarrow \quad \frac{q}{L} = \frac{2\pi(200-20)}{\ln\frac{(5.67/2.5)}{0.17} + \frac{1}{0.0567/(3.0)}} = 105.7w/m$$

Without insulation, the convection from the outer surface of the pipe is

$$\frac{q}{L} = 2\pi rk\,(T_1 - T_0)$$

$= 2\pi 0.025.3\,(200 - 20)$

= 84.8w/m

So, the addition of 3.17cm (5.67 – 2.5) of insulation actually increase the heat transfer by 25%.

<u>Heat – source systems.</u>

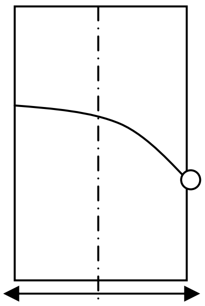
In a number of engineering applications, heat transfer is accompanied with internal heat generation. Examples include application in nuclear reactors, electrical conductors and chemically reacting systems.

Plane Wall.

Consider the plane wall with uniform internal conversion of energy. Assuming constant thermal conductivity and very large dimensions in the y – and t – direction so that the temperature gradient is significant in the x – direction only, the passion equation reduces to

$$\frac{d^2T}{dx^2} + \frac{q^{11}}{k} = 0$$

Which is a second order ordinary differential equation. Two boundary conditions are sufficient in determination of the specific solution for T(x). These are T= T_1 at x = 0 and T = T_2 ad x = 2L.



Integratory equation (1) wrt x yields

$$T \quad = \quad \frac{q^{111}}{2k} x^2 + C_1 x + C_2$$

Using the boundary conditions

$$C_2 = T_1 \; T_1 = \frac{T_2 - T_1}{2L} + \frac{q^{111}}{k}$$

$$:. \; T = \left(\frac{T_2 \qquad T_1}{2L} + \frac{q^{111}}{2k} (2L - x)\right) x + T_1 \qquad \text{……………. (2)}$$

For simple case where $T_1 = T_2 = T_0$

$$\Longrightarrow \quad C_2 = T_s \text{ and } C_1 = \frac{q^{111}L}{k}$$

$$\therefore \; T = T_s + \frac{q^{111}}{2k}(2L - x)x \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Differentiating equ. (3) yields

$$\frac{dT}{dx} = \frac{q^{111}}{k} - \frac{q^{111}x}{k}$$

$$\frac{q^{111}}{k}(L - x)$$

So that the heat flux out of the left face is

$$q = -KA \frac{dT}{dx}/x = 0 = -KA\frac{q^{111}L}{k} = -q^{111}AL$$

Example:

Consider a plate with uniform heat generation q^{111} as discussed in the last section for k = 200w/m.k q^{111} = 40mw/m^3, T_1 = 160^{0}C at x = 0, T2 = 100^{0}C at x = 2L, and a plate thickness of 2cm, determine (a) T(x), (b) q/A at the left face, (c) q/A at the right face, and (d) q/A at tje plate center.

(a) using equ. (3)

$$T = \left[\frac{100-160}{0.02} + \frac{(4 \times 10^7)(0.02 - x)}{2\,(200)}\right]x + 160$$

$$= 160 - 10^3x - 10^5x^2$$

(b) obtain $\frac{dT}{dx}$ at x = 0 and substitute into fourier's law.

$$\frac{dT}{dx}[-10^3 - (2)(10^5)x]$$

$$= \frac{dT}{dx}/x = 0 = -10^3 \text{ k/m}$$

$$\frac{q}{A}/_0 = -k\frac{dT}{dx}/_0 = -\frac{200Ll}{m.k}$$

$$= 200kw/m^2$$

(c) $\frac{dT}{dx}/_{2L} = -10^3 - 2(10^5)(0.02)$

$$= -5.10^3 k/m$$

$$\frac{q}{A}/_{2L} = -k\frac{dT}{dx}/_{2L} = -\frac{200Ll}{m.k} - \frac{5.10^3 k}{m}$$

$$= 1mw/m^2$$

(d) $\frac{dT}{dx}/_L = -10^3 - 2(10^5)(0.01)$

$$= -3.10^3 k/m$$

$$\frac{q}{A}/_L = -\frac{200w}{m.k}\left(\frac{-3 \times 10^3 k}{m}\right)$$

$$= 600kw/m^2$$

Exercise

1. Determine an analytical expression for the dimension less temperature $(T - T_2) / (T_c - T_s)$ distribution in a plane wall with heat generation q^{111} given that T_c is the temperature at the center of the wall.
2. Determine an analytical expression for the dimension less temperature distribution $(T - T_0) / (T_w - T_0)$ in a plane wall with heat generation q^{111} given that $T = T_w$ at $x = \pm L$ and $T = T_0$ at the end plane.

Cylinder with heat sources.

The temperature distribution in 1 – cylindrical wall can also be determined in an analogons fashion to the distribution in plane wall. For a sufficiently long cylinder the temperature may be considered a function of radius only. The appropriate differential equation may be obtained by neglecting the axial atimuth and timo-dependent terms to give

$$= \frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{q^{111}}{k} = 0$$

The boundary conditions are

$T = T_w$ at $r = R$

and heat generated equals heat lost at surface.

$$q\pi R^2 L = -k2\pi RL \left.\frac{dT}{dr}\right|_{r=R}$$

Convective Heat Transfer

- Methods of calculating convection heat transfer
- Ways of predicting the value of convection heat transfer coefficient h.
- To consider the energy balance on the flow system
- To determine the effect of the flow on the temperature the effect of the flow on the temperature gradients in the fluid.
- The analysis of heat transfer by convection demands a thorough understanding of various fluid flow mechanisms. The study of fluid dynamics is left to the fluid mechanics course. In a similar fashion to the study of boundary layer theorem in fluid mechanics, we are also going to look at the thermal boundary layer analysis.

Energy equation of the boundary layer.

The thermal energy equation for an incompressible fluid in Cartesian coordinate is:

$$PCP\left(\cup\frac{dT}{ry}+\cup\frac{dT}{ry}\right)=\frac{\partial}{\partial x}\left(k\frac{dT}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+m\emptyset+q \qquad \text{......... (1)}$$

Where m$\emptyset$ is called the visions dissipation term q is the rate at which energy is generated per unit volume.

The left side of equation (1) represent, the net transport of energy into the control volume, and the right side represents the sum of the net heat conducted out of the control volume, the net visions work done on the element and the energy generated rate per unit volume.

The energy equation of the laminar boundary layer can be obtained from equation (i) by applying the simplifying assumptions to give

$$\cup\frac{dT}{ry}+\cup\frac{dT}{ry}=r\frac{\partial^2 T}{\partial y^2}+\frac{N}{PCP}\left(\frac{\partial u}{\partial y}\right)2 \qquad \text{................. (2)}$$

By involving the order of magnitude analysis on equ. (2) it follows that

$$\cup\sim\cup\,\omega\ and\ y\sim\delta \qquad \text{...............(3)}$$

So that $\gamma \frac{\partial^2 T}{\partial y^2} \sim \delta \frac{T}{\delta^2}$ ....*(4)*

$$\frac{N}{PCP}\left(\frac{\partial u}{\partial y}\right)2 \sim \frac{N}{PCP}\frac{u^2 \omega}{\delta^2}$$ ....*(5)*

$$\frac{N}{PCP}\left(\frac{\partial w}{\partial y}\right)2 \sim \frac{N}{PCP}\frac{w^2 w}{\delta^2}$$

If the ratio of these quantities is small,

$$\frac{N}{PCP\delta}\frac{u^2}{1} \ll 1$$

.....................*(6)*

Then the viscous dissipation is small in comparison with the conduction term. Thus, for low-velocity incompressible flow, we have

The thermal boundary layer

A thermal boundary layer it may be defined as that region where temperature gradients are present in the flow. These temperature gradients would result from a heat exchange process between the fluid and the wall.

The temperature of the wall is T_w, the temperature of the fluid outside the thermal boundary layer is T and the thickness of the thermal boundary layer is designated as .

At the wall, the velocity is zero, and the heat transfer into the fluid takes place by conduction. The heat flux per unit area, q^{11}

$$q^{11} = k\frac{\partial T}{\partial y}/wall \qquad \text{.........................} \; (8)$$

Firm Newton's law of cooking :

$$q^{11} = h(T_w - T_V) \qquad \text{.....................} \; (9)$$

where h is the convection heat transfer coefficient combining equs. (8) and (9), we have

$$h = \frac{-k\left(\frac{\partial T}{\partial y}\right)wall}{T_\omega - T_V} \qquad \text{.....................}(10)$$

so we need only find the temperature gradient at the wall in order to evaluate the heat-transfer co-efficient.

The integral energy equation of the boundary layer for constant properties and constant free-stream temperature T_V

$$\frac{d}{dx}\left[\int_0^4 (T_\infty - T)u\,dy\right] + \frac{N}{PCP}\left[\int_0^H \left(\frac{dx}{dy}\right)^2 dy\right] = \partial\frac{\partial T}{\partial y}/\omega \qquad \text{................}(11)$$

Using the cubic polynomial temperature distribution

$$\frac{\theta}{\theta_\omega} = \frac{T - T_\omega}{T_\infty - T_\omega} = \frac{3}{2}\frac{y}{S_t} - \frac{1}{2}\left(\frac{y}{S_t}\right)^2 \qquad \text{..................}(12)$$

In equation (11) and after simplification, we obtain

$$\gamma = \frac{S_t}{\delta} = \frac{1}{1.026}\,pr^{-\frac{1}{3}}\left[1 - \left(\frac{x_0}{x}\right)^{\frac{3}{4}}\right]^{1/3} \qquad \text{.....................}(13)$$

For the case of x_0 is not equal to zero and $\gamma = \frac{S_t}{\delta} = \frac{1}{1.026}\,pr^{\frac{-1}{3}}$ ....(14)

For the case of $x_0 = 0$

The parameter pr = is called the prand H number. This number relates the relative thicknesses of the hydrodynamic and thermal boundary layers. The kinematic viscosity of a fluid conveys information about the rate of which momentum may diffuse through the fluid because of molecular motion. The thermal diffusivity tells us the same thing in regard to the diffusion of heat in the fluid.

The prand tl numbers of most gasses and liquids are more than 0.7 with the exception of the prand 11 number of liquid metoils which is of the order of 0.01.

Now from equation (10)

$$h = \frac{-k\,(\partial T/\partial y)_{\omega}}{T_{\omega} - T_{\omega}} = \frac{3}{2}\frac{k}{S_t} = \frac{3}{2}\frac{k}{y s} \qquad \text{.......................(15)}$$

Substituting for the hydrodynamic boundary layer thickness.

$$h_x = 0.332k\ pr\,\frac{1}{3}\left(\frac{U_0}{\gamma x}\right)^{1/2}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3} \qquad \text{.......................}(16)$$

The non-divisional heat transfer parameter is called Nusselt number NU

$$NU_x = \frac{hx^n}{k} \qquad \text{.....................} (17)$$

Finally,

$$NU_x = 0.332\ pr^{1/3}\ Re_x^{1/2}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3} \qquad \text{.................}(18)$$

For plate heated from $x = x_0$, or for the plate heated over its entire length, $x_0 = 0$ and

$$NU_x = 0.332\ pr^{1/3}\ Rex^{1/2} \qquad \text{........................(19)}$$

Equations (18) and (19) express the local values of the heat transfer co-efficient in terms of the distance from the leading edge of the plate and the fluid properties for the case the = 0.

$$\bar{h} = \frac{\int_0^L h_x d_x}{\int_0^L dx} = 2h_x = L \qquad \text{.....................(20)}$$

$$\overline{NU_L} = \frac{hL}{k} = 2\ NU_X = L \qquad \text{...............(21)}$$

The above analysis is valid for constant fluid properties. When there is an appreciable variation between wall and free-stream conditions, it is recommended that the properties be evaluated at the so-called film temperature T_f defined as

$$T_f = \frac{T_w + T_\infty}{2} \qquad \text{…………(22)}$$

Constant heat flux

For the constant-heat-flux case the local Nusbelt number is given by

$$NU_X = \frac{h_x x}{k} = 0.453\, Re^{1/2}{}_r\, Pr^{1/3} \qquad \text{................(23)}$$

Nusst it number may be expressed in terms of the wall heat flux and temperature difference as

$$NU_X = \frac{q_w x}{k(T_w - T_0)} \qquad \text{……….(24)}$$

The average temperature difference along the plate may be obtained by performing the integration.

$$T_w - T_\infty = \frac{1}{2} \int_0^2 (T_w - T_\infty)\, dx$$

$$= \frac{q_w\, L/k}{0.6795\, Re_r^{1/2}\quad Pr^{1/3}} \qquad \text{…………(25)}$$

While equation (19) is valid to determine the Nusselt number for friends hanry Prandtl number between 0.6 and 5.0, the following equation is valid for under range of prandtl number.

$$NU_X = \frac{0.3387\, Re_r^{1/2} \quad Pr^{1/3}}{\left[1 + \left(\frac{0.468}{pr}\right)^{2/3}\right]^{1/4}} \; for\; Re_r Pr > 100$$

Example 1

Air at 27^{0}C and 1 atm flows over a flat plate at a speed of 2m/s Assume that the plate is heated over its entire length to a temperature of 60^{0}C calculate the heat transfer in the first 20cm of the plate and the first 40cm of the place. The viscosity of air at 270C is 1.98 x 10-6 kg/ms. Assure unit depth in the z-direction.

Solution:

Required: total heat transfer over a certain length of the plate.

We shall evaluate the properties at the film temperature

Tf = $\frac{27+60}{2}$ = 43.5^0C

From properties table

γ = 17.36×10^{-6} m^2/s

K = 0.02749 w/m^{0}c

pr = 0.7

cp = 1.006 KJ/kg.^{0}C

At x = 20 cm

Re_x = $\frac{U_\infty X}{\gamma} = \frac{2 \, x \, 0.2}{17.36 \, x 10^{-6}}$

= 23.041

NU_x = $\frac{h_x X}{K} = 0.332 \, Rer^{1/2} \quad Pr^{1/3}$

= $0.332 \times 23.041^{1/2} \times 0.7^{1/2}$

= 44.74

h_x = $\frac{NU_x \, K}{x} = \frac{44.74 \, x \, 0.02749}{02}$

h_x = 6.15w/m^2 ^{0}c

h = 2 x 6.15

= 12.3w/m^2 ^{0}C

The heat flow is

Q = $\bar{h}A \, (T_w - T_\infty)$

If we assume unit depth in the z direction,

Q = 12.3 x 0.2 x (60.27)

= 81.18kw

At x = 40cm

$\gamma hc_x = \frac{U_\omega A}{2} = \frac{2 \ x \ 0.4}{17.36 \ x \ 10^{-6}}$

= 46,082

$NU_x = 0.332 \times 46{,}082^{1/2} \times 0.7^{1/3}$

$= 63.28$

$$h_x = \frac{63.28 \times 0.02749}{0.4}$$

$= 4.349\ w/m^{2\ 0}c$

$h = 2 \times 4.349$

$= 8.698 w/m^{2\ 0}c$

$Q = 8.698 \times 0.4 \times (60 - 27)$

$= 114.8W$

Example 2

A 1.01W heater is constructed of a glass plate with an electrically conducting film which produces a constant heat flux. The plate is 60 by 60cm and placed in an airstream at 27^0C, 1 atm with U_0 = 5m/s. Calculate the average temperature difference along the plate and the temperature difference at the trailing edge.

Solution

Properties should be evaluated at the film temperature, but we do not know the plate temperature so for an initial calculation we take the properties at the free-stream conditions of

T_∞ - 270C = 300k

$\gamma = 16.84 \times 10^{-6}\ m^2/s$, pr = 0.708, k = 0.02624 w/m

$$Re_L = \frac{0.6 \times 5}{16.84 \times 10^{-6}}$$

$= 1.78 \times 10^5$

Using equ. (25)

$$T_\omega - T_\infty = \frac{q_\omega L/K}{0.6795 \times Re_L 1/2\, Pr^{1/3}}$$

$$= \frac{\left[1000/(0.6)^2\right] \times 0.6/0.02624}{0.6795 \times (1.78 \times 10^5)^{1/2}(0.708)^{1/3}}$$

$= 248.6^0C$

Now, we go back and evaluate properties at

$T_f \quad = \quad \frac{249.6+27}{2} = 410.8k$

From the properties table, we obtain

$\gamma \quad = \quad 27.18 \times 10^{-6}\ m^2/s$, Pr = 0.687 and k = 00344w/m^{0}c

$= \quad Re_L \quad = \quad \frac{0.6 \times 5}{27.18 \times 10^{6}}$

$= \quad 1.10 \times 10^{5}$

$$T_\omega - T_\infty = \frac{\left[1000/(0.6)^2\right] x\, 3.6 \,/\, 0.0344}{0.6795\ (1.1 x 10^5)^{1/2}\, x\, 0.687^{1/3}}$$

$= \quad 243.6^0C$

At the end of the plate (x = L = 0.6m the temperature difference is obtained from equs. (13) and (25) with the constant 0.453 to give.

$$(T_\omega - T_\infty)_{x=L} = \frac{243 \times 0.6795}{0.453}$$

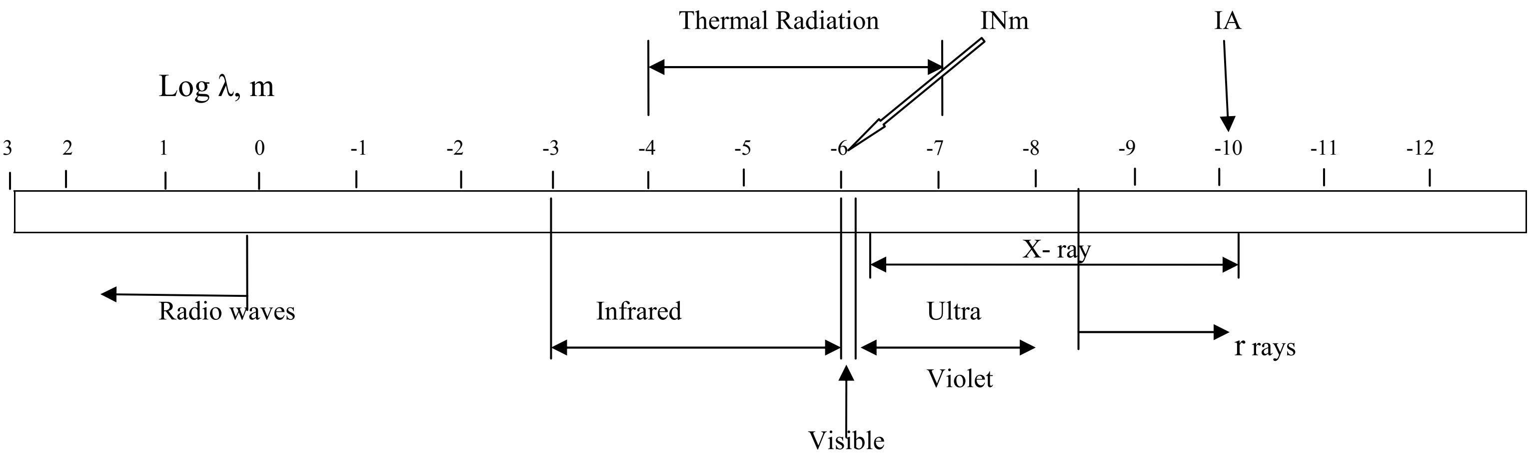
$= \quad 365.4^0C$

Radiation Heat Transfer

Thermal radiation is that electromagnetic radiation emitted by a body as a result of its temperature. There are many types of electromagnetic radiation; thermal radiation is only one. Radiation is propagated at the speed of light, $3x10^8$ m/s. The following relation is valid between the speed of light, c, the wavelength, λ, and frequency, υ.

$$C = \lambda\upsilon$$

The unit for λ may be centimeters, angstroms (IA $=10^{-8}$cm) or micrometers (WM $=10^{-6}$m)



Electromagnetic spectrum

The propagation of thermal radiation takes place in the form of discrete quanta, each quantum having energy of

$$E = h\upsilon$$

Where h = Planck's constant

$= 6.625x10^{-34}$ J.s.

Quantum = α particle = having mass, energy, momentum

Radiation is assumed to be a "photon gas" which may flow from one place to another. Using the relativity relation

$$E = mc^2 = h\upsilon$$

$$m = \frac{h\upsilon}{c^2}$$

$$\text{Momentum} = c \frac{h\upsilon}{c^2} = \frac{h\upsilon}{c}$$

Using the principles of quantum-statistical thermodynamics an expression for the energy density of radiation per unit volume per unit wavelength is given as

$$U_\lambda = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda RT} - I}$$

Where k = bolttmann's constant

= 1.38066×10^{-23} J/molecule.k

Integrating the energy density over all wavelengths, the total energy emitted is proportional to absolute temperature to the fourth power

$$E_b = \sigma T^4$$

The last expression is called the Stefan-Bolttmann's law. E_b is the energy radiated per unit time and per unit area by the ideal radiator

σ = Stefan –Bolttann's constant

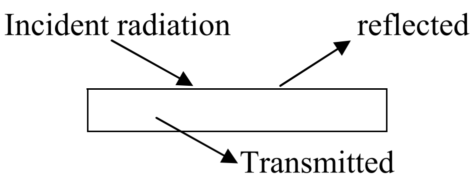
= $5.669 \times 10^{-8} w/m^2\text{-}k^4$

The subscript b denotes that this is the radiation from a blackbody, i.e. body that appears blank to the eye, and which do not reflect any radiation. It is also considered to absorb all radiation incidents upon it. E_b is the emissive power of a blackbody.

<u>Radiation Properties</u>

Reflectivity, ℓ, is the fraction of radiant energy reflected by a surface

Absurptivity, α, is the fraction absorbed, and Transmissivity T, is the fraction transmitted



$\Longrightarrow \ell + \alpha + I = 1$

most solid bodies do not transmit thermal radiation

$\Longrightarrow \ell + \alpha = 1$ for such bodies.

<u>Reflected radiation</u> may be described as <u>specular</u> if the angle of incidence is equal to the angle of reflection, or as <u>diffuse</u> when an incident been is distributed uniformly in all directions after reflection.

<u>The emissive power</u> of a body E is defined as the energy emitted by the body per unit area and per unit time.

A perfectly block enclosure is the one which absorbs all the incident radiation falling upon it.

<u>Blockbody Radiation</u>

A blockbody, or an ideal radiator, is a body which omits and absorbs at any temperature the maximum possible amount of radiation at any given wavelength. The ideal radiator is a theoretical concept which sets an upper limit to the emission of radiation in accordance with the second law of thermodynamics defining.

$E_{b\lambda}$ = Spectral or monochromatic body emissive power, i.e. radiation quantity at a given wavelength.

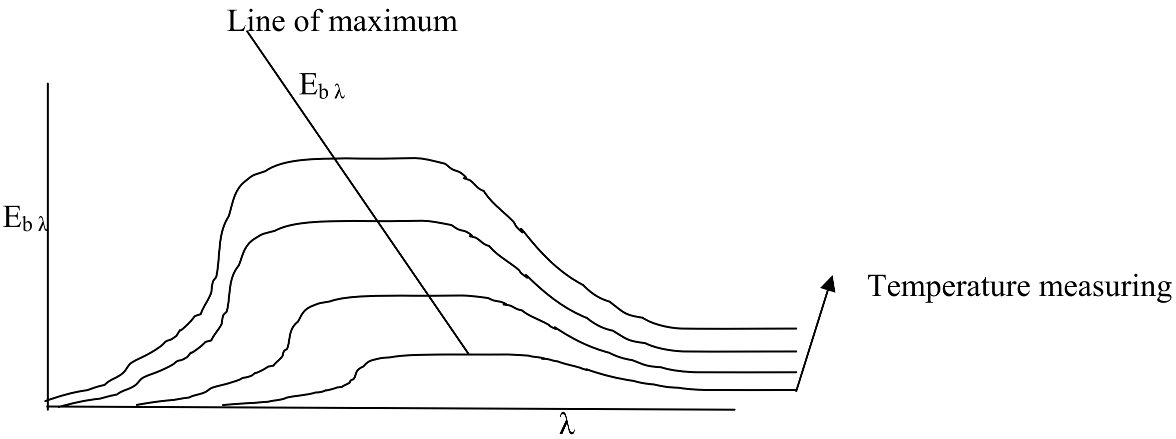
Max plank in 1900 using quantum theory showed that

$$E_{b\lambda} = \frac{C_1}{\lambda^5 (\exp(c_2/\lambda^1)-1)}$$

$E_{b\lambda}$ = monochromatic emissive power of a blackbody at temperature T, kw/m^2

T = absolute temperature of the body

$E_{b\lambda}$ at various temperatures is shown below

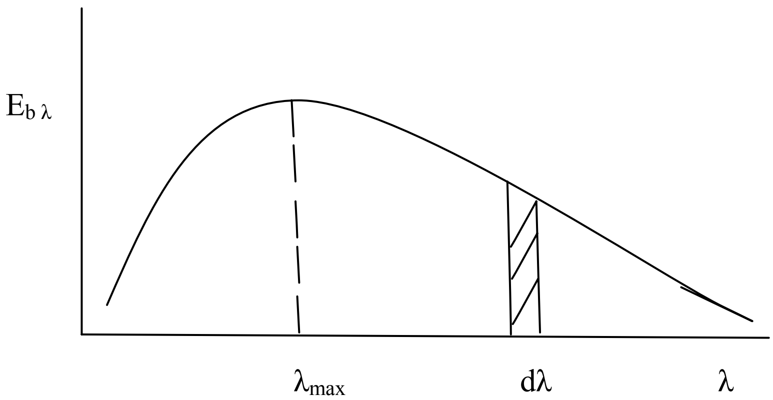


Note that the higher the temperature the higher is the proportion of the energy emitted at shot wavelength and the shorter is the wavelength, λ_{max} for $E_{b\lambda}$ max.

λ_{max} for a given T can be obtained from the

Wien's law:

$\lambda_{max} T = 0.0029$ m k



For a given value of T, the total energy emitted per unit time and unit area of blank surface is given by

$$E_b = \int_0^{\infty} E_{b\lambda}^{(\lambda)}\, d\lambda = \sigma T^4$$

Where σ = stefen Bolttmann's constants

The ratio of the emissive power of a body to the emissive power of a blackbody at the same temperature is equal to the absorptivity of the body. i.e.

$$\alpha = \frac{E}{E_b}$$

This ratio is defined as the emissivity ε of the body:

$$E = \frac{E}{E_b}$$

So that $E = \alpha$

The last expression is called the Kirchhoff's identity.

The emissivity and absorptivity of a body represent the integrated behavior of a material over all wavelengths. Real substances emit less radiation than ideal blank surface as measured by the emissivity of the material. The emissivity of a material varies with temperature and the wavelength of the radiation.

A gray body is defined such that the monochromatic emissivity E_λ of the body is independent of wavelength, i.e. E_λ constant

The monochromatic emissivity is defined as the ratio of the monochromatic-emissive power of the body, to the monochromatic-emissive power of a blackbody at the same temperature and wavelength.

$$E_\lambda = \frac{E_\lambda}{E_{b\lambda}}$$

Total emissivity E is related to monochromatic emissivity by noting that

$$E = \int_0^{\infty} E_\lambda E_{b\lambda}\, d\lambda \qquad \text{and} \qquad E_b = \int_0^{\infty} E_{b\lambda}\, d\lambda\ \sigma T^4$$

So that

$$E = \frac{E}{E_b} = \frac{\int_0^{\infty} E_\lambda E_{b\lambda}\, d\lambda}{\sigma T^4}$$

Where $E_{b\lambda}$ is the emissive power of a blockbody per unit wavelength. If gray body condition is imposed,

$$E_\lambda = E_\lambda$$

The emissivities of various substances vary widely with λ, temperature. The surface functional relation for $E_{b\lambda}$ was derived by max Planck by introducing the quantum concept for electromagnetic energy as:

$$E_{b\lambda} = \frac{u_\lambda c}{4}$$

Or $E_{b\lambda} = \dfrac{c_1 \lambda^{-5}}{e^{c_2/\lambda T} - 1}$

Where λ = wavelength

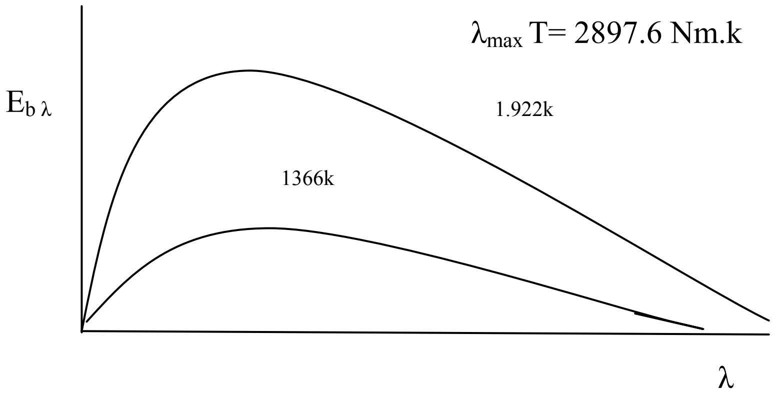
T = temperature

$C_1 = 3.743 \times 10^8$ w.Nm^4/m^2

$C_2 = 1.4387 \times 10^8$ Nm.k

In a plot of $E_{b\lambda}$ as a function of temperature and wavelength, the maximum points in the curves are related by Wien's displacement laws:

λ_{max} T= 2897.6 Nm.k



It will be observed from the curve that the peak of the curve is shifted to the shorter wavelengths for the higher temperatures. This shift in the maximum point of the radiation curve explains the charge in color of a body as it is heated

The concept of a blackbody is an idealization, i.e. a perfect blackbody does not exist all surface reflect radiation to some extent, however slight.

→ As the body is heated, the maximum intensity is shifted to the shorter wavelengths, and the first visible sign of the increase in temperature of the body is a dark- red color, followed by bright red, bright yellow and finally white color with the increase in temperature. The material also appears much brighter at higher temperature since a large portion of the total radiation falls within the visible range.

The fraction of the total energy radiated between wavelengths 0 and λ is given by

$$\frac{E_{b\,o-\lambda}}{E_{b\,o-\infty}} = \frac{\int_0^\lambda E_{b\lambda} - d\lambda}{\int_0^\lambda E_{b\lambda} - d\lambda}$$

But $E_{b\lambda} = \dfrac{c_1 \lambda^{-5}}{\exp(c_2 / \lambda T) - 1}$

Dividing both sides by T^5

$$\frac{E_{b\lambda}}{T^5} = \frac{c_1}{(\lambda T)^5 [\exp(c_2/\lambda T) - 1]}$$

For any temperature, the integrals of equation (*) above may be expressed in terms of λT. The results have been tabulated by profile. For energy radiated between λ_1 and λ_2

$$E_{b\lambda} - \lambda_2 = E_{b\,o} - \infty \quad \frac{(E_{b\,o} - \lambda_2)}{E_{b\,o} - \infty} \quad - \quad \frac{E_{b\,o} - \lambda_1}{E_{b\,o} - \infty}$$

Note that $E_{b\,o} - \infty = \sigma T^4$

= Total radiation emitted over all wavelengths.

Example

Consider the sun as a block surface at 10, 0000 R. Find the fraction of the sun's emitted radiant energy which lies in the visible range, from 0.3 to 0.7 micros.

Solution

At λ = 0.3 micros

Tλ = 0.3 x 10,000 = 3000

From the radiation functions table

At λT = 3000,

$$\frac{E_{b\,o} - \lambda_{=0.3}}{\sigma T^4} = 0.0254$$

At λ= 0.7, λT= 0.7 x 1000^0 = 7000

$$\frac{E_{b\,o} - \lambda_{=0.7}}{\sigma T^4} = 0.4604$$

$$\frac{E_{b\,o}\,3 - 0.7}{\sigma T^4} = 0.4604 - 0.0254$$

= 0.4350

I.e. 43.5% of the sun's emission is in the visible range.

Example 2

A glass plate 30cm square is used to view radiation from a furnace. The transmissivity of the glass is 0.5 from 0.2 to 3.5Nm. The emissivity may be assumed to be 0.3 up to 3.5 Nm and 0.9 above that. The transmissivity of the glass is zero, except in the range from 0.2 to

3.5Nm.Assuming that the furnace is a blackbody at 2000 ^{0}C, calculate the energy absorbed in the glass and the energy transmitted.

Solution

$\lambda = 0.5$ for $0.2, \leq \lambda \leq 3.5$Nm

$E = 0.3$ for $0 \leq \lambda \leq 3.5$ Nm

$E = 0.9$ for $3.5 \leq \lambda \leq \infty$Nm

$T = 0$ for $0 \leq \lambda \leq 0.2$Nm

$T =$ 2000 ^{0}C = 2273k.

$\lambda_1 T$ = 0.2 x 2273 = 454.6Nmk

$\lambda_2 T$ = 3.5 x 2273 = 7955.5

$A = 0.3^{2} = 0.09m^2$

From the radiation fraction table

$$\frac{E_{b\,o} - \lambda_1}{\sigma T^4} = 0 \; ; \qquad \frac{E_{b\,o} - \lambda_2}{\sigma T^4} = 0.85443$$

$\sigma T^4 = 5.669 \times 10^{-6} \times 2273^4$

$= 1513.3\ Kw/m^2$

Total incident radiation is

0.2Nm < > < 3.5Nm =$1.5133 \times 10^{-6} \times (0.85443.0) \times 0.3^2$

= 116.4Kw

Total radiation transmitted = 0.5 x 116.4 = 58.2Kw

Radiation absorbed = {0.3 x 116.4 = 34.92Kw

0.9 x (1-0.85443) x 1513.3 x 1513.3 x 0.09 = 17.84Kw}

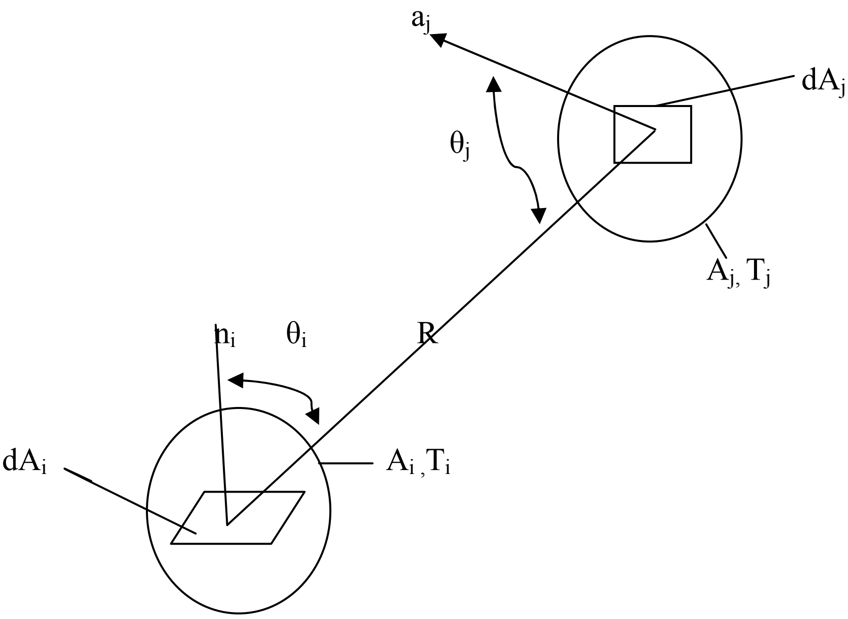
Total radiation absorbed = 34.92 x 17.84 = 52.76Kw.

Radiation Exchange between Surfaces

Radiative exchange between two or more surfaces depends strongly on the surface geometries and orientations, as well as on their radiative properties and temperature.

To compute radiation exchange between any two surfaces we must first introduce the concept of a view factor, also called a configuration or shape factor.

The view factor F_{ij} is defined as the fraction of the radiation leaving surface I that is intercepted by surface j.



Consider an elemental area of surfaces I and j, commented by a line of length R, which forms the polar angles θ_i and θ_j respectively, with the surface normal's n_i and n_j. The values of R, θ_i and θ_j vary with the position of the elemental areas on A_i and A_j

The rate at which radiation leaves dA_i and is

Intercepted by dA_j may be expressed as

$$dq_{i\to j} = I_i \cos\theta_i \, dA_i \, dw_{j-i} \quad (1)$$

Where I_i is the intensity of the radiation leaving surface

i and dw_{j-i} is the solid angle subtended by dA_j when viewed from dA_i with $dw_{j-i} = (\cos\theta_j \, dA_j) / R^2$

$$dq_{i\to j} = I_i \frac{\cos\theta_i \cos\theta_j}{R^2} dA_i \, dA_j \quad (2)$$

If surface $_i$ units and reflects diffusely.

$$I_i = \frac{J_i}{\Pi} \quad (3)$$

Where J_i is the radiative flux called radiosity, where accounts for all the radiant energy leaving a surface. Substituting the last expression in the penultimate one,

$$dq_{i\to j} = J_i \frac{\cos\theta_i \cos\theta_j}{\Pi R^2} dA_i \, dA_j \quad (4)$$

The total rate at which radiation leaves surface I and is intercepted by j is

$$q_{i \to j} = J_i \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\Pi R^2} dA_i \, dA_j \quad \text{-------------------------------} \quad (5)$$

From the definition of the view factor

$$F_{ij} = \frac{q_{i \to j}}{A_i J_i} \quad \text{-----------------------------------} \quad (6)$$

$$F_{ij} = \frac{1}{A_i} \int_{Ai} \int_{Aj} \frac{\cos\theta_i \cos\theta_j}{\Pi R^2} dA_i \, dA_j \quad \text{---------------------} \quad (7)$$

Similarly, the view factor F_{ji} is defined as the fraction of the radiation that leaves A_j and is intercepted by A_i

$$F_{ji} = \frac{1}{A_j} \int_{Ai} \int_{Aj} \frac{\cos\theta_i \cos\theta_j}{\Pi R^2} dA_i \, dA_j \quad \text{---------------------} \quad (8)$$

Equation 7 and 8 may be used to determine the view factor associated with any two surfaces that are diffuse emitters and reflectors and have uniform radiosity.

From the equation 7 and 8. It can be shown that

$$A_i F_{ij} = A_j F_{ji} \quad \text{---} \quad (9)$$

This expression is termed the reciprocity relation. It is useful in determining one view factor from the knowledge of the other.

Relations between shape factors

Another important view factor relation pertains to the surface of an enclosure. From the definition of the view factor, the summation rule

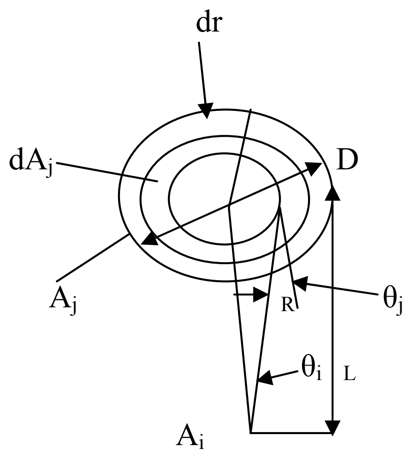
$$\sum_{j=1}^{N} F_{ij} = 1 \quad \text{---} \quad (10)$$

May be applied to each of the N surface of the enclossive. Where the term F_{ii} represents the fraction of the radiation that leaves surface I and is directly intercepted by i. For concave surface, $F_{ii} \neq 0$, but for plane or convex, surface, $F_{ii} = 0$.

Example 3

Consider a diffuse circular disk of diameter D and A_j and a plane diffuse surface of area A_i << A_j. The surfaces are parallel, and A_i is located at a distance L from the center of A_j. Obtain an expression for the view factor F_{ij}.

Solution



The desired view factor may be obtained from eqn. (7)

$$F_{ij} = \frac{1}{A_i} \int_{Ai}\int_{Aj} \frac{\cos\theta_i \cos\theta_j}{\Pi R^2} dA_i\, dA_j$$

Recognizing that θ_i, θ_j and R are approximately independent of position on A_i, this expression reduces to

$$F_{ij} = \int_{Aj} \frac{\cos\theta_i \cos\theta_j}{\Pi R^2} dA_j$$

Or, with $\theta_i = \theta_j = \theta$

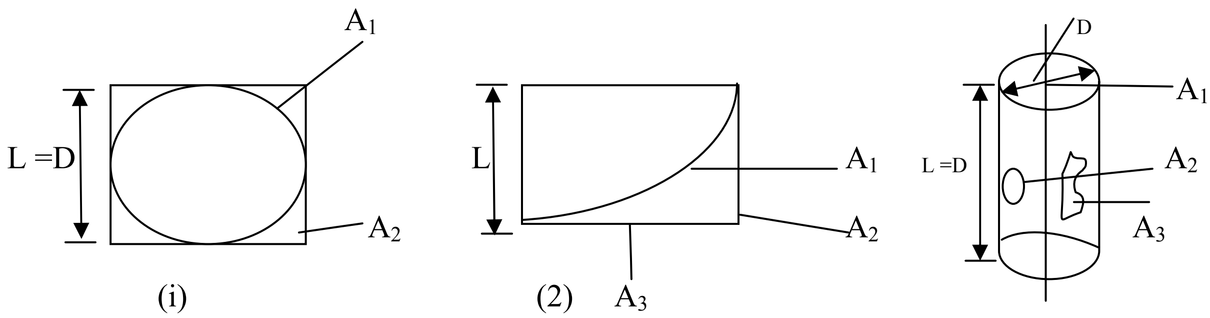
$$F_{ij} = \int_{Aj} \frac{\cos^2\theta}{\Pi R^2} dA_j$$

With $R^2 = r^2 + L^2$, $\cos\theta = L/R$, $dAj = 2\,\Pi rdr$

$$F_{ij} = 2\,L^2 \int_0^{D/2} \frac{rdr}{(r^2 + L^2)^2} = \frac{D^2}{D^2 + 4L^2}$$

Example 4

Determine the view factors F_{12} andF_{21} for the following geometries:



(1) Sphere of diameter D inside a cubical box of length L= D

(2) Diagonal partition within a long square diet\End and side of a circular tube of equal length and diameter.

Solution

The desired view factors may be obtained from inspection, the reciprocity rule, the summation rule, and or use the charts.

1. Sphere within a cube:

   By inspection, $F_{12} = 1$

By reciprocity, $F_{21} = \frac{A1}{A_2}$ $\quad F_{12} = \frac{\Pi D^2}{6 L^2} \times 1 = \frac{\Pi}{6}$

2. Partition within a square duet:

3. From summation rule:

$F_{11} + F_{12} + F_{13} = 1$

$F_{11} = 0$

By symmetry, $F_{12} = + F_{13}$

Hence, $F_{12} = 0.50$

By receptivity,

$F_{21} = \frac{A1}{A2} \quad F_{12} = \frac{\sqrt{2}}{2} \quad L \times 0.5$

$= 0.71$

3 Circular tube:

From figure, with $r_3/_L = 0.5$ and $L/r_1 = 21$

$F_{13} = 0.17$

From summation rule,

$F_{11} + F_{12} + F_{13} = 1$

Or with $F_{11} = 0$, $F_{12} = 1 - F_{13} = 0.83$

From reciprocity,

$F_{21} = \frac{A_1}{A_2} \quad F_{12} \quad = \frac{\Pi D^2/4}{\Pi DL} \times 0.83$

$= 0.21.$

Blackbody radiation Exchange

Consider radiation exchange between two blank surfaces of arbitrary shape. Defining

$q_{i\to j} = (A_i J_i)\, F_{ij}$ ------------------------(11)

Or since radiosity equal emissive power for a blank surface ($J_i = E_{bi}$)

$\Rightarrow q_{0\to j} = A_i F_{ij} E_{bi}$ -------------------------(12)

Similarly, $q_{j\to i} = A_i F_{ij} E_{bi}$ -------------------------(13)

The net radiative exchange between the two surfaces may then be defined as

$q_{ij} = q_{o\to i} - q_{j\to c}$ ------------------------(14)

From which we obtain

$q_{ij} = A_i F_{ij} E_{bi} - A_j F_{ji} E_{bj}$ ------------------------(15)

$A_i F_{ij} \sigma (T_i^4 - T_j^4)$ ------------------------(16)

Equation (16) provides the net rate at which radiation leaves surface i as a result of its interaction with j, which is equal to the net rate at which j gowns radiation due to its interaction with i. The net radiation transfer from an enclosure of black surfaces with N surfaces maintained at different temperatures may be expressed as

$q_i = \sum_{j=1}^{N} A_i F_{ij} \sigma (T_i^4 - T_j^4)$ -----------------------(17)

Example 5

Two parallel plates 0.5 by 1.0m are spaced 0.5m apart. One plate is maintained at 1000^0c and the other at 500^0c. What is the net radiant heat exchange between the two plates?

Solution

The ratios for use with radiation shape factor for radiation between parallel rectangles are

$$\frac{y}{D} = \frac{0.5}{0.5} = 1.0; \quad \frac{x}{D} = \frac{1.0}{0.5} = 2.0$$

So that $F_{12} = 0.285$

The heat transfer is calculated from

$q = A_1 F_{12} (E_{bi} - E_{b2})$

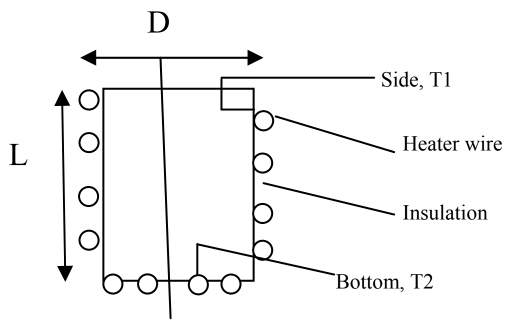
$= \sigma A_1 F_{12} (T_1^4 - T_2^4)$

$= 5.669 \times 10^{-8} \times 0.5 \times 0.285 (1273^4 - 773^4)$

$\Sigma = 18.33$kw

Example 6

A furnace cavity which is in the form of a cylinder of 75mm diameter and 150mm length is open at one end to large surroundings that are at 27^0c. The sides and bottom may be approximated as blacksides, are heated electrically are well insulated, and are maintained at temperatures of 1350 and 1650^{0}c , respectively.



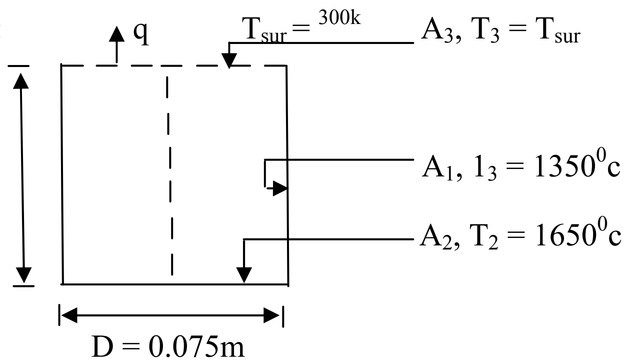
How much power is required to maintain the furnace conditions?

Solution

Known: Surface temperature of cylinder furnace

Find: Power required maintaining prescribed temperatures.

Schematic:



Assumptions:

1. Interior surfaces behave as blackbodies
2. Heat transfer by convection is iregligible
3. Outer surface of furnace is adiabatic.

Analysis:

The power needed to operate the furnace at the prescribed conditions must balance heat losses from the furnace. Subject to the foregoing assumptions, the only heat loss is by radiation through the opening, which may be treated as a hypothetical surface of area A_3. Because the surroundings are large, radiation exchange between the furnace and the surface as a blackbody at $T_3 = T_{sur}$. The heat loss may then be expressed as

$$q = q_{13} + q_{23}$$

$$q = A_1F_{13}\,\sigma\,(T_1^4 - T_3^4) + A_2F_{23}\,\sigma\,(T_2^4 - T_3^4)$$

From the figure for the view factor for coaxial parallel disks, it follows that, with r_j/L = 0.0375/0.15 = 0.25 and L/r_i = 0.15/0.0375 = 4 , F_{23} = 0.06.

From the summation rule

$$F_{21}\ 1 - F_{23} = 1 - 0.06 = 0.94$$

And from reciprocity

$$F_{12} = \frac{A_1}{A_2} F_{21} = \frac{\Pi (0.075)^2/4}{\Pi (0.075 \times 0.15)} \times 0.94 = 0.118$$

From symmetry, $F_{13} = F_{12}$

$\Rightarrow$ $q = (\Pi \times 0.075 \times 0.15) \times 0.118 \times 5.67 \times 10^{-8} \times 10.^{-8} \times (1623^4 - 300^4)$

$+ \frac{\Pi}{4} \times 0.075^2 \times 0.06 \times 5.67 \times 10^{-8} \times (1923^4 - 300^4)$

$= 1639 + 205$

$Q = 1844w$

Heat exchange between non blackbodies

Radiation heat transfer involving non blackbodies is much more complex, for all the energy striking a surface will not be absorbed; part will be reflected back to another heat-transfer surface, and part may be reflected out of the system entirely. The problem can become complicated because the radiant energy can be reflected back and forth between the heat-transfer surfaces several times. The analysis of the problem must take into consideration these multiple reflection of correct conclusions are to be drawn.

We assume that all surfaces considered in our analysis are diffuse that all uniform in temperature and that the and emissive properties are constant over all the surface. Two new terms may be defined:

G: Irradiation

= total radiation incident upon a surface per unit time and per unit area.

J: radiosity

= total radiation which leaves a surface per unit time and per unit area.

It is also assured that the radiosity and irradiation are uniform over each surface. The radiosity is the sum of the energy emitted and the energy reflected when no energy is transmitted,

$$J = \varepsilon E_b = \ell^{G} \qquad \text{------------------- (18)}$$

Assumed that T = 0

$\Rightarrow \ell = 1 = \alpha = 1 - \varepsilon$

So that

$$J = \varepsilon E_b + (1-\varepsilon) G \qquad \text{------------------- (19)}$$

The net energy leaving the surface is the difference between the radiosity and the irradiation: (see figure behind this page)

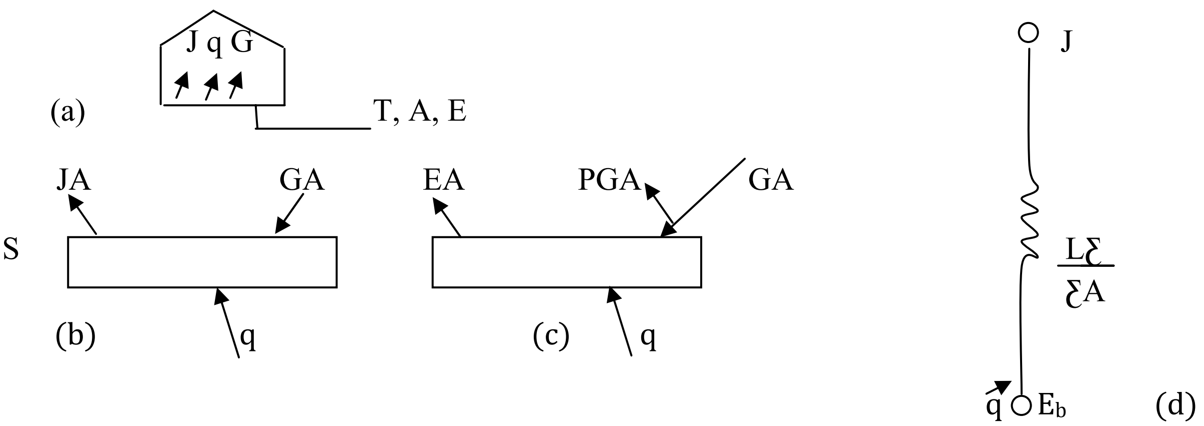
$$q = A\,(J - G) = A\,(E_b - \alpha G) \qquad \text{-------------------- (20)}$$

Solving for G in term of J from eqn (19) and substituting in (20)

$$\Rightarrow q = \frac{\varepsilon A\,(E_b - J)}{1 - \varepsilon}$$

$$q = \frac{(E_b - J)}{(1 - \varepsilon)/\,\varepsilon A} \qquad \text{------------------- (21)}$$

This transfer, which may be represented by the network element as shown in the figure below, is associated with the during potencial.



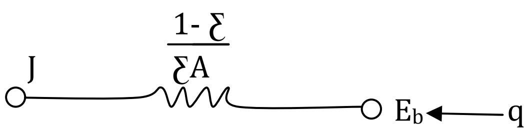
Radiation exchange in an enclosure of diffuse, gray surfaces with a non participating medium. (a) Schematic of the enclosure

(b)Radiative balance according to equation (20a)

(c) Radiative balance according to equation (20b)

(d) Network element representing to equation the net radiation transfer from a surface.

$(E_b - J)$ and a surface radiative resistance of the form $(1 - \varepsilon)/\,\varepsilon A$. Hence if the emissive power that surface would have if it were black exceeds its radiosity, there is net radiation heat transfer from the surface; if the inverse is time, the net transfer is to the surface.



Radiation Exchange between surfaces

Now consider the exchange of radiant energy by two surfaces $A_1$and A_2. Of that total radiation which leaves surface I, the amount that reaches surface 2 is

$$J_1 A_1 F_{12}$$

And of that total energy leaving surface 2, the amount that reaches surface 1 is

$$J_2 A_2 F_{21}$$

The net interchange between the two surfaces is

$$q_{12} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$$

But $\quad A_1 F_{12} = A_2 F_{21}$

So that $\quad q_{12} = (J_1 - J_2) A_1 F_{12} = (J_1 - J_2) A_2 F_{21}$

$$\boxed{q_{12} = \frac{J_1 - J_2}{1/ A_1 F_{12}}} \qquad \text{-------------------- (22)}$$

Equation (21) may be written in a general form to determine q_i, which is the rate at which radiation leaves surface i, as

$$\boxed{q_i = \frac{Eb_i - J_i}{(1/ \varepsilon_i) / \varepsilon_i A_i}} \qquad \text{---------------------- (23)}$$

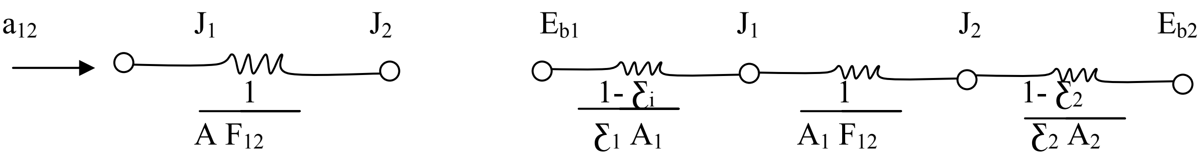
and equation (22) as

$$\boxed{q_i = \sum_{j=1}^{N} A_i F_{ij} (J_1 - J_2) = \sum_{j=1}^{N} q_{ij}} \qquad \text{----------------------- (24)}$$

This result equation the net rate of radiation transfer from surface I, q_i, to the sum of components q_{ij} related to radiative exchange with the other surfaces. Each component may be represented by a network element for which $(J_i - J_j)$ is the driving potential and $(A_i - F_{ij})^{-1}$ is a space or geometrical resistance combining equation (23) and (24)

$$\Rightarrow \quad \frac{Eb_i - J_i}{(1/ \varepsilon_i) / \varepsilon_i A_i} = \sum_{j=1}^{N} \frac{(J_1 - J_j)}{(A_i F_{ij})^{-1}}$$

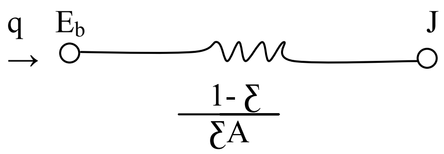
The two surface enclosure



(a) Element representing space resistance in radiation- network method

(b) Element representing surface resistance in radiation- network method

(c) Radiation network for two surfaces which see each other and nothing else.



The two network elements shown in figures (b) and (b) above represent the essentials of the radiation- network method. To construct a network for a particular radiation heat-transfer problem we need only connect a surface resistance (1- Ɛ)/ ƐA to each surface and a space resistance 1/ A_i F_{ij} between the radiosity potentials. For example, two surfaces which exchange heat with each other and nothing else would be represented by the network shown in figure (c). In this case the net heat transfer would be the overall potential difference dividing by the sum of the resistances:

$$q_{net} = \frac{E_{b1} - E_{b2}}{(1-\varepsilon_1) / \varepsilon_2 A_1 + 1/A_1F_{12} + (1-\varepsilon_2)/ \varepsilon_2 A_2}$$

$$q_{net} = \frac{\sigma (T_1^4 - T_1^4)}{(1-\varepsilon_1) / \varepsilon_1 A_1 + 1/A_1F_{12} + (1-\varepsilon_2)/ \varepsilon_2 A_2}$$

A three- body problem is shown in the figure below.

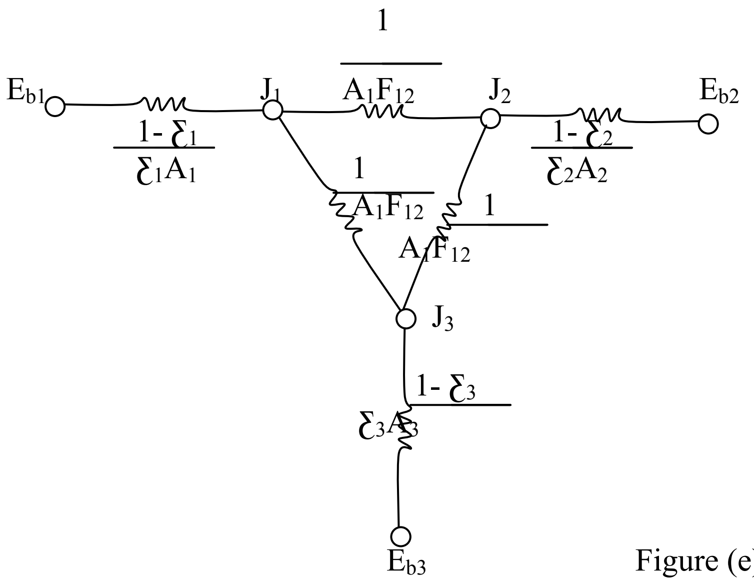


Figure (e)

In this case each of the bodies exchanges heat with other two. The heat exchange between body 1 and 2 would be

$$a_{12} = \frac{J_1 - J_2}{1/A_1 F_{12}}$$

And that between body 1 and 3

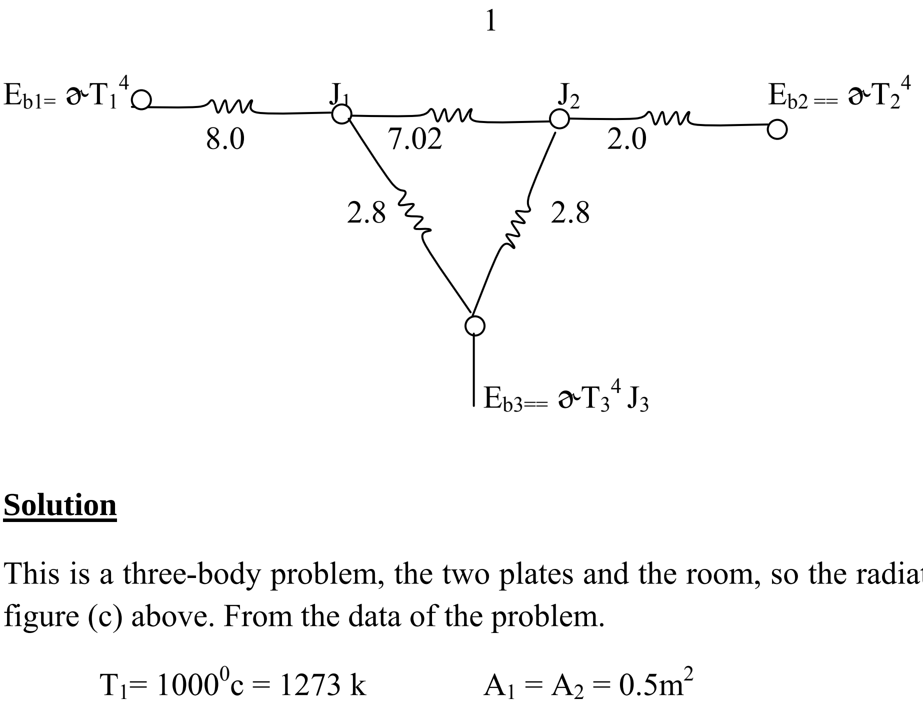
$$a_{13} = \frac{J_2 - J_3}{1/A_1 F_{13}}$$

To determine the heat flows in a problem of this type, the values of radiosities must be calculated. This may be accomplished by performing standard methods of analysis used in the circuit theory. The most convenient method is an application of Kirchhoff's current law to the circuit, which states that the sum of the current entering a node is zero. The following example illustrates the use of the method for the three-body problem.

Example:

Two parallel plates 0.5 by 1.0m are spaced 0.5m apart. One plate is maintained at 1000^0c and the other at 500^0c. The emissivities of the plates are located in a very large room, the walls of which are maintained at 27^0c. The plates exchange heat with each other and with the room, but only the plate surfaces facing each other are to be considered in this analysis. Find the net transfer to each plate and to the room.

Solution



Solution

This is a three-body problem, the two plates and the room, so the radiation network is shown figure (c) above. From the data of the problem.

$T_1 = 1000^0c = 1273$ k $\qquad A_1 = A_2 = 0.5m^2$

$T_2 = 500^0c = 773$ k $\qquad \varepsilon_1 = 0.2$

$T_3 = 27^0c = 300k$ $\qquad \varepsilon_2 = 0.5$

Because the area of the room A_3 is very large, the resistance $(1-\varepsilon_3) / \varepsilon_3 A_3$ may be taken as zero and we obtain $E_{b3} = J_3$. The shape factor was given example 5 as

$F_{12} = 0.285 = F_{21}$

$F_{13} = 1 - F_{12} = 0.715$

$F_{23} = 1 - F_{21} = 0.715$

The resistances in the network are calculated as

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{1-0.2}{0.2 \times 0.5} = 8.0$$

$$\frac{1-\varepsilon_2}{\varepsilon_2 A_2} = \frac{1-0.5}{0.5 \times 0.5} = 2.0$$

$$\frac{1}{A_1 F_{12}} = \frac{1}{0.5 \times 0.285} = 7.018$$

$$\frac{1}{A_1 F_{13}} = \frac{1}{0.5 \times 0.715} = 2.797$$

$$\frac{1}{A_2 F_{23}} = \frac{1}{0.5 \times 0.715} = 2.797$$

Taking the resistance $(1-\varepsilon_3) / \varepsilon_3 A_3$ as zero, we have the network as shown. To calculate the heat flows at each surface we must determine the radiosities J_1 and J_2. The network is solved by setting the sum of the heat currents entering nodes J_1 and J_2 to zero:

Node J_1: $$\frac{Eb_i - J_1}{8.0} + \frac{J_2 - J_1}{7.017} + \frac{Eb_3 - J_1}{2.797} = 0 \quad \text{(a)}$$

Node J_2: $$\frac{J_1 - J_2}{7.018} + \frac{Eb_3 - J_2}{2.797} + \frac{Eb_2 - J_2}{2.0} = 0 \quad \text{(b)}$$

Now $E_{b1} = \sigma T_1^4 = 148.87$ kw/m^2

$$E_{b2} = \sigma T_2^4 = 20.241 \text{ kw/m}^2$$

$$E_{b3} = \sigma T_3^4 = 0.4592 \text{ kw/m}^2$$

Inserting the values of Eb_1, Eb_2 and Eb_3 into eqns. (a) and (b), we have two equations and two unknowns J_1 and J_2 which may be solved simultaneously to give

$$J_1 = 33.469 \text{ kw/m}^2$$

$$J_2 = 15.054 \text{ kw/m}^2$$

The total heat loss by plate 1 is

$$q_1 = \frac{Eb_1 - J_1}{(1-\epsilon_1) / \epsilon_1 A_1} = \frac{148.87 - 33.469}{8.0}$$

$$q_1 = 14.425 \text{ kw}$$

And the total heat lost by plate 2 is

$$q_2 = \frac{Eb_2 - J_2}{(1-\epsilon_2) / \epsilon_2 A_2} = \frac{20.241 - 15.054}{2.0}$$

$$q_2 = 2.594 \text{ kw}$$

The total heat received by the room is

$$q_2 = \frac{J_1 - J_3}{1 - A_1 F_{13}} = \frac{J_2 - J_3}{1 - A_2 F_{23}}$$

$$q_2 = \frac{33.469 - 0.4592}{2.797} + \frac{15.054 - 0.4592}{2.797}$$

$$= 17.020 \text{ kw}$$

From an overall- balance stand point we must have

$z_2 = a_1 + a_2$ because the net energy lost by both plates must be absorbed by the room

Radiation shields — Gas radiation

Radiation network for an absorbing and transmitting medium

The reradiating surface — Radiation exchange with specular surfaces

Radiating exchange with transmitting, reflecting, and absorbing media

Multimode Heat Transfer — Solar radiation

Radiation properties of the environment

Heat Exchanger

The application of the principles of heat transfer to the design of equipment to accomplish a certain engineering objective is of extreme importance because it leads to product development for economic gain. A heat exchanger is a device used for the process of heat exchange between two fluids that are at different temperatures and separated by a solid wall, which occurs in many engineering applications.

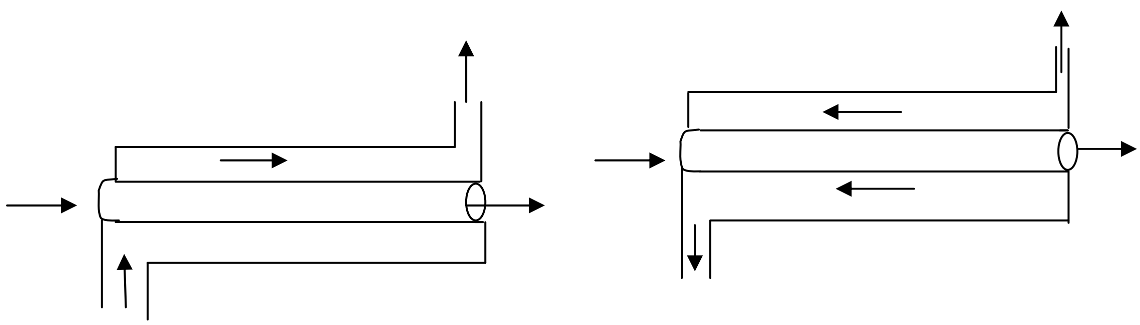
Heat exchangers find applications in space heating and air conditioning, power production, waste heat recovery, and chemical processing, aeronautical applications, etc. Economics plays a key role in the design and selection of heat- exchange equipment. The weight and site are important cost factors in the overall applications of heat exchanger and these may be considered as economic variables. We shall concern ourselves with

- Method of predicting heat- exchanger performance
- Method used to estimate the heat- exchange site and type necessary to accomplish a particular task.

Heat Exchanger Types

Heat exchangers are typically classified according to flow arrangement and type of construction.

(i) Concentric tube exchanger: In a concentric tube heat exchanger, the hot and cold fluids move in either the same or opposite direction called parallel flow or counter flow arrangement respectively.



(a) (b)

Concentric tube heat exchangers (a) parallel flow (b) counter flow.

(ii) Cross- flow heat exchangers, i.e. fluids more perpendicular to each other
   (a) Finned with both fluids unmixed

(b) Unfinned with one fluid mixed and the other unmixed

(iii) Shell- and- tube heat exchanger

Specific forms differ according to the number of shell-and-tube passes. The simplest form involves single tube and shell passes. Baffles are usually installed to increase the convection coefficient of the shell-side fluid by inducing turbulence and a cross flow velocity component.

(iv) Compact heat exchangers – These devices have dense arrays of finned tubes or plates and are typically used when at least one of the fluids is a gas, and is hence characterized by a small convection coefficient.

Heat Exchanges Analysis: Use of the Log Mean Temperature Difference.

To design or to predict the performance of a heat exchanger it is essential to relate the total heat transfer rate to the quantities such as the inlet and outlet fluid temperatures, the overall heat transfer coefficient, and the total surface area for heat transfer. This is done by applying overall energy balances to the hot and cold fluids. This leads to:

$$q = M_h CP_h (Th_{iL} - Th_{io})$$

$$q = M_c CP_{ic} (T_{cio} - T_{ciL})$$

Where i is the fluid inlet conditions

o is the fluid outlet =

h refers to hot and c to cold fluid. M is the mass flow rate, C_p is the fluid specific heat capacity, T is the fluid temperature at any particular cross-section of the channel.

The temperature difference between the hot and cold fluids is determined from the

$$\Delta T = T_h - T_c$$

However, T varies with position in the heat exchange, so it is necessary to work with a rate equation of the form.

$$q = U A \Delta T_m$$

where ΔT_m is referred to as the log mean temperature difference, and U as the overall heat transfer coefficient.

It can be shown that the log means temperature difference is determined from the expression:

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\gamma n(\Delta T_2/\Delta T_1} = \frac{\Delta T_1 - \Delta T_2}{\gamma n(\Delta T_1/\Delta T_2}$$

Note that for parallel flow exchanger

$$\Delta T_1 = T_{h,1} - T_{h,1} = T_{h,i} - T_{ci}$$

$$\Delta T_2 = T_{h,2} - T_{h,2} = T_{h,o} - T_{c,o}$$

For counter flow exchanger:

$$\Delta T_1 = T_{h,1} - T_{c,1} = T_{h,i} - T_{c,o}$$

$$\Delta T_2 = T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i}$$

The above derivation for LMTΔ involves two important assumptions: (i) the fluid specific heats do not vary with temperature, and (2) the convection heat-transfer co-efficient are constant throughout the local exchanger. The second assumption is usually the more serious one because of entrance effects. Fluid viscosity and thermal-conductivity changes, etc.

If a heat exchanger other than the double-pipe type is used, the heat transfer is calculated by using a correction factor applied to the LMTΔ for a counter flow double-pipe arrangement with the same hot and cold fluid temperatures. The heat-transfer equation then takes the form.

$$q = UAF\Delta T_{2m}$$

Values of the correction factor F are posted in different charts for several different types of habit exchangers. When change in phase as in condensation or boiling (evaporation, the fluid normally remains at essentially constant temperature and the relations are simplified. For this condition

F = 1.0 for boiling or condensation.

Example 1

Water at the rate of 68kg/min is heated from 35 to 75^{0}C by an oil having a specific heat of 1.9KJ/kg.^{0}C. The fluids are used in a counter flow double-pipe heat exchanger, and the oil enters the exchanger at 110^{0}C and leaves at 75^{0}C. The overall heat-transfer coefficient is 320W/m^2. Calculate the heat-exchange area.

Solution:

The total heat transfer is determined from the energy absorbed by the water:

$$q = M_w C_{p,w} T_w = 68 \text{ X } 4180 \text{ X } (75 - 35)$$

$$= 11.37 \text{ MJ/Min}$$

$$= 189.5\text{KW}$$

Since all the fluid temperatures are known, the LMT∆ can be calculated by using the expression.

$$\Delta T_m = \frac{(T_{h,i} - T_{c,o}) - (T_{h,o} - T_{c,i})}{\gamma n\, [(T_{h,i} - T_{c,o}) / (T_{h,o} - T_{c,i})]}$$

$$\Delta T_m = \frac{(110 - 75) - (75 - 35)}{\gamma n\, [(110 - 75) / (75 - 35)]} = 37.44^0\text{C}$$

From the expression

$$q = UA\Delta T_m$$

$$A = \frac{q}{\Delta T_m} = \frac{1.895 \text{ X } 10^5}{320 \text{ X } 37.44}$$

$$A = 15.82\text{m}^2$$

<u>Examples 2</u>

Instead of the double-pipe to use a shell-and tube exchanger with the water making one shell pass and the oil making two tube passes. Calculate the area required for this exchanger, assuming that the overall heat transfer co-efficient remains at 320W/M^2,^{0}C.

Solution:

To save this problem, we determine a correction factor from the curves on correction factor plot for exchanger with one shell pass and two, four, or any multiple of tube passes".

To be used with the LM$T\Delta$ calculated on the basis of a counter flow exchange. The parameters according to the nomenclature of the figure are:

T_i = 35^0C, T_2 = 75^0C, $t_1 = 110^0$C, t_2 = 75^0C

$$P = \frac{t_2 - t_1}{T_2 - t_1} = \frac{75-110}{35-140} = 0.467$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{35-75}{75-110} = 1.143$$

So the correction factor is

F = 0.81

And the heat transfer is

$$q = UAF\Delta T_m$$

$$\text{so that } A = \frac{1.895 \times 10^5}{7320 \ X\ 0.81 \ X\ 37.44}$$

$$A = 19.53M^2$$

Examples 3

Water at the rate of 3.783kg/s is heated from 37.78 to 54.44^0C in a shell and tube heat exchanges. On the shell side one pass is used with water as the heating fluid, 1.892kg/s, entering the exchanger at 93.33^0C. The overall heat-transfer coefficient is $14.19W/M^2\ ^0C$, and the average water velocity in the 1.905cm diameter tube is 0.366 M/S. Because of space limitations the tube length must not be longer than 2.438m. Calculate the number of tube passes, the number of tubes per pass, and the length of the tubes, consistent with this restriction. The specific heat capacity of water is 4.1820KJ/kg 0C.

Solution:

We first assume one tube pass and check to see if it satisfies the conditions of this problem. The exit temperature of the hot water is calculated from

$q = M_c CP_c \Delta T_c = M_h CP_h \Delta T_h$ ....(a)

$$\Delta T_h = \frac{3.783 \times 4.1870 \times (54.44 - 37.78)}{1.892 \times 4.1870}$$

$= 33.33^0C$

So Th, exit $= 93.33 - 33.33$

$= 60^0C$

The total required heat transfer is obtained from equ (a) for the cold fluid

$q = 3.783 \times 4182 \times (54.44 - 37.78)$

$= 263.6KW$

For a counter flow exchange, with the required temperature

$$LMT\Delta = \Delta T_m = \frac{(93.33-54.44)-(60-37.78)}{\ln[(93.33-54.44)/(60-37.78)]} =$$

$\Delta T_m = 29.78^0C$

$q = UA\Delta T_m$

$$A = \frac{2.636 \times 10^5}{1419 \times 29.78}$$

$A = 6.238M^2$ (b)

Using the average water velocity in the tubes and the flow rate, we calculate the tube flow area with

$M_c = PAV$

$$A = \frac{3.783}{1000 \times 0.366}$$ (c)

$= 0.01034M^2$

This area is the product of the number of tubes and the flow area per tube:

$$0.01034 = n\frac{\pi d^2}{4}$$

$$n = \frac{4 \ X \ 0.01034}{\pi \ (0.01905)^2}$$

$$n = 36.3$$

or n = 36 tubes. The surface area per tube per meter of length is

$$\pi d = \pi \ (0.01905) = 0.0598 M^2 / \ tube.m$$

We recall that the total surface area required for a one-tube pass exchanger was calculated in equation (b) as 6.238m^2. We may thus compute the length of tube for this type of exchange firm

$$n\pi dL = 6.238$$

$$L = \frac{6.238}{36 \ X \ 0.0598}$$

$$L = 2.898m$$

This length is greater than the allowable 2.438m, so we must use more than one tube pass. When we increase the number of passes, we correspondingly increase the total surface are required because of the reduction in LMTΔ caused by the correction factor F. We next try two tube passes. From the appropriate chart fig. 10 – 8, F= 0.88, and thus

$$A_{total} = \frac{q}{UF\Delta T_m} = \frac{2.636 \ X \ 10^5}{Gx1419 \ X \ 0.88 X \ 29.78}$$

$$= 7.089m^2$$

The number of tubes per pass is still 36 because of the velocity requirement. For the two-tube-pass exchanger the total surface area is now related to the length by

$$A_{total} = 2n\pi dL$$

So that
$$L = \frac{7.089}{2 \ X \ 36 \ X \ 0.0598}$$

$$L = 1.646m$$

This length is within the 2.438rm requirement, so the final design choice is

Number of tubes per pass	=	36
Number of passes	=	2
Length of tube per pass	=	1.646m

Example 4

A cross flow heat exchanger with one fluid mixed and one unmixed is used to heat an oil in the tubes (cp = 1.9 KJ/kg.^{0}C) from 15^{0}C to 85^{0}C. Blowing across the outside of the tubes is steam which enters at 130^{0}C and leaves at 110^{0}C with a mass-flow of 5.2kg/s. The overall heat-transfer coefficient is 275 W/M2 ^{0}C and cp for steam is 1.86KJ/kg ^{0}C. Calculate the surface area of the heat exchanger.

Solution:

The total heat transfer may be obtained from an energy balance on the steam.

q = m,cp,$_s$ ΔT_s = 5.2 X 1.86 X (130 – 110)

= 193KW

We can solve for the area from equation

q = UAFΔT_m

the values of ΔT_m is calculated as if the exchanger were counter flow double pipe thus:

$$\Delta T_m = \frac{(130-85)-(110-15)}{\gamma n\,[(130-85)/(110-15)]}$$

= 66.9^{0}C

Now, from figure 10-11, t1 and t2 will represent the unmixed fluid (the oil) and T_1 and T_2 will represent the mixed fluid (the steam) so that

T_1 = 130, T_2 = 110, t_1= 15 and t_2 = 85^{0}C and we can calculate

R = $\frac{130-110}{85-15} = 0.286, P = \frac{85-15}{130-15} = 0.609$

Consulting figure 10 -11, we find

F = 0.97

So the area is calculated from

$$A = \frac{q}{UF\Delta T_m} = \frac{193{,}030}{275 \text{ X } 0.97 \text{ X } 66.9}$$

$$A = 10.82m^2$$

$$q = 0.1 \text{ x } 2131 \text{ x } (100 - 60)$$

$$= 8524W$$

For water on that temperature

$$q = M_c\, C_{p,c}\, (T_{c,o} - T_{c,i})$$

$$T_{c,o} = \frac{q}{M_c C_{pc}} + 30$$

$$T_{c,o} = 40.2^0C$$

Accordingly, use of Tc = 350C to evaluate the water properties was a good choice.

The required heat exchanger length may now be obtained from equation

$$q = UA\Delta T_m$$

where $A = \pi\Delta_i L$

$$\Delta T_m = \frac{(T_{h,i} - T_{c,o}) - (T_{h,o} - T_{c,i})}{\gamma n\,[(T_{h,i} - T_{c,o})/(T_{h,o} - T_{c,i})]}$$

$$= \frac{59.8 - 30}{\gamma n\,(59.8/30)}$$

$$\Delta T_m = 43.2^0C$$

<u>Example 5</u>

A counter flow, concentric tube heat exchanger is used to cool the lubricating oil for a large industrial gas turbine engine. The flow rate of cooling water through the inner tube (Δ_i=25mm) is 0.2kg/s, while the flow rate of oil through the outer annulus (Δ_o=45mm) is 0.1kg/s. The oil and water enter at temperatures of 100 and 30^{0}C respectively. How long must the tube be made if the outlet temperature of the oil is to be 60^{0}C ?

Solution

Properties: From the table on the thermo physical properties to saturated fluids

For unused engine oil

At Th = 80^0C = 353 K

Cp = 2131 J/kg.k, ai = 3.25×10^{-2} N.S/m^2

K = 0.138 W/M.k

For water

Tc = 35^0C, cp = 4178 J/kg, k, ai = 725×10^{-6} N.S/m^2

K = 0.625 W/m.k. pr = 4.85

Analysis: The required heat transfer rate may be obtained from the overall energy balance for the hot fluid

q = $M_h C_{p,h} (T_{hi} - T_{h,o})$

the overall heat transfer coefficient is

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}}$$

For water flow through the tube

$$Re_D = \frac{4m_c}{\pi \Delta_i m} = \frac{4 \times 0.2}{\pi (0.025)p \times 725 \times 10^{-6}}$$

Re_D = 14,050

Accordingly, the flow is turbulent and the convection coefficient may be computed from

$NU_D = 0.023 Re_D^{4/5} Pr^{0.4}$

$= 0.023 \times (14,050)^{4/5} \; 4.85^{0.4}$

= 90

Hence

$$hi = NU_D \frac{k}{\Delta_i} = \frac{90 \times 0.625}{0.025}$$

= 2250

For the flow of oil through the annulus, the hydraulic diameter is, from equ

$$\Delta_h = \frac{4A_c}{p} = \frac{4 \times \text{flow cross sectional area}}{\text{wetted perimeter}} = \frac{4(\pi/4)(\Delta_o^2 - \Delta_i^2)}{\pi\Delta_o + \pi\Delta_i}$$

$$= \quad \Delta_o - \Delta_i$$

$$\Delta_h = \Delta_o \quad \Delta_i = 0.02\text{m}$$

And the reyholds number is

$$Re_D \quad = \quad \frac{\rho u m \Delta_h}{N} = \frac{\rho(\Delta_o - \Delta_i)}{N} \; x \; \frac{M_h}{\rho\pi(\Delta_o{}^2 - \Delta_i{}^2)/4}$$

$$Re_D \quad = \quad \frac{4m_h}{\pi(\Delta_o - \Delta_i)m} = \frac{4 \; x \; 0.1}{\pi(0.045+0.025) \; x \; 3.25 \; x \; 10^{-2}}$$

$$= \quad 56.0$$

The annuler flow is therefore laminar. Assuming uniform temperature along the inner surface of the annulus and a perfectly insulated outer surface, the convection coefficient at the inner surface may be obtained from table of Nasselt number for fully developed laminar flow

With Δ_i/Δ_o = 0.56, linear interpolation provides

$$NU_i = \frac{h_o \; \Delta_h}{k} = 5.56$$

And $h_o = \frac{5.56 \; x \; 0.138}{0.020}$

$$= \quad 38.4\text{w/m}^2\text{.k}$$

The overall convection coefficient is then

$$U \quad = \quad \frac{1}{\frac{1}{2250} + \frac{1}{38.4}}$$

$$= \quad 37.8\text{w/m}^2\text{.k}$$

And thus from the rate equation, it follows that

$$L \quad = \quad \frac{q}{U\pi\Delta_i\Delta T_m} = \frac{8524}{37.8 \; x \; 0.025 \; x \; 43.2}$$

$$= \quad 66.5\text{m}$$

<u>Comments:</u>

1. The hot side convection coefficient controls the rate of heat transfer between the two fluids, and the low value of h_0 is responsible for the large value of L. A spiral tube arrangement would be needed.

2. Because $h_i \gg h_o$, the tube wall temperature will follow closely that of the coolant water. Accordingly, the assumption of uniform wall temperature used to obtain h_o is reasonable.

Nusselt number for fully developed Laminar flow in a circular tube annulus with one surface insulated and the other at constant temperature.

Λ_i/Λ_o	NN_i	NV_o
0	-	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

Firm handbook of Hea Transfer, Chapter 7 W.M. kays and H.C. per kins, Eds. W.M. Kolisenoro and J.P. Hartwest