**COURSE CONTENT**

- Radioactivity and useful effects of Radiation

- Introductory atomic Physics

The structure of the atom: The idea that atoms were very small particles was first suggested by the Greeks some two thousand years ago, but it was not until the nineteenth century that any ideas about the inside of an atom were proposed. The English Scientist, J.J. Thompson suggested that an atom was a neutral particle made of positive charge with lumps of negative charges within it. The model was called the plum pudding model of the atom, the positive charge being the "pudding" and the negative particles the "plums".

In 1906, Rutherford proposed a classic experiment, to investigate the structure of the atom, which Geiger and Marsden carried out few years later. A very thin sheet of gold (about 1μm thick) was bombarded with alpha particles and the resulting paths of the alpha- particles recorded. As was expected, many of the alpha particles passed straight through the foil, meaning that they had not suffered any collisions with the gold atoms. Some of the alpha particles made very large change of directions.

This can only be explained by assuming that the atom has a small but heavy central nucleus with a positive charge surrounded by a cloud of negative charge which is the electrons. See figure.
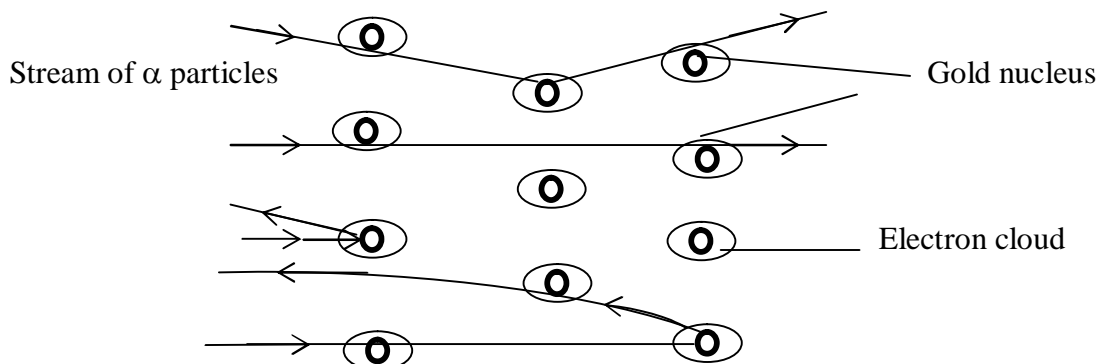
The Structure of the Nucleus: The atomic nucleus itself was found to consist of two types of particles:

(a) the proton – a positively charged stable particle;

(b) the neutron- a neutral particle which is stable

While the electrons revolve round the nucleus. All nuclei (except that of hydrogen, which consists of a single proton) contain both types of particle, there are usually more neutrons than protons.

- They are held together in the nucleus by the strong nuclear force that acts only over the very small distances in the nucleus

- At these small distances the nuclear force is great enough to overcome the electrostatic repulsion between the protons, which would otherwise force the nucleus apart

- The number of protons in the nucleus of an atom is known as the proton number (Z) (previously called the 'atomic number')

- The number of protons plus neutrons in the nucleus of an atom is known as the nucleon number (A) (previously called the 'mass number')

- For heavily nuclei, the number of neutrons exceeds the number of protons, the excess increasing with nucleon number.

- when an electron is accelerated through a potential difference of one volt, the energy gained is known as an electron volt (eV) $(= 1.6 \times 10^{-19} J)$

- $1\ MeV = 10^6\ eV\ and\ 1\ GeV = 10^9\ eV$

Thermionic emission- In 1880, Edison noticed that current could flow in an evacuated bulb from a glowing filament to another filament if the hot filament was negatively charged; this did not happen if the filament was positively charged.

- This emission of electrons was called thermionic emission

- Thermionic emission is the emission of free electrons from a hot metal surface

- Many of the basic properties of the electron may be studied using thermionic emission

- The emitting surface was usually a metal plate in an evacuated value indirectly heated with a hot wire

- The cloud of electrons produced near the plate could be accelerated away by placing a second plate in the valve and applying a potential difference between the two plates forming a simple thermionic diode. See figure below.
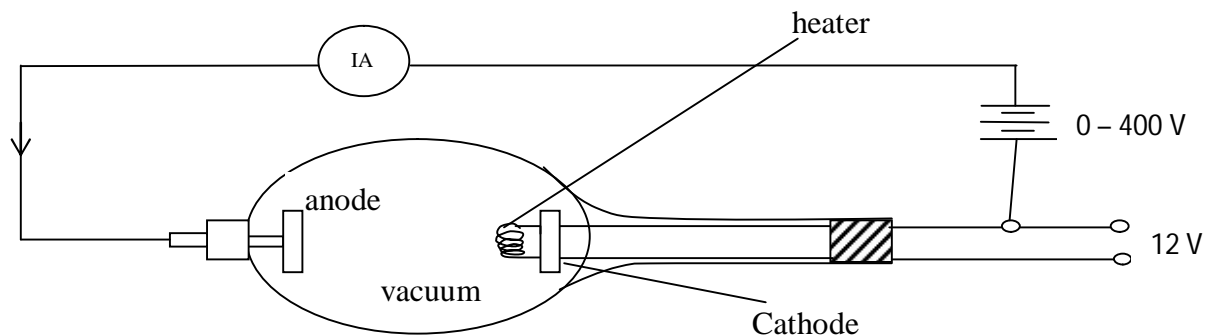
- Current was found to flow across the valve if the anode was positive with respect to the cathode and the cathode was hot.

- The hotter the cathode, the greater the current, since more electrons are emitted per sec.

- The greater the potential difference across the tube (i.e the greater the anode potential) the greater the electron velocity will be.

- Thermionic emission occurs because the free electrons within the metals are given sufficient energy to escape from the surface.

- This energy is provided by the heater of the filament.

- The fact that the streams of electrons were emitted from the cathode led to them being called cathode rays

- The cathode rays have the following properties:

    (a) rectilinear propagation

    (b) they cause fluorescence

    (c) they are deflected by electric and magnetic fields travelling in circles in magnetic fields at right angles to their motion and in parabolas in electric fields at right angles to their motion- the nature of the deflection shows that they have negative charges

    (d) they posses kinetic energy which is changed to heat when they are brought to rest

    (e) they can produce X- rays if they are of sufficiently high energy.

Emission of radiation from atoms

At the beginning of this century scientists believed that the atom was composed of a heavy central nucleus, positively charged and a swarm of negatively charged electrons orbiting round it (as stated earlier by Rutherford). In 1910, Rutherford then showed that such a system would be unstable which is a failure. Naturally, an atom is mechanically stable as the Coulomb attractive force between the nucleus and the electrons is balanced by the Centripetal force. But Rutherford argued that according to electromagnetic theory, an accelerated charged particle must radiate energy in the form of electromagnetic radiation continuously. That an electron moving round the nucleus in circular orbit is subjected to a continuous acceleration of constant magnitude directed towards the nucleus and because of this, it is radiating energy (i.e lossing energy) and as a result energy is reducing as it is moving in the circular orbit and therefore move in a reduced spiral motion until it spiral into the nucleus and collapse. In the process of the spiral motion, it would

emit radiation of continuously increasing frequency and hence would give rise to continuous spectrum. This means that atoms are unstable and exist only for a fraction of a second $(\sim 10^{-8}\ Sec)$.

Contrary to the above, atoms are known to be stable and they show no tendency to collapse. Moreover, atoms of any element emit radiation of discrete wavelengths which form the line spectrum of the element rather than a continuous spectrum suggested by Classical theory.

However in 1913, Niels Bohr overcame this difficulty by applying the quantum theory to the problem. This theory was developed in 1900 by Max-Planck to explain the black body radiation curves.

Plank stated that the electron's energy was quantized, that is, that the electron could only have certain distinct values of energy. That electron could only revolve about a nucleus in the allowed stationary orbit,

- moving in the allowed orbit, electrons do not emit nor absorb radiation
- although they are accelerated, atom is still stable
- the allowed electron's orbit are those for which the angular momentum of the electron about the nucleus is an integral multiple of $\hbar$ i.e

$$mvr = n\hbar$$
$$L = n\hbar$$
$$L = \frac{nh}{2\pi}$$

The atom radiates or absorbs energy when leaving an allowed orbit and the change in energy is $\quad E_i - E_f = h\nu$

THE HYDROGEN SPECTRUM:

For simplicity we shall consider only the hydrogen atom with its single electron

- Taken an electron that is at rest outside the atom to have zero energy, and so the levels, within the atom have negative energy values

- Each energy level is given a number i.e the quantum number (n) for that level, with the higher numbers representing the states of greater energy (e.g n= 4, E= - 0.85, n = 1, E = - 13.60 with -0.85> -13.60).

- When an electron in an atom falls from one of the upper energy levels, to one lower down, (e.g n= 3 to n = 1), energy is emitted in the form of radiation: the bigger the energy difference, the greater will be the energy of the emitted quantum. The frequency f of the emitted radiation is given by Planck's formula:

$E = hf$

E is the energy difference and h is Planck's Constant $(6.67 \times 10^{-34} \ JS)$

Each energy change gives rise to a quantum of radiation of frequency, f and therefore to a line in the hydrogen spectrum. A diagram of the energy levels in the hydrogen spectrum is shown in the figure below
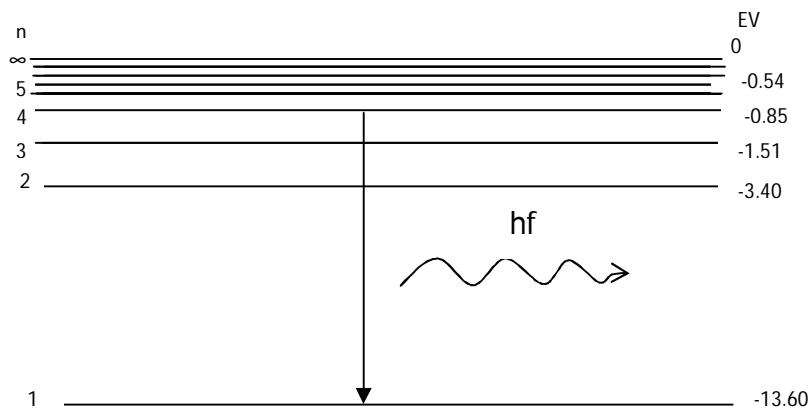
The values of the energy levels can be calculated from Bohr's formula.

- The lowest level, with n= 1, is called the ground state. The electron will always occupy this lowest level unless it absorbs energy. This is also known as the lowest state for the atom

- The other levels are called the excited state and the top level, with n= ∞, is the ionization state. An electron raised to this level will be removed from the atom.

- When an electron in a high energy state falls back to a lower level, energy is emitted in the form of radiation. This type of emission of radiation is called spontaneous emission. The loss of energy of an atom is a random process and we cannot tell when a particular electron within an atom may fall to a lower energy level. Turning the energy level diagram above on its side will give a diagram that represents the line spectrum of Hydrogen, see below.
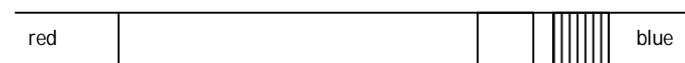
The lines get closer together towards the blue end of the spectrum, corresponding to the closer energy levels near n = ∞

Calculation of wavelengths from the energy level diagram

Knowing the values of the energy levels gives us a way of calculating the frequency and wavelength of a given transition.

It is important that the energy values used in calculations is always in Joules

Example:

Calculate the energy released and the wavelength of the emitted radiation when an electron falls from level $n = 3 \ (-1.51 \ eV)$ $to \ n = 2 \ (-3.4 \ eV)$

Solution

Energy difference $(E) = -1.51 - 3.4 = 1.89 eV$

But an $eV = 1.6 \times 10^{-19} J$

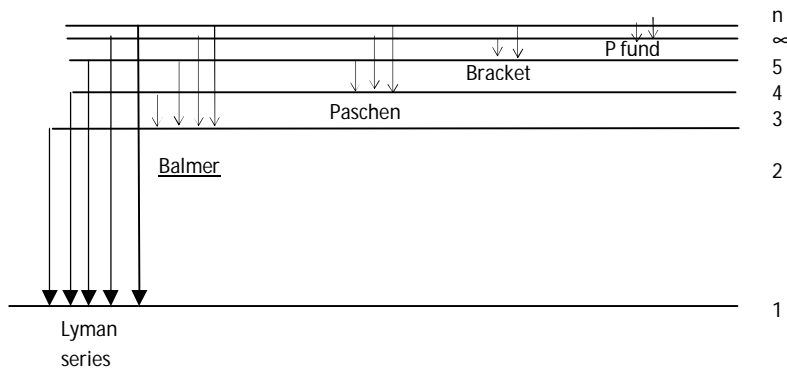$\therefore E = 1.89 \times 1.6 \times 10^{-19} J = 3.0 \times 10^{-19} J$

Therefore wavelength emitted $= \dfrac{hC}{E}$

Since $V = f \lambda, = c = v \lambda, v = \dfrac{c}{\lambda}$ but $E = hv$

$\therefore E = \dfrac{hc}{\lambda}$, $\therefore \lambda = \dfrac{hc}{E}$, $\lambda = \dfrac{6.6 x 10^{-34} \, x 3 x 10^{8}}{3.0 x 10^{-19}} = 6.6 x 10^{-7} \, m \equiv 660 nm$.

Spectral series of Hydrogen

The spectrum of hydrogen contains distinct group of lines known as spectral series. These are shown in the figure below:

They represent groups of electron transitions that end on levels n= 1,2,3,4 etc

The series ending on n=1 shows the largest energy transitions and gives lines in the ultraviolet region of the spectrum (invisible). This is known as the Lyman series. The series ending on n=2 lies mostly in the visible region of the spectrum and is called the Balmer series. Other series

ending on n=3 and above lie in the infrared region. Notice how the lines in a given spectral series get closer together as the wavelength decreases reaching a so-called series limit at the short-wavelength end.

Types of spectrum: There are two main types of spectrum

  (a) Emission spectra, where light is given out by a source and

  (b) Absorption spectra, where light from a source is absorbed when it passes through another material, usually a gas or a liquid (e.g when light from a white light source is passed through a gas)

Bohr's equation for the Hydrogen atom:

Bohr derived an equation to give the values of the energy levels in the hydrogen atom

He showed that the energy levels were proportional to $1/n^2$, where n is an integer

i.e $E \propto \dfrac{1}{n^2}$, n = 1,2, ………………………….

When an electron falls from one energy level E_2 to another level E_1, radiation of frequency f will be emitted, Bohr showed that the energy difference ($E_2 - E_1 = hf$) was given by the equation:

$$E_2 - E_1 = hf = \frac{me^4}{8\varepsilon_0{}^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Where n_1 and n_2 are integers (1,2,3…………..) and the other constants have their usual meanings

The equation can be expressed in the form of frequency (v) as:

$$v = \frac{me^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

And the equation can be expressed in the form of wave number $\left(1/\lambda\right)$ as:

$$\frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0{}^2 ch^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

The term $\frac{me^4}{8\varepsilon_0{}^2 ch^3}$ is known as Rydberg's constant (R):

R= $1.097 \times 10^7 \ m^{-1}$

Putting n_1= 1,2 or 3 will give us three series of energy changes and therefore three series of wavelengths.

$n_1 = 1$ gives the Lyman series (Ultraviolet)

$n_1 = 2$ gives the Balmer series (visible);

$n_1 = 3$ gives the Paschen series (infrared)

Quantum numbers: Every electron within an atom is considered to have four quantum numbers:

(a) The Principal quantum number (n), represents the energy level and characterizes the electron shells

(b) The orbital quantum number (l), which may have an integral value from 0 to n-1

(c) The magnetic quantum number (m), which may have any integral value from -L to +L

(If n=1, L=0, m=0, If n=2, L=0,1, m= -1,0,+1, If n = 3, L= 0,1,2, m = -2,-1,0,+1,+2 electronic states.)

(d) The spin quantum number (S), which may have values of $-\frac{1}{2}$ or $+\frac{1}{2}$.

Related to these quantum numbers is the Pauli Exclusion Principle which states that no two electrons in an atom may exist in the same quantum state. This important statement may be used to predict the numbers of electrons in the shells of an atom.

Consider the following illustration in the K-shell, n = 1

The only possible values for l and m are 0, (i.e when n= 1, l= 0 ,and m = 0). S can be $+^1/_2$ or $-^1/_2$; and so only two electrons can exist in this shell.

THE PHOTOELECTRIC EFFECT: This is a process in which radiation (i.e electromagnetic wave) is used to knock out electrons from a metal surface.

- When an electromagnetic radiation of sufficiently high frequency such as ultraviolet radiation or X-rays is incident on a metal surface, electrons are emitted from it. This phenomenon is known as the photoelectric effect, and the emitted electrons are called photo-electrons.

- Photoelectric effect is a quantum process i.e it can only be explained by considering energy to be discrete and not continuous.

- For the process to occur the energy of the incident light (i,e the frequency) must be equal to or exceeds the amount of energy that binds the electrons to the metal surface (i.e the threshold frequency of the metal).

- For each metal, there is a well defined frequency known as the threshold frequency, f_0 which must be exceeded before the process can occur, i.e any given frequency of radiation less than f_0 will not cause photoelectric emission.

  Radiations like X-rays and gamma rays which propagate at high frequencies will cause photoelectric emissions in many metals.

UV

quate window D
to transmit UV

A

c   --   evacuated bulb   +
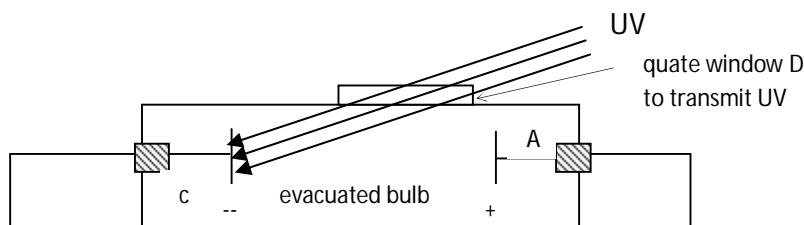
Experimental study

A photosensitive metal plate c, the cathode, is mounted opposite to another metal plate A, the anode in a highly evacuated tube B. The plates A and C from two electrodes to which a variable potential difference can be applied.

Ultraviolet light from a source is transmitted through the quartz window and made to fall on the surface C.

- When the potential difference between A and C is such that C is at a negative potential with respect to A, negative ions (photoelectrons are emitted from C and accelerated towards A). The resulting photoelectric current, I flowing in the circuit is measured by either micro-ammeter or galvanometer and the accelerated potential difference is measured by the vacuum tube voltmeter v.

- As a photon of radiation hits the metal surface, it loses its kinetic energy to an electron

- Part of which is used to overcome the binding energy on the electron, the rest becomes the kinetic energy of the electron. Thus we have the following equations

$$hf = E_k + hf_0$$

h= plank constant = 6.63 x 10^{-34}JS

f= frequency of the radiation

hf= energy of the photon

Ek = kinetic energy of the electron

f_o = threshold frequency and

$hf_{0=}$ is known as the work function. It is equal to the binding energy on the electron

Increasing the radiation intensity at constant frequency gives rise to increased photocurrent (i.e more photoelectrons are produced).

Increasing the frequency of the radiation at constant intensity, increases the kinetic energy of the photoelectrons.

WORKED EXAMPLE OF PHOTOELECTRIC EFFECT

1. Calculate the stopping voltage for a photocell containing a cesium emitting surface if light of wavelength 500nm is shown on it. The work function for cesium is 3.0 x10^{-19}J.

Solution

The stopping voltage can be found from Einstein's equation in the form

$E_k = hf - hf_o$ and

$eV = h(f - f_0)$

Therefore the stopping voltage $V = \dfrac{h(f - f_0)}{e}$

But work function is given by

$W = hf_0 = 3.0 \times 10^{-19}$

Therefore $f_0 = \dfrac{3.0 x 10^{-19}}{6.63 x 10^{-34}}$

$f_0 = 4.52 \times 10^{14}$ Hz.

Frequency of incident light $f = c/\lambda$ (i.e from $C = f\lambda$)

So $f = \dfrac{C}{\lambda} = \dfrac{3 \times 10^8}{500 \times 10^{-9}} = 6 \times 10^{14}\ Hz$.

Therefore the stopping voltage $V = \dfrac{h(f - f_0)}{e}$

$$= \dfrac{6.63 \times 10^{-34} \times (6 \times 10^{14} - 4.52 \times 10^{14})}{1.6 \times 10^{-19}}$$

$V = \dfrac{6.63 \times 10^{-34} \times 1.48 \times 10^{14}}{1.6 \times 10^{-19}} = 0.61V.$

X- RAYS: In 1895, Roentgen discovered that X-rays were invisible electromagnetic waves with their wavelengths shorter than that of visible light.

He realized that X-rays were produced when a beam of high energy electrons hit a metal target: the greater the electrons energy, the higher the frequency of the X-rays

X-ray tube

Filament   EH$_1^{-\ +}$ Copper anode
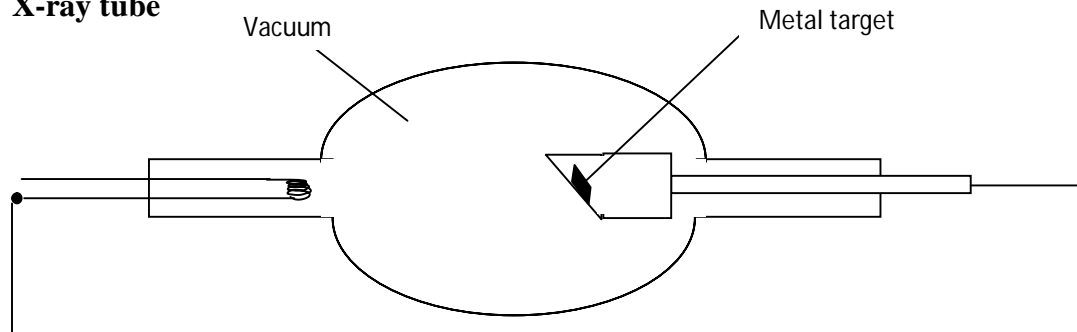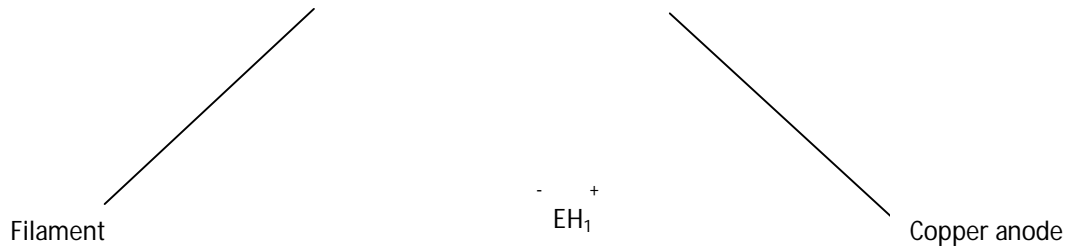
A diagram of a modern X-ray tube is shown above

This type of tube was discovered by Coolidge in 1913. It can operate with either a hot or a cold cathode. In the hot-cathode tube, electrons are emitted by thermionic emission and then accelerated by voltages usually of the order of 20kv, giving relatively long wavelength X-rays called 'soft' X- rays.

With a cold cathode, however, the voltages required to cause electron emission are much greater around 100Kkv, and these tubes produce 'hard' X-rays of much shorter wavelength, between 10^{-9} and 10^{-13}m, depending on the voltages used. For some applications, potential differences of up to 1000 000V are used.

The intensity of the X-ray beam depends on the number of electrons striking the target per second and in the hot-cathode tubes this is controlled by the heater current. The wavelength depends on the voltage across the tube.

The penetrating power of the X-rays is thus dependent on the accelerating voltage and the intensity of the beam of the heater voltage.

When the electrons collide with the target anode they lose their kinetic energy, some of this energy is converted into X-radiation, but much of it produces heat. In fact less than 0.05 per cent of the kinetic energy of the electrons becomes X-ray energy.

To prevent damage to the anode, it has to be cooled, either by air cooling, using fins, or by pumping cooling liquid through it. It may even be rotated during use to spread the wear over a larger area.

ABSORPTION OF X-RAYS: When X-rays pass through matter such as a human body they will lose energy in one or more of the following ways:

(a) The photoelectric effect: an X-ray photon transfers all its energy to an electron which then escapes from the atom

(b) Compton scattering: an X-ray photon collides with a loosely-bound outer electron. At the collision the electron gains some energy and a scattered X-ray photon is produced travelling in a different direction from the incident photon and with a lower energy.

(c) Pair production: an X-ray photon with energy greater than 1.02MeV enters the intense electric field of the nucleus. It may be converted into two particles, a positron and an electron.

USES OF X-RAYS

- In engineering: for checking metal castings for defects, crystal analysis.

- In art; for detecting covered paintings.

(a) Diagnostic: In a simple form, this would be the detection of a broken bone or a tooth cavity. With the addition of an absorber such as barium or iodine, X-rays may be used to check respiratory or digestive disorders.

(b) Therapeutic: This use is almost restricted to the treatment of malignant cancers

- It is greatly important in the use of X- rays for medical purposes that the dose given to both the patients and the operator is carefully controlled. X-rays can damage living tissue hence their use for the destruction of tumours.

PROPERTIES OF X-RAYS

(a) They pass through many materials more or less unchanged (but put in mind their absorption discussed earlier)

(b) They cause fluorescence in materials such as rock salt, calcium compounds or uranium glass;

(c) They affect photographic plates, causing fogging,

(d) They cannot be refracted

(e) They are unaffected by electric and magnetic fields

(f) They discharged electrified bodies by ionizing the surrounding air

(g) They can cause photoelectric emission

(h) They are produced when a beam of high energy electrons strike a metal target (the higher the nucleon number of the target the greater the intensity of the X-rays produced).

CALCULATION OF X-RAY WAVELENGTH

If electrons are accelerated to a velocity v by a potential difference V and then allowed to collide with a metal target, the maximum frequency of the X- rays emitted is given by the equation

$\frac{1}{2}mv^2 = eV = hf$

$$f = \frac{eV}{h}$$

This shows that the maximum frequency is directly proportional to the accelerating voltage

Example:

Calculate the minimum wavelength of X-rays emitted when electrons accelerated through 30Kv strike a target.

Solution

$$f = \frac{eV}{h}$$

$$= \frac{1.6 \times 10^{-19} \times 3 \times 10^{4}}{6.63 \times 10^{-34}}$$

V = potential difference, h = Planck constant, e = charge of electron.

∴ the wavelength (λ = c/f) is 0.41 x 10^{-10}m = 0.041nm.

NUCLEAR STABILITY: RADIOACTIVITY

The nucleus is the central region of an atom and contains protons and neutrons. It has an average diameter of 10^{-15}m

- The force binding the nucleons together is known as the nuclear force which is a short range attractive force which overrides the Coulomb's force of repulsion between two like charges like the protons.

- Nuclear forces decreases as particle separation increases and it vanishes when particle separation exceeds 5 x 10^{-15}m.

- The nucleons then tend to pull apart and is said to be unstable.

Heavy elements therefore stand the risk of nuclear instability on account of their long nuclear diameters.

- All elements with atomic number greater than 82 have unstable nuclei.

- Apart from nuclear diameter, other factors exist which determine nuclear instability because there are also unstable light elements.

Radioactivity: is the process by which unstable nuclei undergo spontaneous disintegrations by emitting certain radiation until they attain stability.

- The radiation commonly emitted in this process are named alpha (α) , beta (β) and gamma (γ) radiations.

- The table below shows the main properties of these radiations.

	Alpha particle	Beta particle	Gamma radiation
Penetration	small	Large	Very large
Range in air	Few Cm	Tens of Cm	Many Meters
Charge	Positive	Negative	No charge
Deflection by electric field or magnetic field	Some	Large	None
Ionization	Highest ionization	Higher ionization	High ionization
Velocity	0.05C	0.9C	C
Mass	Heavy	Light	None
Nature	Helium nuclei	Electron	Electromagnetic radiation

Elements with unstable nuclei are said to be radioactive

Safety precautions and the biological effects of radiation

With all kinds of radioactivity, shielding is important, the thickness and type of shielding depends on the type of radiation.

- Alpha particles are not penetrating and aluminum sheet will stop or shield against them

- Their only biological effects are to the surface of the skin, with the production of radiation 'burns'

- The penetrating power of beta-particles is greater, thicker aluminum is needed to shield them

- Gamma radiation is intensely penetrating and many centimeters of lead are required to reduce the intensity from a large source to safe level.

- It affects the internal organs of the body due to its high penetrating power and it obeys the inverse square law in air, it is better to be as far away from its source as possible.

UNITS OF RADIATION:

The unit of activity of radioactive source used to be the curie (3.7×10^{10} disintegration per sec) but it is now Becquerel (Bq). A source has an activity of 1Bq if it emits 1 particle per sec, thus a source of 1 curie has an activity of 3.7×10^{10} Bq.

Radiation dose is the energy liberated by radiation within a material. Its unit is Grays

IGy = energy liberated by radiation of 1J per kilogram of the material.

Effective dose = radiation dose x RBE (Relative Biological Effects).

Radioactive decay law: To derive the law relating to the decay of a radioactive sample. Assume that the number of radioactive nuclei (dN) decaying in a time dt is proportional to the number (N) present at that instant. i.e $\frac{d_N}{dt} \propto N$

We can write this as $\frac{dN}{dt} = -\lambda N \; or \; dN = -\lambda N dt$ (*)

λ = constant called the radioactive decay constant or disintegration constant.

The negative sign shows that the number of nuclei decreases with time.

$\frac{dN}{dt}$ is called the activity of the sample and it is measured in Becquerels.

If there are N_o undecayed nuclei at time t =0 then $\frac{dN}{N} = -\lambda dt$ which when integrated becomes

$N = N_0 e^{-\lambda t}$

The constant λ may be written as $\lambda = -\frac{dN}{Ndt}$ (from (*).

Half – life of a source: which is the time it takes for the activity of the sample to fall to half its original value. It is also the time it takes for the number of radioactive nuclei in the sample to reduce to half.

From N (t) = $N_0 e^{-\lambda t}$

When t $=T_{1/2}$

Then N (t) $=N_{0/2}$ (**)

∴ From (**)

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{\frac{1}{2}}}$$

$$\frac{1}{2} = e^{-\lambda t_{\frac{1}{2}}} \qquad e^{+\lambda t_{\frac{1}{2}}} = 2, \quad \lambda t_{\frac{1}{2}} = \ln 2, \qquad , \quad t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}, \quad \frac{0.693}{\lambda}$$

USES OF RADIOACTIVE ISOTOPES

These materials have a variety of uses and a selection of these are listed below

(a) Dating geological specimens, using uranium

(b) Dating archaeological specimens, using carbon-14

(c) Thickness measurement by back-scattering beta-radiation

(d) Treatment of tumours

(e) Sterilization of foodstuffs

(f) Liquid flow measurement

(g) Tracing sewage or slit in the sea or river

(h) Checking blood circulation and blood volume

(i) Checking the silver content of coins

(j) Radiographs of teeth

(k) Testing for leaks in pipes

(l) Tracing phosphate fertilizers using phosphorus 32

(m) Sterilization of insects for pest control

Radioactive dating: e.g carbon dating

Every living thing absorbs carbon, including the relatively rare isotope ^{14}C (about 1% of all carbon isotopes). Carbon -14 is radioactive by beta-emission, with a half-life of about 5700 years. When the specimen dies, it stops absorbing carbon, and so by measuring the amount of ^{14}C present in a sample, and comparing that with a comparable living specimen, one can get an estimate of the sample's age.

Binding energy:

The neutrons and protons in a stable nucleus are held together by nuclear forces and energy is needed to pull them apart. This energy is called the binding energy of the nucleus, the greater the binding energy, the more stable is the nucleus. This energy shows itself as a difference between the mass of the nucleus and the sum of the masses of the nucleons within it. For example, consider the alpha- particle, or helium -4 nucleus, which contains two protons and two neutrons.

Mass of proton = 1.007276amu

Mass of neutrons = 1.008665amu

Mass of two protons plus two neutrons = 4.031882amu

But the mass of a helium nucleus = 4.001508amu

Therefore the binding energy (the difference between the mass of the nucleons and the nucleus)

= 4.031882 – 4.001508

= 0.030374amu

To change from amu – MeV

$= 0.030374$ x $931.5 = 28.3\text{MeV}$

Since $1\text{amu} = 1.6606$ x 10^{-27}kg. The amount of energy equivalent to this mass is approximately 931.5MeV.

PHS 106 TUTORIAL QUESTIONS

1.   The energy of a particular state of an atom is 5.36eV and the energy of another state is

3.45eV. Find the wavelength of the light emitted when the atom makes a transition from one

state to the other.

Solution: E_1 = 5.36eV, E_2 = 3.45eV

Applying $\Delta E = h\nu$ i.e $E_{1-} E_2 = h\nu$

ν = frequency

ΔE = Change in energy

But $\nu = \dfrac{C}{\lambda}$

$\Rightarrow E_1 - E_2 = \dfrac{hc}{\lambda}$ $\therefore \lambda = \dfrac{hc}{E_1 - E_2}$

$\lambda = \dfrac{6.626 \times 10^{-34} \times 3 \times 10^8}{(5.36 \times 1.602 \times 10^{-19}) - (3.45 \times 1.602 \times 10^{-19})}$

$\Rightarrow = 6496A^0$

Note: $1A^0 = 1 \times 10^{-10}$m.

2.   Calculate the energy of the electron in the second, third and fourth orbits given that the

total energy of the electron in the first Bohr orbit of the hydrogen atom is -13.6eV

Solution: E1 = -13.6eV

$$En = \frac{E_1}{n^2} \text{ when n} = 2, E_2 = \frac{E_1}{2^2}, \text{when n} = 3, E_3 = \frac{E_1}{3^2} \text{ when n} = 4, \text{ven } E_4 = \frac{E_1}{4^2}$$

$$E_2 = \frac{-13.6}{2^2} = \frac{-13.6}{4} = -3.4eV$$

$$E_3 = \frac{-13.6}{3^2} = \frac{-13.6}{9} = -1.51eV$$

$$E_4 = \frac{-13.6}{4^2} = \frac{-13.6}{16} = -0.85eV$$

3.   Calculate the wavelength and frequency of Hα line of the Balmer series of hydrogen spectrum, given $R_H = 1.097 \times 10^7 m^{-1}$, and $C = 3 \times 10^8 m/s$.

Solution

$$\frac{1}{\lambda_\alpha} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), n_f = 2 - Balmer, \quad n_i = 3$$

$$\frac{1}{\lambda_\alpha} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2}\right) \equiv 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\lambda_\alpha = 6.5634 \times 10^{-7} m$$

Applying $c = \upsilon\lambda$, c = velocity, υ = frequency and λ = wavelength

$$=> \upsilon = \frac{c}{\lambda} \text{ i.e } \upsilon = \frac{3 \times 10^8}{6.5634 \times 10^{-7}} \equiv 4.571 \times 10^{14} \text{ HZ}$$

4.   Find the maximum energy of the photo-electrons in eV when light of frequency 1.5 x 10^9 MHz falls on a metal surface for which the threshold frequency is 1.2 x 10^9 MHz.

Solution: $E_{max} = ?$, $\upsilon = 1.5 \times 10^{15}$ Hz, $\upsilon_0 = 1.2 \times 10^{15}$ Hz

v = frequency of the light and v_0 = threshold frequency of the metal.

Hence $h\upsilon = h\upsilon_0 + E_{max} \implies E_{max} = h(\upsilon - \upsilon_0)$

i.e. $E_{max} = 6.626 \times 10^{-34} (1.5 - 1.2) \times 10^{15} J$

$= 6.626 \times 0.3 \times 10^{-19} J = 1.987 eV$

5.   A photon of energy 10eV falls on molybdenum whose work function is 4.15eV. Find the stopping potential.

Solution: $h\upsilon - W_0 = eVs$ …………. (1)

From ($h\upsilon = W_0 + \frac{1}{2}mv^2$ max $\equiv h\upsilon = w_0 + eV_s$max)

Hence from equation (1)

$10 - 4.15 = eVs \Rightarrow eVs = 5.85\ eV$

$\Rightarrow Vs = 5.85\text{volts.}$

$Vs = $ stopping potential.

6.   Light of wavelength $2000A^0$ falls on a silver surface with work function 4.2eV. Calculate (i) the threshold wavelength, and (ii) the stopping potential.

Solution:

(i) Threshold wavelength, $\lambda = ?$

$W_0 = h\upsilon_0,\ \upsilon_0 = \dfrac{C}{\lambda_0}$ Where $\upsilon_0 = $ threshold frequency

$W_0 = \dfrac{hc}{\lambda_0}$ $\lambda_0 = $ threshold wavelength

$\therefore \lambda_0 = \dfrac{hc}{Wc} = 2952A^0$ Note $1A^0 = 1\text{x}10^{-10}\text{m}$

(ii) $hv = W_0 + eV_s \Rightarrow eV_s = hv - W_0 \Rightarrow eV_s = \dfrac{hc}{\lambda} - W_0$

$V_s = \dfrac{hc}{e\lambda} - \dfrac{W_0}{e}$ $\therefore V_s = \dfrac{2.0eV}{e} = 2.0Volts$

7.   The electronic configuration of an atom is described using $n\ell^x$ notation, where X denotes the number of electrons occupying the sub shell e.g. Write the electronic configuration of the following atoms.

(i)   Sodium, Na: $Z = 11 => 1s^2, 2s^2, 2p^6, 3s$

(ii)   Nitogen, N: $Z = 7 => 1s^2, 2s^2, 2p^3$

(iii)   Potassium, K: $Z = 19 => 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s$

(iv)   Polonium, Pa: $Z = 84 => 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6 5s^2\ 4d^{10}\ 5p^6\ 6s^2\ 4f^{14}\ 5d^{10}\ 6p^4$

(v)   Boron, B: $Z = 5 => 1s^2, 2s^2, 2p^1$

VOTE: The sequence of energy state in order of increasing energy of the orbital of a multi electron atom is in the following order.

S<2s<2p<3s<3p<4s<3d<4p<5s<4d<5p<6s<4f<5d<6p

8.   Estimate the value of Planck constant if the cut off wavelength is measured to be $1.18A^0$ for 10-Kev electrons striking a target in a Coolidge tube.

Solution: $eV = \frac{hc}{\lambda} \therefore h = \frac{ev\lambda}{c}$ => $\dfrac{1.6 \times 10^{-19} \times 10000 \times 1.18 \times 10^{-10}}{3 \times 10^8}$

$$=6.3 \times 10^{-34} Js$$

9.   Plutonium decays by the following reaction with a half –life of 240 years.

$$^{239}_{94} Pu \rightarrow ^{235}_{92} U + ^{4}_{2} He$$
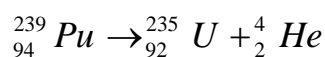
If plutonium is stored for 73200 years, what fraction of it remains?

Solution: $\dfrac{73200}{24000}$ i.e. $\dfrac{\text{number of years stored}}{\text{Half-life of Pu}}$

$=> \cong 3$ half $-$ lives, then $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ (fraction of plutonium left).

10.   Determine the cut off wavelength (in A^0) of X-rays produced by 50 – KeV electrons in a Coolidge tube.

Solutions: The minimum wavelength is the cut-off wavelength; so

λcutoff $= \frac{hc}{eV}$ since $eV = hf$ and

$f = \frac{c}{\lambda_{cut\,off}}$ $=>$ λcutoff $= \dfrac{6.3 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 50000}$

$= 0.248 A^0$

11.   Prove that $N(t) = N_o e^{-\lambda t}$

Solution: $\dfrac{dN}{dt} \alpha N$

$\dfrac{dN}{dt} = -\lambda N$

$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{o}^{t} dt$

$In \dfrac{N}{N_o} = -\lambda t$

$N(t) = N_o e - \lambda t$

12.   Prove that the half life, $T_{1/2} = \dfrac{0.693}{\lambda}$

Solution: $N(t) = N_o e - \lambda t$

When $t = T_{1/2}$, $N(t) = No/2$

$\dfrac{N_o}{2} = Noe^{-\lambda T1/2}$ {Since t is now $T_{1/2}$ then $N(t) \equiv \dfrac{N_o}{2}$}

$\dfrac{1}{2} = e^{-\lambda T1/2}$

$e^{+\lambda T1/2} = 2$

$\lambda T_{1/2} = ln2$

$T_{1/2} = \frac{ln2}{\lambda} = \frac{0..693}{\lambda}$

13.   A radioactive substance gives 378 counts/per minute at a certain time, 24hours later the count rate had fallen to 58 per minute. Calculate the half life of the material.

Solution:

Activity at time t = 0, A_0 = 378 counts/min.

Activity at time t = 24hr, A(t) = 58 counts/min

$A(t) = A_o e - \lambda t$

$(58) = 378 e^{-\lambda t}$

$\frac{58}{378} = e^{-24\lambda}$

$e^{+24\lambda} = \frac{378}{58}$

Taking ln of both side

$24\lambda = \ln\left(\frac{378}{58}\right)$

$\lambda = 0.078 ln^{-1}$

Half life $(T_{1/2}) = \left(\frac{0.693}{\lambda}\right) = 8.88$ hours

$T_{1/2} = 8.88h$

14.   A living wood was found to have a count of 15.3per min per gram of 14C. (Carbon-14). An evacuated wood handle has a count of 12.6/min/gram of 14c. Calculate the age of the wood handle if the half-life of 14c is 5730yrs.

Solution:

Present activity of the wood handle,

A(t) = 12.6/min/gram.

Since the living wood has an activity of 15.3/min/gram it follows that the wood handle also had this amount of activity while it was a living wood.

∴ Original activity of wood handle

A_0 = 15.3/min/gram

We want to determine the time, t, it took the activity to drop from 15.3 to 12.6/min/gram. This gives the age of the dead wood handle.

$T_{1/2}$ of ^{14}c = 5730yrs

$\lambda = \frac{0.693}{5730}$ (y^{-1}), since $\lambda = \frac{ln2}{T_{1/2}}$

= 1.209 x 10^{-4} per year

$A(t) = Aoe^{-\lambda t}$

$12.6 = 15.3e^{-1.209 \times 10^{-4}t}$

$e^{1.209 \times 10^{-4}t} = \frac{15.3}{12.6}$

$1.209 \times 10^{-4}t = ln\frac{15.3}{12.6}$

$t = \frac{ln (15.3/12.6)}{1.209 \times 10^{-4}}$

∴ t = 1606yrs

15.   Calculate the binding energy of hydrogen atom if the atomic mass of hydrogen in 1.00783a.m.u; mass of electron, m_e = 0.0005a a.m.u, mass of proton = 1.0073 a.m.u.

[1 a.m.u. = (atomic mass unit) = $931.5M_{ev}$]

Solution:

There is 1 electron and 1 proton in a hydrogen atom.

Total mass of constituent particles = Mp + Me

$= (1.0073 + 0.00055)$ a.m.u

$= 1.00785$ a.m.u

Atomic mass $= 1.00783$ a.m.u

Mass different, $\Delta m = 1.00785 - 1.00783$

$= 0.00002$ a.m.u

∴ The binding energy $= 0.00002 \times 931.5$ MeV

$= 0.01863$ M.V

$= 18630$ eV