COURSE TITLE: **SERVOMECHANISM AND CONTROL**

COURSE CODE: **ELE 403**

COURSE UNIT: **3**

LECTURER: **NUGA O.O.**

COURSE CONTENT: Control system concept: open and closed loop control systems, block diagrams. Resume of Laplce transform. Transfer functions of electrical and control systems. Electromechanical devices: Simple and multiple gear trains, electrical and mechanical analysis. Error detector and transducer in control systems. The amplidyne: AC and DC tachogenerator and servomotors, rotary and translational potentiometers. Hydraulic and pneumatic servomotors and controllers. Dynamics of simple servomechanism. Steady state error and error constants, the use of non-dimensional notations and the frequency response test. Log and polar plots of control systems. Basic stability concepts in control systems.

CONTROL SYSTEM CONCEPT

An automatic control system is a combination of components that act together in such a way that the overall system behaves automatically in a prespecified desired manner.

A close examination of the various machines and apparatus that are manufactured today leads to the conclusion that they are partially or entirely automated, e.g., the refrigerator, the water heater, the clothes washing machine, the elevator, the TV remote control, the worldwide telephone communication systems, and the Internet.

Industries are also partially or entirely automated, e.g., the food, paper, cement, and car industries. Examples from other areas of control applications abound: electrical power plants, reactors (nuclear and chemical), transportation systems (cars, airplanes, ships, helicopters, submarines, etc.), robots (for assembly, welding, etc.), weapon systems (fire control systems, missiles, etc.), computers (printers, disk drives, magnetic tapes, etc.), farming (greenhouses, irrigation, etc.), and many others, such as control of position or velocity, temperature, voltage, pressure, fluid level, traffic, and office automation, computer-integrated manufacturing, and energy management for buildings. All these examples lead to the conclusion that automatic control is used in all facets of human technical activities and contributes to the advancement of modern technology.

The distinct characteristic of automatic control is that it reduces, as much as possible, the human participation in all the aforementioned technical activities. This usually results in decreasing labor cost, which in turn allows the production of more goods and the construction of more works. Furthermore, automatic control reduces work hazards, while it contributes in reducing working hours, thus offering to give people a better quality of life (more free time to rest, develop hobbies, have fun, etc.).

Automatic control is a subject which is met not only in technology but also in other areas such as biology, medicine, economics, management, and social sciences.

In particular, with regard to biology, one can claim that plants and animals owe their very existence to control. To understand this point, consider for example the human body, where a tremendous number of processes take place automatically: hunger, thirst, digestion, respiration, body temperature, blood circulation, reproduction of cells, healing of wounds, etc. Also, think of the fact that we don’t even decide when to drink, when to eat, when to go

to sleep, and when to go to the toilet. Clearly, no form of life could exist if it were not for the numerous control systems that govern all processes in every living organism.
It is important to mention that modern technology has, in certain cases, succeeded in replacing body organs or mechanisms, as for example in replacing a human hand, cut off at the wrist, with an artificial hand that can move its fingers automatically, as if it were a natural hand. Although the use of this artificial hand is usually limited to simple tasks, such as opening a door, lifting an object, and eating, all these functions are a great relief to people who were unfortunate enough to lose a hand.

The Basic Structure of A Control System

A system is a combination of components (appropriately connected to each other) that act together in order to perform a certain task. For a system to perform a certain task, it must be excited by a proper input signal. Figure 1.1 gives a simple view of this concept, along with the scientific terms and symbols. Note that the response *y(t)* is also called system's behavior or performance.
Symbolically, the output *y(t)* is related to the input *u(t)* by the following equation

$$y(t)=Tu(t) \quad (1.1)$$

where T is an operator. There are three elements involved in Eq. (1.1): the input *u(t)*, the system T, and the output *y(t)*. In most engineering problems, we usually know (i.e., we are given) two of these three elements and we are asked to find the third one. As a result, the following three basic engineering problems arise:

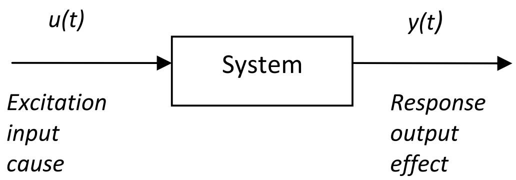

Figure 1.1 Schematic diagram of a system with its input and output.

1. The analysis problem. Here, we are given the input *u(t)* and the system T and we are asked to determine the output *y(t)*
2. The synthesis problem. Here, we are given the input *u(t)* and the output *y(t)* and we are asked to design the system T.
3. The measurement problem. Here, we are given the system T and the output *y(t)* and we are asked to measure the input *u(t)*.

Control systems can be divided into two categories: the open-loop and the closed-loop systems.

An open-loop system (Figure 1.2a) is a system whose input *u(t)* does not depend on the output *y(t)*, i.e., *u(t)* is not a function of *y(t)*.
A closed-loop system (Figure 1.2b) is a system whose input *u(t)* depends on the output *y(t)*, i.e., *u(t)* is a function of *y(t)*.

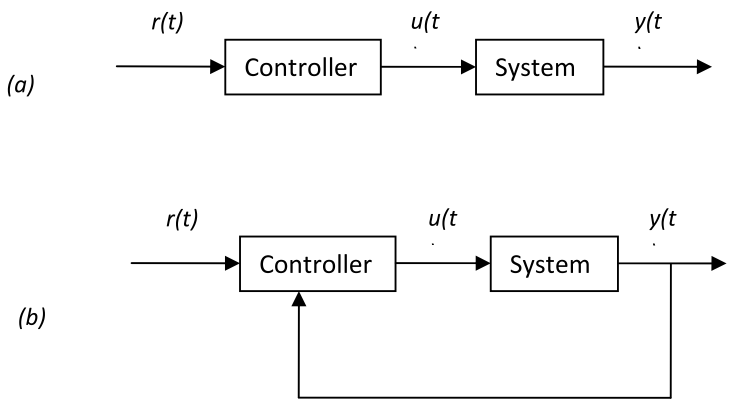

Figure 1.2 Two types of systems: (a) open-loop system; (b) closed-loop system.

In control systems, the control signal *u(t)* is not the output of a signal generator, but the output of another new additional component that is added to the system under control. This new component is called controller (and in special cases regulator or compensator). Furthermore, in control systems, the controller is excited by an external signal *r(t)*, which is called the reference or command signal. This reference signal *r(t)* specifies the desired performance (i.e., the desired output *y(t)*) of the open- or closed-loop system. That is, in control systems, we aim to design an appropriate controller such that the output *y(t)* follows the command signal *r(t)* as close as possible. In particular, in open-loop systems (Figure 1.2a) the controller is excited only by the reference signal *r(t)* and it is designed such that its output *u(t)* is the appropriate input signal to the system under control, which in turn will produce the desired output *y(t)*. In closed-loop systems (Figure 1.2b), the controller is excited not only by reference signal *r(t)* but also by the output *y(t)*. Therefore, in this case the control signal *u(t)* depends on both *r(t)* and *y(t)*. To facilitate better understanding of the operation of open-loop and closed-loop systems the following introductory examples is presented below:

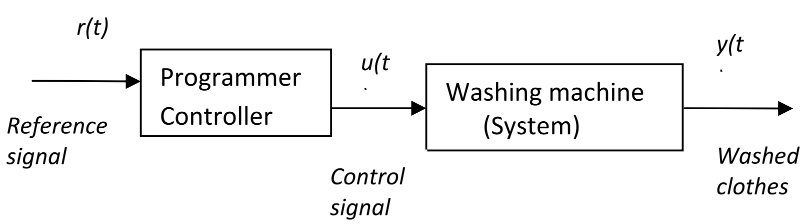

Figure 1.3 The clothes washing machine as an open-loop system.

A very simple introductory example of an open-loop system is that of the clothes washing machine (Figure 1.3). Here, the reference signal *r(t)* designates the various operating conditions that we set on the ‘‘programmer,’’ such as water temperature, duration of various washing cycles, duration of clothes wringing, etc. These operating conditions are carefully chosen so as to achieve satisfactory clothes washing.

The controller is the ‘‘programmer,’’ whose output *u(t)* is the control signal. This control signal is the input to the washing machine and forces the washing machine to execute the desired operations assigned in the reference signal *r(t)*, i.e., water heating, water changing, clothes wringing, etc. The output of the system *y(t)* is the ‘‘quality’’ of washing, i.e., how well the clothes have been washed. It is well known that during the operation of the washing machine, the output (i.e., whether the clothes are well washed or not) it not taken into consideration. The washing machine performs only a series of operations contained in *u(t)* without being influenced at all by *y(t)*. It is clear that here *u(t)* is not a function of *y(t)* and,

therefore, the washing machine is a typical example of an open-loop system. Other examples of open-loop systems are the electric stove, the alarm clock, the elevator, the traffic lights, the worldwide telephone communication system, the computer, and the Internet.
A very simple introductory example of a closed-loop system is that of the water heater (Figure 1.4). Here, the system is the water heater and the output *y(t)* is the water temperature. The reference signal *r(t)* designates the desired range of the water temperature. Let this desired temperature lie in the range from 65 to 70^{o}C. In this example, the water is heated by electric power, i.e., by a resistor that is supplied by an electric current. The controller of the system is a thermostat, which works as a switch as follows: when the temperature of the water reaches 70^{o}C, the switch opens and the electric supply is interrupted. As a result, the water temperature starts falling and when it reaches 65^{o}C, the switch closes and the electric supply is back on again. Subsequently, the water temperature rises again to 70^{o}C, the switch opens again, and so on. This procedure is continuously repeated, keeping the temperature of the water in the desired temperature range, i.e., between 65 and 70^{o}C.
A careful examination of the water heater example shows that the controller (the thermostat) provides the appropriate input *u(t)* to the water heater. Clearly, this input *u(t)* is decisively affected by the output *y(t)*, i.e., *u(t)* is a function of not only of *r(t)* but also of *y(t)*. Therefore, here we have a typical example of a closed-loop system.
Other examples of closed-loop systems are the refrigerator, the voltage control system, the liquid-level control system, the position regulator, the speed regulator, the nuclear reactor control system, the robot, and the guided aircraft. All these closed-loop systems operate by the same principles as the water heater presented above.
It is remarked that in cases where a system is not entirely automated, man may act as the controller or as part of the controller, as for example in driving, walking, and cooking. In driving, the car is the system and the system's output is the course and/or the speed of the car. The driver controls the behavior of the car and reacts accordingly: he steps on the accelerator if the car is going too slow or turns the steering wheel if he wants to go left or right. Therefore, one may argue that driving a car has the structure of a closed-loop system, where the driver is the controller.
Similar remarks hold when we walk. When we cook, we check the food in the oven and appropriately adjust the heat intensity. In this case, the cook is the controller of the closed-loop system.

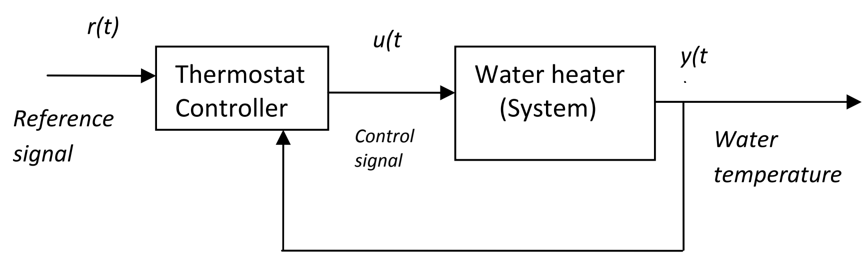

Figure 1.6 The water heater as a closed-loop system.

From the above examples it is obvious that closed-loop systems differ from open-loop systems, the difference being whether or not information concerning the system's output is fed back to the system's input. This action is called feedback and plays the most fundamental role in automatic control systems.

THE LAPLACE TRANSFORM

To study and design control systems, one relies to a great extent on a set of mathematical tools. These mathematical tools, an example of which is the Laplace transform, facilitate the engineer's work in understanding the problems he deals with as well as solving them.

For the special case of linear time-invariant continuous time systems, which is the main subject of the book, the Laplace transform is a very important mathematical tool for the study and design of such systems. The Laplace transform is a special case of the generalized integral transform presented below.

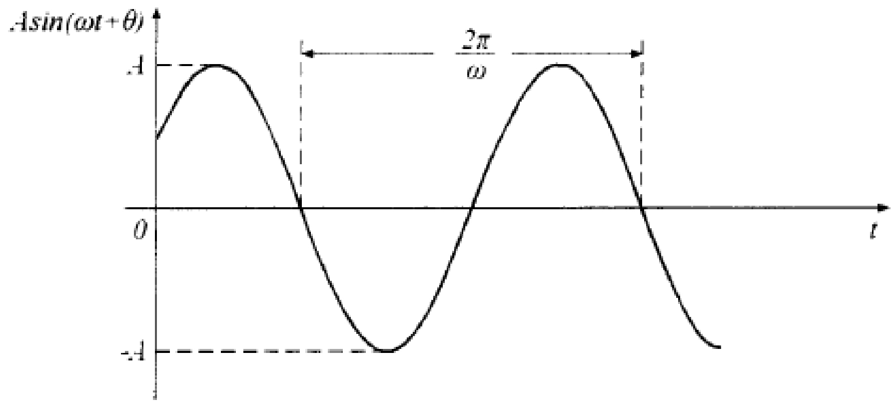

Figure 2.1 The sinusoidal function

Introduction to Laplace Transform

The Laplace transform is a linear integral transform with kernel $k(s, t) = e^{-st}$ and time interval (0,1). Therefore, the definition of the Laplace transform of a function f (t) is as follows:

$$L\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

where *L* designates the Laplace transform and *s* is the complex variable defined as $s = \sigma + j\omega$. Usually, the time function *f (t)* is written with a small *f*, while the complex variable function *F(s)* is written with a capital *F*.

For the integral to converge, *f (t)* must satisfy the condition

$$\int_0^\infty |f(t)|e^{-\sigma t}\,dt \leq M$$

Where σ and M are finite positive numbers.

Let $L\{f(t)\} = F(s)$. Then, the inverse Laplace transform of *F(s)* is also a linear integral transform, defined as follows:

$$L^{-1}\{F(s\} = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(s)e^{st}ds = f(t)$$

where L^{-1} designates the inverse Laplace transform, $j = \sqrt{-1}$, and c is a complex constant.

Properties and Theorems of the Laplace Transform

The most important properties and theorems of the Laplace transform are presented below.

1. Linearity
2. The Laplace Transform of the Derivative of a Function

3. The Laplace Transform of the Integral of a Function
4. Time Scaling
5. Shift in the Frequency Domain
6. Shift in the Time Domain
7. The Initial Value Theorem
8. The Final Value Theorem
9. Multiplication of a Function by t
10. Division of a Function by t
11. Periodic Functions

APPLICATIONS OF THE LAPLACE TRANSFORM

This section presents certain applications of the Laplace transform in the study of linear systems.

Example 1

Determine the voltage across the capacitor of the circuit shown in Figure 2.2. The switch S closes when t = 0. The initial condition for the voltage capacitor is zero, i.e. $Vc\,(0) = 0$.

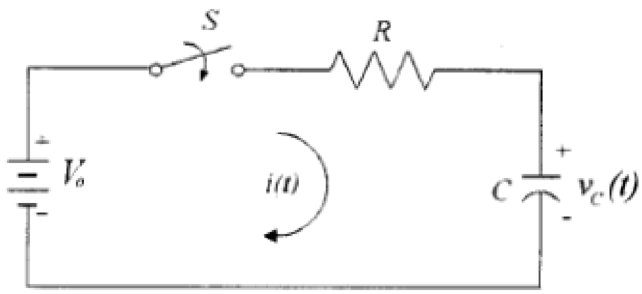

Figure 2.2 RC network

Solution

From Kirchhoff's voltage law we have

$$Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)dt = V_0$$

Applying the Laplace transform to both sides of the integral equation, we get the following algebraic equation

$$RI(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{i^{(-1)}(0)}{s}\right] = \frac{V_0}{s}$$

Where $I(s)=L\{i(t)\}$ and $i^{(-1)}(0) = \int_{-\infty}^{0} i(t)dt = Cv_c(0) = 0$. Replacing $i^{(-1)}(0) = 0$ in the above equation, we have

$I(s)\left[\frac{1}{Cs} + R\right] = \frac{V_0}{s}$ or $I(s) = \frac{v_0/R}{s+1/RC}$

The inverse laplace transform of $I(s)$ is as follows

$$i(t) = L^{(-1)}\{I(s)\} = \frac{V_0}{R}e^{-t/RC}$$

Hence, the voltage $v_c(t) = V_0 - Ri(t) = V_0 - V_0e^{-t/RC} = V_0[1 - e^{-t/RC}]$

TRANSFER FUNCTION

In contrast to the differential equation method which is a description in the time domain, the transfer function method is a description in the frequency domain and holds only for a restricted category of systems, i.e., for linear time-invariant systems having zero initial conditions. The transfer function is designated by *H(s)* and is defined as follows:

The transfer function H(s) of a linear, time-invariant system with zero initial conditions is the ratio of the Laplace transform of the output y(t) to the Laplace transform of the input u(t), i.e.,

$$H(s) = \frac{L\{y(t)\}}{L\{u(t)\}} = \frac{Y(s)}{U(s)}$$