UNIVERSITY OF AGRICULTURE, ABEOKUTA

COLLEGE OF NATURAL SCIENCES

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DEPARTMENT OF COMPUTER SCIENCE

2009/2010 FIRST SEMESTER EXAMINATION

TITLE OF THE COURSE: CSC 251-NUMERICAL ANALYSIS I
TIME: 2 HOUR
INSTRUCTION: ANSWER ANY THREE QUESTIONS

Question 1

(a) If the exact answer is A and the computed answer is $\bar{A}$, find the relative error when

(a) A = 10.147 $\bar{A}$ = 10.159

(b) A = 0.0047 $\bar{A}$ = 0.0045

(c) $A = 0.671\times10^{12}$, $\bar{A} = 0.669\times10^{12}$

(b) Use the bisection method to approximate √3 to 2 decimal places. Use $f(x) = x^2 - 3$ with $f(0) = -3$ and $f(2) = 1$ as the starting point.

Question 2

(a) Let $a = 0.471\times10^{-2}$ and $b = -0.185\times10^{-1}$. Use 3 digits floating point arithmetic to compute $a+b, a-b, a\times b$ and a/b. Find the rounding error in each case.

(b)(i) Express the numbers x = 12.74, y = 0.0025 and z = -12.55 as three digits floating point numbers.

(ii) Compute the following expression using the three floating point arithmetic.

$$\frac{x-y}{x+z}$$

(iii) Identify the rounding errors at each step of the calculation, including the representation of x, y, and z .

(iv) Calculate the total error due to rounding in the calculation.

Question 3

Sketch the cubic polynomial

$$f(x) = 4x^3 - 10x^2 + 2x + 5$$

to get a rough estimate of its roots. Use the Newton Raphson method to approximate each root to 4 decimal places.

(b) With x in radians, show graphically that there are infinitely many solutions of

$$x = \tan(x)$$

(ii) Use the Newton Raphson method to find the smallest positive roots.

Question 4

(a) Describe briefly how the Gauss Seidel method differs from the Jacobi Method.

(b) Carry out two iterations of the Jacobi method for solving the following with initial estimate $x = 0$.

$$\begin{bmatrix} 10 & -1 & -1 \\ 1 & 10 & -2 \\ 3 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

Question 5

(a) The Newton's form of Lagrange interpolation expresses

$$L_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \ldots + a_n(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

If one impose the interpolation constraints,

$$L_n(x_i) = f(x_i) \textit{ for } i = 0 : n, \textit{ then}$$

$$f(x_0) = f_0 \Rightarrow a_0 = f_0$$

$$f(x_1) = f_1 \Rightarrow a_0 + a_1(x_1 - x_0) = f_1$$

And if $a_1 = \dfrac{f_1 - f_0}{x_1 - x_0}$ **proof that**

$$a_2 = \frac{\dfrac{f_2 - f_0}{x_2 - x_0} - \dfrac{f_1 - f_0}{x_1 - x_0}}{x_2 - x_1}$$

Given

$$L_n(x_2) = f(x_2) = f_2 \Rightarrow a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$$

(b) Complete the divided difference table for a function f(x) giving the divided differences to four decimal places.

i	x_i	$f_i = f[x_i]$	$f[x, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$	$f[x_i, x_{i+1}, x_{i+2}, x_{i+3}]$
0	*0.0*	*0.633*			
			?		
1	*0.2*	*0.932*		?	
			?		?
2	*-0.4*	*0.389*		?	
			?		
3	*0.4*	*0.783*			