University of Agriculture, Abeokuta,

Department of Mathematics

2009/2010 First Semester Examination July 2010

MTS323 - Real Analysis II

INSTRUCTION: Answer Any Four Questions Time: $2\frac{1}{2}$ Hours

1. (a) Let $f : \Re^n \rightarrow \Re^m$ be defined at a point $a \in \Re^n$ as well as in some neigborhood of a, and u is a vector in $\Re^n$. Define the directional derivative of f at a in the direction of u.

   Let $f : \Re^3 \rightarrow \Re$ be defined by $f(x, y, z) = 2x^2 - y + 6xy - z^3 + 3z$. calculate the directional derivative of f at the origin in the direction of the vector $u = (1, 2, 3)$. Find $D_1 f(0, 0, 0), D_2 f(0, 0, 0)$ and $D_3 f(0, 0, 0)$.

   (b) Show that the existence of partial derivatives does not mean that it is differentiable at the same point by considering the function $f : \Re^2 \rightarrow \Re$ defined by

$$f(x, y) = \begin{cases} \frac{xy^2}{x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

2. (a) i. Obtain the Maclaurin's series of the function $f(x, y) = e^{x+y} cosy$ up to the second degree.

   ii. Find and classify the extreme value (if any) of $f(x, y) = x^2 + y^2 + x + y + xy$.

   iii. By using the method of Lagranges's multiplier, find the points on the sphere $x^2 + y^2 + z^2 = 36$ that are closest and farthest from (1,2,2).

   (b) i. When is a real valued function g defined on $\Re^n$ said to be a quadratic function?

   ii. When is a quadratic function $g : \Re^n \rightarrow \Re$ said to be positive definite, negative definite?

   iii. Verify that the origin is a critical point of the function $f(x, y) = 2x^5 y + 3xy^5 + xy$. Determine the behaviour of this critical point using the theorem on positive and negative definite.

ii. Prove that if $f'(x)$ exists and is bounded on [a,b], then f is of bounded variation on [a,b].

(b) Let

$$f(x) = xsin\frac{\pi}{x} \quad if \quad 0 < x \leq 2 \quad and \quad f(x) = 0 \quad if \quad x = 0$$

Is f of bounded variation variation on [0,2]?

(a) Let $\{f_n\}$ be a sequence of real valued functions defined on the subset D of $\Re$. When is $\{f_n\}$

i. said to converge pointwise to f on D?

ii. said to converge uniformly to f on D?

iii. said to be uniformly Cauchy on D?

iv. State the Uniform Cauchy Criterion for the sequence $\{f_n\}$

(b) Let $f_n(x) = \frac{x^n}{2+x}$ for $x \in [0,4]$.

i. Find the set $D \subset [0,4]$ for which $f(x) = lim_{n \to \infty} f_n(x)$ is defined as a real valued function.

ii. Show that if $0 < a < 1$, the convergence is uniform on [0,a].

iii. Show that the convergence is not uniform on [0,1]

For each positive integer n, define $f_n : [0,1] \to \Re$ by $f_n(x) = \frac{x}{3+nx}$ for every $x \in [0,1], n \geq 1$. Show that $\{f_n\}$ converges uniformly on [0,1].