LECTURE NOTE

NAME OF COURSE: APPLIED MECHANICS (STATICS)

COURSE CODE: MCE 201

UNITS: 2

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INTRODUCTION

The science which describes and predicts the conditions of rest or motion of bodies under the action of forces is known as **mechanics.**

Mechanics of rigid bodies is subdivided into statics and dynamics; the former dealing with bodies at rest, the latter with bodies in motion.

In this aspect of the study of mechanics, bodies are assumed to be perfectly rigid, though, actual structures and machines are not absolutely rigid, they deform under the loads to which they are subjected.

The deformations are usually small and do not appreciably affect the conditions of equilibrium or motion of the structure under consideration. They are important as far as the resistance of the structure to failure is concerned and are studied in mechanics of materials, which is a part of the mechanics of deformable bodies. The purpose of mechanics is to explain and predict physical phenomena and thus to lay the foundations for engineering applications.

DIMENSIONS AND COORDINATE IN SPACE

Dimensions

All variables can be derived from quite a small number of certain basic entities. These are called dimensions, for example, length, breadth and height. They are measured properties used to describe the physical state of the body or system. In describing dimensions, agreeable units of measurement are used. The fundamental quantities and their S. I. units and symbols are given below.

Quantity	Units	Dimension
Length	Metres	L
Mass	Kilogramme	M
Time	Second	T
Force	Newton	F
Temperature	Kelvin	O

When the thermodynamic effects are not required, the four dimensions of length, mass, time and force are sufficient to describe mechanics behaviour. The four dimensions do not exist independently but are related through Newton's second law.

The dimensions of all other variables can be expressed in terms of these fundamental dimensions. For instance velocity has the dimension of L/T as it is the ratio of distance travelled to the time taken.

Coordinates in Space

Coordinates are planes used to describe a body's position in space. **Space** is the geometric region occupied by bodies. The position of a particular body are described by linear and angular measurements relative to a coordinate system.

The dimensions of a body are important to the description of its position. For three-dimensional problems, three independent coordinates are needed. For two-dimensional problems, only two coordinates are required.

A body may be treated as a particle when its dimensions are considered to be near zero so that we may analyze it as a mass concentrated at a point.

VECTORS AND VECTOR ALGEBRA

Vectors

In general, any quantity that has direction and magnitude is a **vector** quantity. It must have a direction to be complete. The direction may be explicitly stated or may be implied in the statement of the problem.

Scalar quantities are quantities that have magnitude only. They are complete without a direction. Some measures are normally scalar. Examples include area, density, temperature and power. Other measures are normally vector. Examples include force and acceleration.

Vectors are represented by a line with an arrowhead at one end. The length of the line should be proportional to the magnitude of the quantity represented. The heading of the line and the location of the arrow head indicate direction.

Vector Algebra

The direction of the vector $\mathbf{v}$ ($\vec{v}_1$)may be measured by an angle O from some known reference direction. The negative of $\mathbf{v}$ is a vector $-\mathbf{v}$. It has the same magnitude as $\mathbf{v}$ but directed in the sense opposite to $\mathbf{v}$.

It is important for vectors to obey the parallelogram law of combination. This law states that two vectors $\mathbf{v_1}$ and $\mathbf{v_2}$, treated as free vectors may be replaced by their

equivalent vector v, which is the diagonal of the parallelogram formed by $\mathbf{v_1}$ and $\mathbf{v_2}$ as its two sided as shown below. This combination is called the vector addition and is represented by the vector equation

$$\vec{v}=\vec{v_1}+\vec{v_2}$$
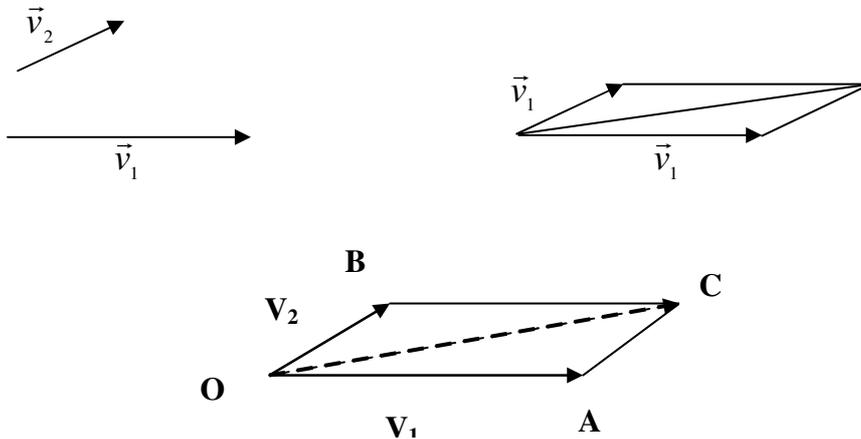
FORCES, COUPLES AND THEIR SYSTEMS

Forces

A force is defined as an action of one body on another. It is a vector quantity because its effect depends on the direction as well as on the magnitude of the action. Because force is a vector quantity, forces may be combined according to the parallelogram law of vector addition. Consider the figure below, the force vector is **F**, and magnitude F. the effect of the action of **F** depends on F, the angle Θ, and the location of the point of application A.
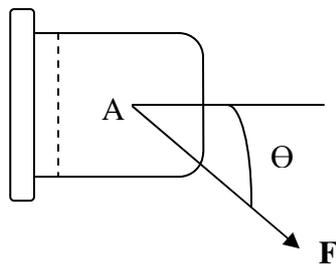
Therefore, the complete specification of the action of a force must indicate its magnitude, direction, and point of application and it must be treated as a fixed vector. Force is a vector quantity, it is important for forces to obey the parallelogram law of combination.

F_2

F_1
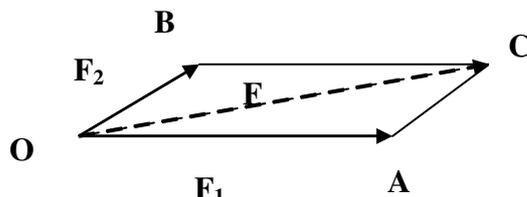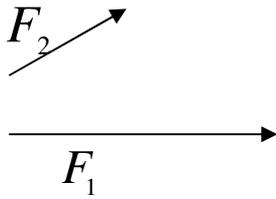
$$\mathbf{F_1 + F_2 = F}$$

**F** is the sum of the two forces and it is otherwise known as **resultant**.

Couple

When two equal unlike parallel forces are brought together no resultant can be formed instead further pairs of equal unlike parallel forces will be produced, each pair being equivalent to the original set. Such pairs of forces therefore have no resultant. A force set of this kind is termed a **couple**.

COMPOSITION, RESOLUTION, VARIGNON'S THEOREM, EQUIVALENCE AND REDUCTION OF SYSTEMS, WRENCH, RIGID BODIES AND EQUILIBRIUM
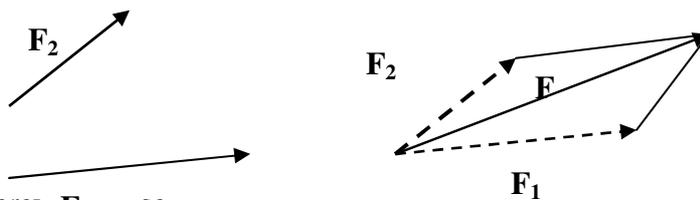
Composition and Resolution of Forces into Components

A force can be resolved into components for two-dimensional cases or three-dimensional cases. Example of 2-dimensional cases is given in the figure below. Force **F** is resolved into **F₁** and **F₂**.

F

F₁   **F**

F₂
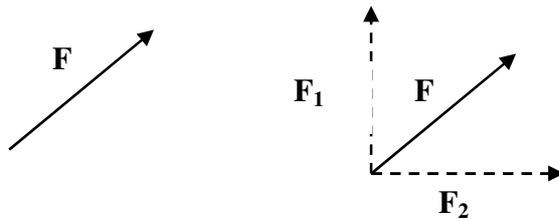
Let us consider the case of **F₁** and **F₂** considered earlier

F₂

F₂   F

F₁

In the prev **F₁**   se

$$\mathbf{F_1 + F_2 = F}$$

while in the new case

F =F₁+F₂

i.e. **F** is resolved into **F₁** and **F₂**

The most common two-dimensional resolution of a force vector is into rectangular components
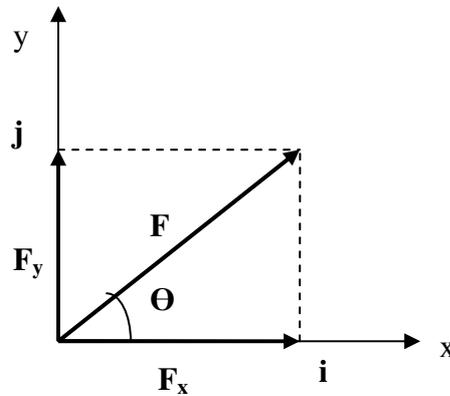
From the parallelogram rule,

F=Fₓ+F_y

Where **Fₓ** and **F_y** are vector components of **F** in the x- and y- directions.

In terms of the unit vectors i and j ,

Fₓ=Fₓi,   F_y=F_yj

Thus, **F = Fₓi+F_yj**

Where the scalars F_x and F_y are the x and y scalar components of the vector **F.**

A unit vector is given by

V=nV, n is the unit vector having a magnitude of one unit and the same direction with **V**, while V is a scalar component of **V**.

So . the scalar components can be positive or negative, depending on the quadrant into which **F** points. In the figure above, x and y scalar components are both positive.

$$F_x = F \cos\theta , \qquad F_y = F \sin\theta$$

and $\quad F = \sqrt{F_x^{\,2} + F_y^{\,2}} , \quad \theta = \tan^{-1}\left(\frac{F_y}{F_x} \right)$

Moment

A force can tend to rotate a body about an axis in addition to the tendency to move a body in the direction of its application. The axis may be any line which neither

intersects nor is parallel to the line of action of the force. This rotational effect of the force is known as **moment**. The magnitude of the moment produced is given as

$$M = Fd$$

where d is the moment arm which is the perpendicular distance from the axis to line of action of the force. The moment is a vector **M** perpendicular to the plane of the body. The sense of **M** depends on the direction in which **F** tends to rotate the body.

Varignon's Theorem

One of the most useful principles of mechanics is Varignon's theorem, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

Reduction of a system of forces to a single force

Systems of forces can be reduced to a single force. The systems of forces which can be reduced to a single force or resultant are the systems for which the force **F** and the couple vector are mutually perpendicular. This is not generally satisfied by systems of forces in space, but will be satisfied by systems consisting of

   (i) Concurrent forces

   (ii) Coplanar forces

   (iii)Parallel forces

Reduction of a system of forces to a wrench

The systems of forces for which the force F and the couple vector are not perpendicular, and neither of which is zero can not be reduced to a single force or a single couple, rather the couple vector can be replaced by two other couple vectors obtained by resolving the couple vector M_0 into a component M_1 along **F** and M_2 in a plane perpendicular to **F**. The couple vector M_2 and force **F** can then be replaced by a single force **F** acting along a new line of action. This particular force-couple system is called a wrench., and the resulting combination of push and twist is found in drilling and tapping operations and in tightening and loosening of screws.
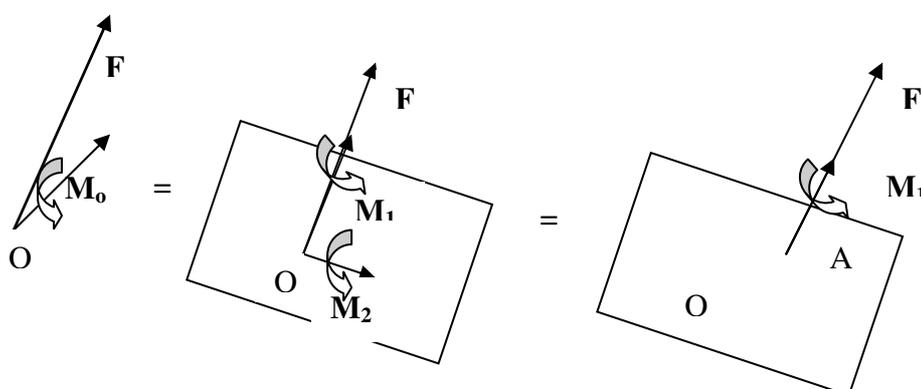
Rigid body

A rigid body is defined as one which does not deform.

Equilibrium of a body

The external forces acting on a body can be reduced to a force-couple system at some arbitrary point O. When the force and the couple are both equal to zero, the external forces form a system equivalent to zero, and the rigid body is said to be in equilibrium.

CENTRES OF GRAVITY, CENTROIDS AND THEIR APPLICATIONS

Centre of Gravity

We have assumed all along that the attraction exerted by the earth on a rigid body could be represented by a single force W. This force is called the force of gravity. In actual sense, the earth exerts a force on each particles forming the body. The action of the earth on a rigid body can thus be represented by a large number of small forces distributed over the entire body. All these small forces can be replaced by a single equivalent force, W, that is the resultant.

Centroids

STRUCTURES AND MACHINES

Structures

Frames and Machines

Frames and machines are two common types of structures which are often composed of pin-connected multiforce members, i.e., members that are subjected to more than two forces. While frames are generally stationary and are used to support loads, machines contain moving parts and are designed to transmit and alter the effect of forces.

FRICTION

When two surfaces are in contact, tangential forces, called friction forces, will always develop if one attempts to move one surface with respect to the other. On the other hand, these friction forces are limited in magnitude and will not prevent motion if sufficiently large forces are applied. The distinction between frictionless and rough surfaces is thus a matter of degree.

MOMENTS OF INERTIA

VIRTUAL WORK

Sometimes bodies are composed of interconnected members which can move relative to each other. Thus, various equilibrium configurations are possible and must be examined. Force- and moment- equilibrium equations, although valid and adequate are often not the most direct and convenient approach for problems of this type. Virtual work, a method of analyzing the equilibrium of a body on the concept of the work done by forces on the body is more direct to tackle the issue.

The Principle of Virtual Work

The principle of virtual work for a particle states that "if a particle is in equilibrium, the total virtual work of the forces acting on the particle is zero for any virtual displacement of the particle".