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MTS 103: VECTOR AND GEOMETRY

Lecture Guide on Elementary Vector Operations

Definition

Scalars and vectors: Scalar is a single real number called magnitude and is not related to any direction in space. The vector is a quantity which has a magnitude as well as a definite direction in space. The speed of a bus is a scalar quantity but the velocity is a vector quantity.

Line vectors: The vector $\overline{AB}$, A is called the origin and B the terminus. The magnitude of the vector is given by the length AB and its direction is from A to B. These vectors are called line vectors.

Equal vectors: Two vectors are said to be equal when they have the] same length (magnitude) and are parallel having the same sense of direction. The equality of two vectors is written as $\bar{a} = \bar{b}$.

Zero vectors: If the origin and terminal points of a vector are same, then it is said to be a zero vector. Evidently its length is zero and its direction is indeterminate.

Unit vector: A vector is said to be a unit vector if its magnitude be of unit length.

Position vector: The position vector of any point P, with reference to an origin O is the vector $\overline{OP}$. Thus taking O as origin we can find the position vector of every point in space. Conversely, corresponding to any given vector $\bar{r}$ there is a point P such that $\overline{OP} = \bar{r}$

Addition of two vectors: Let $\bar{a}$ and $\bar{b}$ be two vectors with respect to the origin O. The sum of these two vectors is given by $\bar{a} + \bar{b}$.

The unit vectors $\bar{i}, \ \bar{j}, \ \bar{k}$ **:** The vectors $\bar{i}, \ \bar{j}, \ \bar{k}$ have unit magnitude and they lie on the *x*, *y* and *z* axes respectively. We can expresses any vector in terms of these three unit vectors $\bar{i}, \ \bar{j}, \ \bar{k}$ **.**

Collinear vectors: Two vectors $\bar{a}$ and $\bar{b}$ are said to be collinear if $\bar{a} = \lambda \bar{b}$, for some scalar λ, i.e., two vectors are collinear if the coefficient of $\bar{i}, \ \bar{j}$ and $\bar{k}$ are proportional.

Magnitude of a vector: Let $\bar{a} = a_1 i + a_2 j + a_3 k$. Then the magnitude or length of the vector $\bar{a}$ is denoted by $|\bar{a}|$ or a and is defined as $|\bar{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Distance between two points: Let P_1 and P_2 be two points whose position vectors are respectively $\bar{a} = a_1 i + a_2 j + a_3 k$ and $\bar{b} = b_1 i + b_2 j + b_3 k$. Then the vector $\overline{P_1 P_2}$ = position vector of P_2 - position vector of $P_1 = (b_1 i + b_2 j + b_3 k) - (a_1 i + a_2 j + a_3 k)$

Then the distance between two points P_1 and P_2 is the magnitude of the vector $\overline{P_1 P_2}$.

*** The unit vectors in the direction of a vector $\bar{a}$ are given as $\pm \dfrac{\bar{a}}{|\bar{a}|}$.

Examples:

1. Find the value of *q* if $\bar{a} = 2i + 5j + qk$. If the magnitude of $\bar{a}$ is 9

2. Given vectors $\bar{a} = 5i + j + 3k$, $\bar{b} = i - 3j + 4k$ and $\bar{c} = 7i + 2j - 3k$. find the unit vector in the direction of $\bar{a} - \bar{b} + 2\bar{c}$.

3. Find the distance between A and B whose-position vectors are $\bar{a} = 5i + j + 3k$, $\bar{b} = i - 3j + 4k$ respectively.

4. Prove by vector method that the three points *A (2*, 3, 4), *B (1*, 2, 3) and *C (4*, 2, 3) form a right-angled triangle.

5. Show that the three points *-3i – 6j + 21k, 9i +3k and 15i + 3j – 6k* are collinear.

Scalar Product or Dot Product

The scalar or dot product between vectors $\overline{a}$ and $\overline{b}$ is denoted by $\overline{a} \bullet \overline{b}$ and defined as

$\overline{a} \bullet \overline{b} = |\overline{a}||\overline{b}|\cos\theta$, where θ is the angle between $\overline{a}$ and $\overline{b}$. The value of $\overline{a} \bullet \overline{b}$ is a scalar quantity.

*** Two vectors $\overline{a}$ and $\overline{b}$ are perpendicular if and only if $\overline{a} \bullet \overline{b} = 0$

Examples:

1. Given vectors $\overline{a} = 5i + j + 3k$, $\overline{b} = i - 3j + 4k$ and $\overline{c} = 7i + 2j - 3k$.

Find (i) $\overline{a} \bullet \overline{b}$ (ii) $(\overline{a} \bullet \overline{b}) \bullet \overline{c}$ (iii) angle between $\overline{a} - \overline{c} + 2\overline{b}$ and $\overline{a} + \overline{b} + \overline{c}$

2. Find the value of q for which the two vectors are $\overline{a} = 5i + qj + 3k$ and $\overline{b} = i - 3j + 4k$ are perpendicular to each other.

Vector Product or Cross Product

The vector product or cross product between two vectors $\overline{a}$ and $\overline{b}$ is denoted by $\overline{a} \times \overline{b}$ and is defined by $\overline{a} \times \overline{b} = |\overline{a}||\overline{b}|\sin\theta$, where θ is the angle between $\overline{a}$ and $\overline{b}$.

NOTE:

(i) $\overline{a} \times \overline{b} = -\overline{b} \times \overline{a}$

(ii) If $\overline{a} \times \overline{b} = 0$, then $\overline{a}$ and $\overline{b}$ are parallel or collinear

(iii) $\overline{a} \times \overline{a} = 0$

(iv) If $\overline{a}$ and $\overline{b}$ represent the adjacent sides of a parallelogram then its area is $|\overline{a} \times \overline{b}|$.

(v) If $\overline{a}$ and $\overline{b}$ represent any two sides of a triangle then its area is $\frac{1}{2}|\overline{a} \times \overline{b}|$

(vi) If $\overline{a} \times \overline{b} = \overline{c}$ then $\overline{c}$ is perpendicular to both $\overline{a}$ and $\overline{b}$.

(vii) $\overline{a} \times (\overline{b} \times \overline{c}) = (\overline{a} \bullet \overline{c})\overline{b} - (\overline{a} \bullet \overline{b})\overline{c}$

(viii) Three vectors $\overline{a}$, $\overline{b}$ and $\overline{c}$ are said to be coplanar, if $\overline{a} \bullet (\overline{b} \times \overline{c}) = 0$.

Examples

1. If $\overline{a} = 5i + j + 3k$ and $\overline{b} = i - 3j + 4k$. Find $\overline{a} \times \overline{b}$

2. If $\overline{a}$ and $\overline{b}$ are two vectors such that $|\overline{a}| = 16$, $|\overline{b}| = 12$ and $\overline{a} \bullet \overline{b} = 0$ Find $|\overline{a} \times \overline{b}|$.

3. Find the area of the triangle two of whose sides are given by the vectors $\overline{a} = 5i + j + 3k$ and $\overline{b} = i - 3j + 4k$.

4. Find the area of the parallelogram formed by two vectors $\overline{a} = 5i + j + 3k$ and $\overline{b} = i - 3j + 4k$.

5. Find the unit vector perpendicular to each of the vectors $\overline{a} = 5i + j + 3k$ and $\overline{b} = i - 3j + 4k$.

6. Find a vector of magnitude 9 perpendicular to both the vectors $\overline{a} = 5i + j + 3k$ and $\overline{b} = i - 3j + 4k$.

7. Show 'that the vectors $\overline{a} = 4i + 2j + k$, $\overline{b} = 2i - j + 3k$ and $\overline{c} = 8i + 7k$ are coplanar.

8. If a force given by $\overline{F} = 5i + j + 3k$ displaces a particle from the position B to C whose position vectors are $\overline{b} = i - 3j + 4k$ and $\overline{c} = 7i + 2j - 3k$ respectively. Find the work done by the force.

- ***B. I OLAJUWON, 2011***