UNIVERSITY OF AGRICULTURE, ABEOKUTA (UNAAB)

DEPARTMENT OF PHYSICS

STUDY MATERIAL

PHS451 NUCLEAR PHYSICS

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2011

NUCLEAR PHYSICS

PREFACE

This study material was originally written for students learning physics in the Open and Distance Learning Education Programme, so a great deal of efforts is made for the material to be interactive and self-instructional. But it will also be useful as teaching/learning material in the normal face-to-face learning mode. The content is an appropriate introductory course for physics major or for other areas of nuclear science and technology.

The overall aim of this course is to understand the properties of the nucleus, particularly in terms of the types of the constituents of the nucleus and types of interactions among the constituents. The interactions within the nucleus cannot be understood with the laws of classical physics. Therefore, as a student, you will be expected to possess an introductory knowledge of quantum physics. Apart from this, knowledge of differential equations would be sufficient to follow most of the concepts presented in this material.

Some of the information in this material could also be found, either in greater depth or with more general treatments, in other textbooks. At the end of each chapter a list of such textbooks is provided for further readings.

TABLE OF CONTENT

GENERAL INTRODUCTION

At this stage you have studied various modules of physics and come across various laws of physics; those that deal with phenomena involving large objects, such as planets, and those that deal with small objects, e.g., atoms. In this course, emphasis will be on the nucleus. The nucleus is the core of the atom; it contains almost the entire mass of the atom. But it occupies a very tiny component of the atom; its dimension is about 10^{-14}m in comparison to 10^{-10}m of the atom (see Figure 1.1). Nuclear Physics is the study of this complex system.

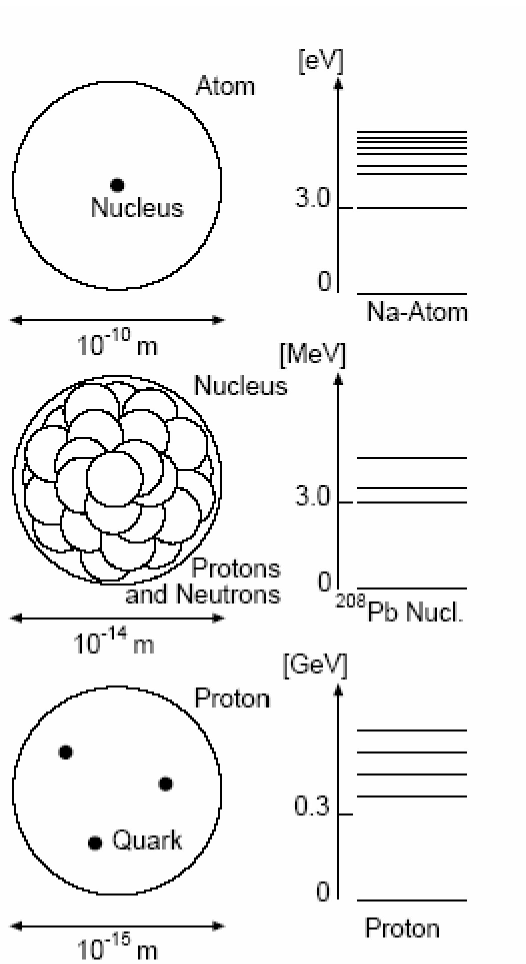


Figure 1.1 A comparison of atomic, nuclear and sub-nuclear dimensions (Adapted from Kaiser (2004)).

The purpose of this study material is to explain all aspects of the nucleus, its structure, its behaviour under various conditions, and its effect on nature and mankind.

Course Objectives

1. Explain some of the nuclear terminologies, including Isotope, Isobar, Atomic mass, Mass number, etc.
2. Describe the methods of evaluating the nuclear radius
3. Explain the inter-relationship between nuclear binding energy and nuclear stability
4. Distinguish between nuclear force and the other forces of nature; namely the gravitational and electromagnetic forces
5. Enumerate the importance of nuclear models and describe the successes and failures of the different models
6. Define Radioactivity and differentiate between nuclear decay and nuclear fission
7. Discuss some of the practical applications of nuclear physics in industries, medicine, etc.
8. List the unique properties of the major elementary particles.

Lecture 1: Properties of the nucleus

Introduction

In this lecture we will study some of the most basic properties of the nucleus; its mass, size, shape, and other externally observable properties. We will also familiarize with some of the nomenclature and terminologies that are commonly used in nuclear physics. Finally, you will be introduced to the units and dimensions that are peculiar to nuclear system, and we will describe how to convert from the nuclear units to the S.I. unit and vice versa.

Objectives

At the end of this lecture you should be able to

1. Explain nuclear terminologies such as ***nucleon, mass number, atomic mass, isotopic mass, nuclear binding energy, atomic mass unit, etc.***
2. State the general properties of stable nuclides
3. Describe at least one experimental setup for determining the density distribution of the nucleus and hence the radius of the nucleus
4. Convert from Atomic Mass Unit to kg and vice versa
5. Calculate the binding energy for a specified nucleus

1.1. Constituents of the Nucleus

We will start the discussion by reviewing some fundamental facts that you are probably already familiar with. The nucleus is made up of NEUTRONS and PROTONS. A proton is about 1840 times more massive than the electron, and the neutron is slightly heavier than the proton. Protons and neutrons are referred to as ***nucleons***.

ATOMIC AND MASS NUMBERS

The number of protons in the nucleus is the ***Atomic Number***, denoted by $\boldsymbol{Z}$. The total number of nucleons is called ***Mass Number*** and denoted by $\boldsymbol{A}$. Therefore the number of neutrons is $\boldsymbol{A-Z}$.

ISOTOPE

The atoms which make up a chemically pure substance of an element do not all have the same mass. For example, when we analyze samples of many naturally occurring elements, we find they contain different atoms all having the same atomic number $\boldsymbol{Z}$ but different mass number $\boldsymbol{A}$, i.e., different neutron numbers. Such atoms are called isotopes.

For example, the element chlorine has two isotopes; ^{35}Cl, ^{37}Cl that are stable against radioactive decay, and many that are artificially produced in nuclear reactions.

NUCLIDE

Nuclide is a specific combination of numbers of protons and neutrons. Generally a nuclide is represented by the symbol $^{A}_{Z}X_{N}$, where ***X*** is the chemical symbol of the element and ***N*** (= ***A-Z***) is the number of neutrons. Examples of nuclides are $^{1}_{1}H_{0}$, $^{238}_{92}U_{146}$, $^{56}_{26}Fe_{30}$, etc. Sometimes it is cumbersome and unnecessary to indicate the ***N*** and ***Z***, i.e. it may be sufficient to write ^{1}H, ^{238}U, and ^{56}Fe.

ISOTONE

Nuclides with the same ***N*** but different ***Z*** are called ***isotones***. For example, ^{2}H and ^{3}He are isotones with $N = 1$.

ISOBAR

Isobars are nuclides with the same mass number, e.g., ^{3}He and ^{3}H.

Activity

Give FIVE examples of (i) Nuclides; (ii) Isotopes, (iii) Isotones, and (iv) Isobars.

ISOMER

Nuclei can have the same ***A***, ***N***, and ***Z***, but different internal energies. Such nuclei are said to be in different states. Nucleus having the lowest internal energy is said to be in ***ground state***, while nuclei with higher internal energies are said to be in ***excited states***. Excited nuclei are unstable and they transit to lower excited (more stable) states by emitting high-energy photons (gamma rays). A nucleus X in an excited state is denoted by X*. Generally, nuclei do not exist in excited states for any appreciable time. However, some excited nuclei have lifetime of several hours. Such a long-lived excited state is known as an ***isomer***.

THE ATOMIC NUCLEUS (RUTHERFORDS SCATTERING EXPERIMENT)

Rutherford (1911) bombarded thin foils with alpha particles, which he had previously shown to be doubly ionized helium atoms, and observed the angles of deflection due to the scattering in the foils. He attributed the effective deflecting force to the Coulomb repulsion $2Ze^2/r^2$, where Z is the atomic number of the scattering element. Furthermore, some of the scattering angles indicated that the alpha particle had passed within a distance of about 10^{-14} m from the centre of an atom. This showed that they had passed right through the atom, the radius of which is about 10^{-10} m. On the other hand, for almost head-on collisions, for which the distance of the closest approach was even less than 10^{-14} m, deviations from the Coulomb law were observed. Hence the nucleus must have a finite size. Also, the nucleus had to contain almost the whole mass of the atom, since otherwise the alpha particles would have been deflected by the outer regions of the atom. Lastly, the whole atom was neutral, and since the nucleus had a charge +Ze, it had to be surrounded by Z electrons which circled around the nucleus in radii of the order of 10^{-10} m. The emptiness of the atom is almost unimaginable. The atom is often compared to the planetary system. But if the nucleus were enlarged to the size and mass of the sun, then the mass of an electron would be that of the earth and its distance from the nucleus ten times greater than that of the farthest planet from the sun. The variations in density and distance are thus vastly greater in the atom than in the solar system .

Excerpt from Elton L.R.B. (1958)

ATOMIC MASS UNIT (*u*)

Atomic mass unit is $(1/12)^{th}$ of the mass of the carbon (^{12}C) atom. $1u = 931.481\ MeV/c^2$ $= 1.66042 \times 10^{-27}\ kg$.

Relationship between a.m.u and MeV

.
By definition, the ^{12}C atom has a mass of exactly 12 a.m.u. Since its gram atomic weight is 12 g, it follows that 1 a.m.u. = $1/(6.02\times10^{23}) = 1.66\times10^{-24}$ g = 1.66×10^{-27} kg.
Using the Einstein relation and $c = 3\times10^{8}$ ms^{-1}, we obtain
1 a.m.u. = $(1.66\times10^{-27})(3\times10^{8})^{2}$
= 1.49×10^{-10} J = 1.49×10^{-10} J /(1.6×10^{-13}) J MeV^{-1} = 931 MeV.
More precisely, 1 a.m.u. = 931.49 MeV.

ISOTOPIC MASS
The average mass of all the isotopes of an element considering their relative abundance. For example, the two isotopes of Chlorine are present in the approximate ratio of three atoms of $^{35}Cl_{17}$ to one of $^{37}Cl_{17}$. Therefore the (average) isotopic mass of Chlorine is 35.5 u.

ATOMIC MASS (M)
This is the *exact* mass of an isotope (comprising A nucleons and A-Z electrons) as would be measured experimentally, e.g. with a mass spectrograph.

Example
Chlorine is found to have two naturally occurring isotopes: $^{35}Cl_{17}$, which is 76% abundant, and $^{37}Cl_{17}$, which is 24% abundant. The atomic weights of the two isotopes are 34.97 and 36.97. Show that this isotopic composition accounts for the observed atomic weight of the element.

Solution
Taking the weighted average of the atomic weights of the two isotopes, we find for the atomic weight of Cl, $0.76 \times 34.97 + 0.24 \times 36.97 = 35.45$, as observed.

1.2. Binding Energy

MASS EXCESS
Mass excess (Δm_e) is the difference between the exact mass M (for a neutral atom) and its mass number A:

$$\Delta m_e = M - A \qquad 1.1$$

Table 1.1 Examples of mass number, atomic mass and mass excess in atomic mass units (u)

Nuclide	M	A	Δm_e
^{1}H	1.007825	1	0.007825
^{4}He	4.002603	4	0.002603
^{16}O	15.994915	16	-0.005505
^{35}Cl	34.968851	35	-0.031149
^{120}Sn	119.902198	120	-0.097802

Note that Δm_e is zero for ^{12}C atom.

MASS DEFFECT AND BINDING ENERGY

The particles that constitute a stable nucleus are held together by strong attractive forces, and therefore work must be done in separating them from each other until they are at 'large distances' apart. In other words, energy must be supplied to the nucleus to separate it into its individual constituents. It also means that the total energy of the constituents when separated by 'large distances' is greater than the energy of the nucleus formed. This apparent loss in energy as a nucleus is formed is called the binding energy of the nucleus, and it is given in terms of the mass-energy relation of the special theory of relativity,

$$E = \Delta m_d c^2 \qquad 1.2$$

where E is the binding energy of the nucleus, Δm_d is the mass defect and c is the velocity of light in vacuum. Δm_d is the difference between the mass of the nucleus M and sum of the masses of all the constituents of the nucleus.

$$\Delta m_d = \{Zm_H + (A-Z)m_N\} - M \qquad 1.3$$

Example

Find the binding energy of the nuclide $^{24}Na_{11}$. The masses in a.m.u. of the constituents are 1.0073, 1.0087, 0.000555, and 23.991 a.m.u. for proton, neutron, electron and the nuclide $^{24}Na_{11}$, respectively.

Solution

One can work in terms of either a.m.u. or MeV. The atom consists of 11 protons, 13 neutrons, and 11 electrons. Therefore the binding energy is:

BE = 11(1.0073) + 13(1.0087) + 11(0.00055) -23.991 a.m.u. = 24.199 – 23.991 = 0.208 a.m.u. = 194 MeV.

BINDING ENERGY PER NUCLEON

If we divide the binding energy of a nucleus by the number of protons and neutrons (number of nucleons), we get the **binding energy per nucleon**. This is the common term used to describe nuclear reactions because atomic numbers vary and total binding energy would be a relative term dependent upon that. The following figure, called the binding energy curve, shows a plot of nuclear binding energy as a function of mass number.

The peak is at iron (Fe) with mass number equal to 56. The eventual dropping of the binding energy curve at high mass numbers tells us that nucleons are more tightly bound when they are assembled into two middle-mass nuclides rather than into a single high-mass nuclide. In other words, energy can be released by the nuclear fission, or splitting, of a single massive nucleus into two smaller fragments.

The rising of the binding energy curve at low mass numbers, on the other hand, tells us that energy will be released if two nuclides of small mass number combine to form a single middle-mass nuclide. This process is called nuclear fusion.

Another striking feature of the B/A curve is the approximate constancy at ~ 8 Mev per nucleon, except for the very light nuclei. It is instructive to see what this behavior implies. If the binding energy of a pair of nucleons is a constant, say C, then for a nucleus with A nucleons, in which there are A(A-1)/2 distinct pairs of nucleons, the B/A would be ~ C(A-1)/2. Since this is not what one sees in the figure of B/A vs A, one can surmise that a given nucleon is not bound equally to all the other nucleons; in other words, nuclear forces, being short-ranged, extend over only a few neighbors. The constancy of B/A implies a saturation effect in nuclear forces, the interaction energy of a nucleon does not increase any further once it has acquired a certain number of neighbors. This number seems to be about 4 or 5.

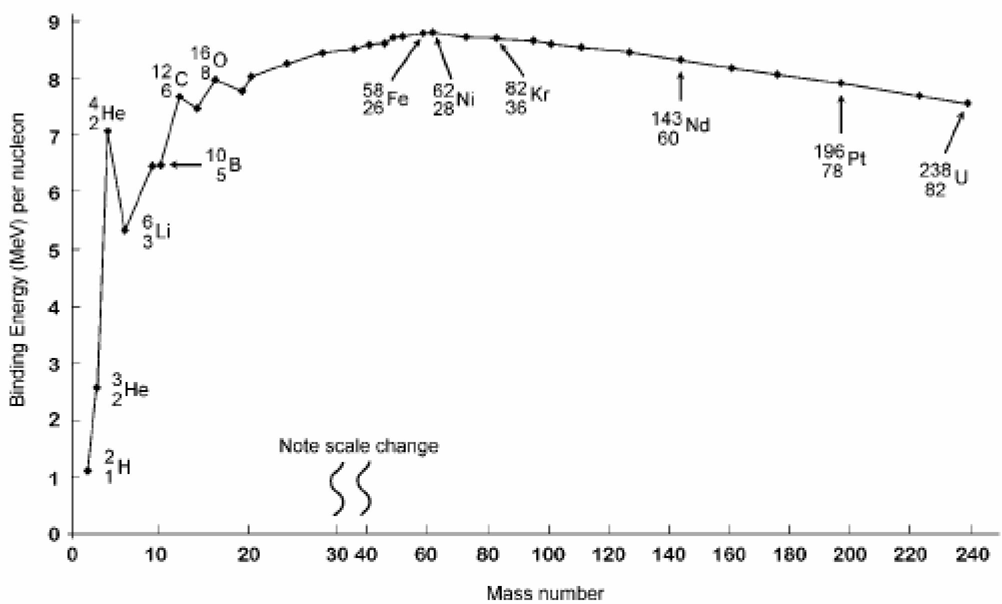


Figure 1.2. Binding energy per nucleon versus number of nucleon (Adapted from Kaiser, 2004)

One can understand the initial rapid increase of B/A for the very light nuclei as the result of the competition between volume effects, which make B increase with A like A, and surface effects, which make B decrease (in the sense of a correction) with A like $A^{2/3}$. The latter should be less important as A becomes large, hence B/A increases (see the discussion of the semi-empiricial mass formula and the liquid drop model in chapter three). At the other end of the curve, the gradual decrease of B/A for A > 100 can be understood as the effect of Coulomb repulsion which becomes more important as the number of protons in the nucleus increases.

As a quick application of the B/A curve we make a rough estimate of the energies release in fission and fusion reactions. Suppose we have symmetric fission of a nucleus with A ~ 240 producing two fragments, each A/2. The reaction gives a final state with B/A of about 8.5 Mev, which is about 1 Mev greater than the B/A of the initial state. Thus the energy released per fission reaction is about 240 Mev. (A more accurate estimate gives 200 Mev.) For fusion reaction we take $H^2 + H^2 \rightarrow He^4$. The B/A values of H^2 and He^4 are 1.1 and 7.1 Mev/nucleon respectively. The gain in B/A is 6 Mev/nucleon, so the energy released per fusion event is ~ 24 Mev.

Separation Energy

Recall the definition of binding energy involves an initial state where all the nucleons are removed far from each other. One can define another binding energy where the initial state is one where only one nucleon is separated off. The energy required to separate particle *a* from a nucleus is called the separation energy S_a. This is also the energy released, or energy available for reaction, when particle *a* is captured.

This concept is usually applied to a neutron, proton, deuteron, or α -particle. The energy balance in general is

$$S_a = \left[M_a(A',Z') + M(A-A',Z-Z') - M(A,Z)\right]c^2 \qquad 1.4$$

where particle *a* is treated as a 'nucleus' with atomic number Z' and mass number A'.

For a neutron,

$$S_n = \left[M_n + M(A-1,Z) - M(A,Z)\right]c^2 \qquad 1.5$$

$$= B(A,Z) - B(A-1,Z) \qquad 1.6$$

S_n is sometimes called the binding energy of the *last neutron*. Clearly S_n will vary from one nucleus to another. In the range of A where B/A is roughly constant we can estimate from the B/A curve that $S_n \sim S_p \sim 8$ Mev. This is a rough figure, for the heavy nuclei S_n is more like 5 – 6 Mev. It turns out that when a nucleus M(A-1,Z) absorbs a neutron, there is ~ 1 Mev (or more, can be up to 4 Mev) difference between the neutron absorbed being an even neutron or an odd neutron (see Figure 1.2). This difference is the reason that U^{235} can undergo fission with thermal neutrons, whereas U^{238} can fission only with fast neutrons (E > 1 Mev).

Generally speaking the following systematic behavior is observed in neutron and proton separation energies,

$S_n(\text{even } N) > S_n(\text{odd } N)$ for a given Z

$S_p(\text{even } Z) > S_p(\text{odd } Z)$ for a given N

This effect is attributed to the pairing property of nuclear forces – the existence of extra binding between pair of identical nucleons in the same state which have total angular momenta pointing in opposite directions. This is also the reason for the exceptional stability of the α -particle. Because of pairing the even-even (even Z, even N) nuclei are more stable than the even-odd and odd-even nuclei, which in turn are more stable then the odd-odd nuclei.

1.3. Nuclear Stability

One can construct a stability chart by plotting the neutron number N versus the atomic number Z of all the stable nuclides. The results, shown in Figure 1.3, show that N ~ Z for low A, but N > Z at high A.

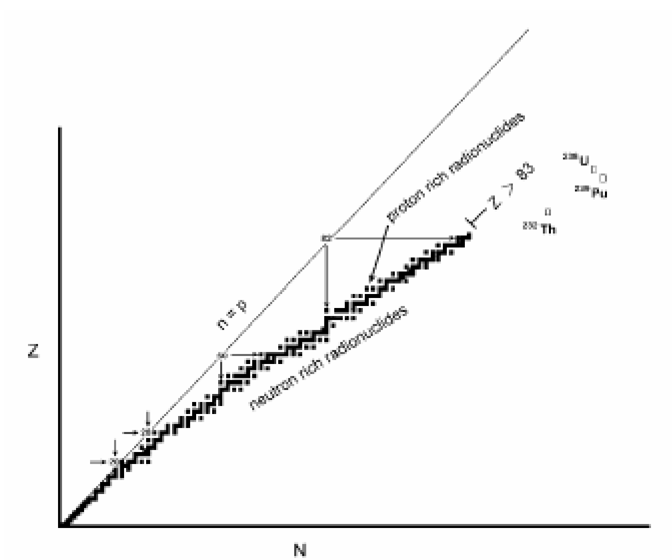


Figure 1.3. Plot of Z versus N for illustrating nuclear stability (adapted from Martin, 2006)

Binding energy is also a measure of the stability; a nucleus is stable against break-up if its mass is less than the combined mass of the fragments. For example, if we imagine that $^6Li_3$ can split up into $^2H_1$ + $^4He_2$. The masses involved are ^{6}Li – 6.0170, ^{2}H – 2.0147, and

^4He – 4.0039, respectively. The mass defect of ^6Li against this reaction is 0.0016, hence ^6Li is stable.

Another way to summarize the trend of stable nuclides is shown in the table below.

Table 1.1: Systematics of stability trends in nuclei (Adapted from Meyerhof)

A	Z	N	Type	Alternative Designation	Number of Stable + Long lived Nuclides	Degree of Stability	Usual number of isotopes/ elements
Even	Even	Even	e-e	Even mass Even N	166 + 11 =177	Very Pronounced	Several (2 and 3)
Odd	Even	Odd	e-o	Odd mass, Odd N	55 + 3 = 58	Fair	1
Odd	Odd	Even	o-e	Odd mass, Even N	51 + 3 = 54	Fair	1
Even	Odd	Odd	o-o	Even mass Odd N	6 + 4 = 10	Low	0
					278 + 21 = 299		

Also, one can readily understand that in heavy nuclei the Coulomb repulsion will favor a neutron-proton distribution with more neutrons than protons. It is a little more involved to explain why there should be an equal distribution for the light nuclides (see the following discussion on the semi-empirical mass formula). We will simply note that to have more neutrons than protons means that the nucleus has to be in a higher energy state, and is therefore less stable. This symmetry effect is most pronounced at low A and becomes less important at high A.

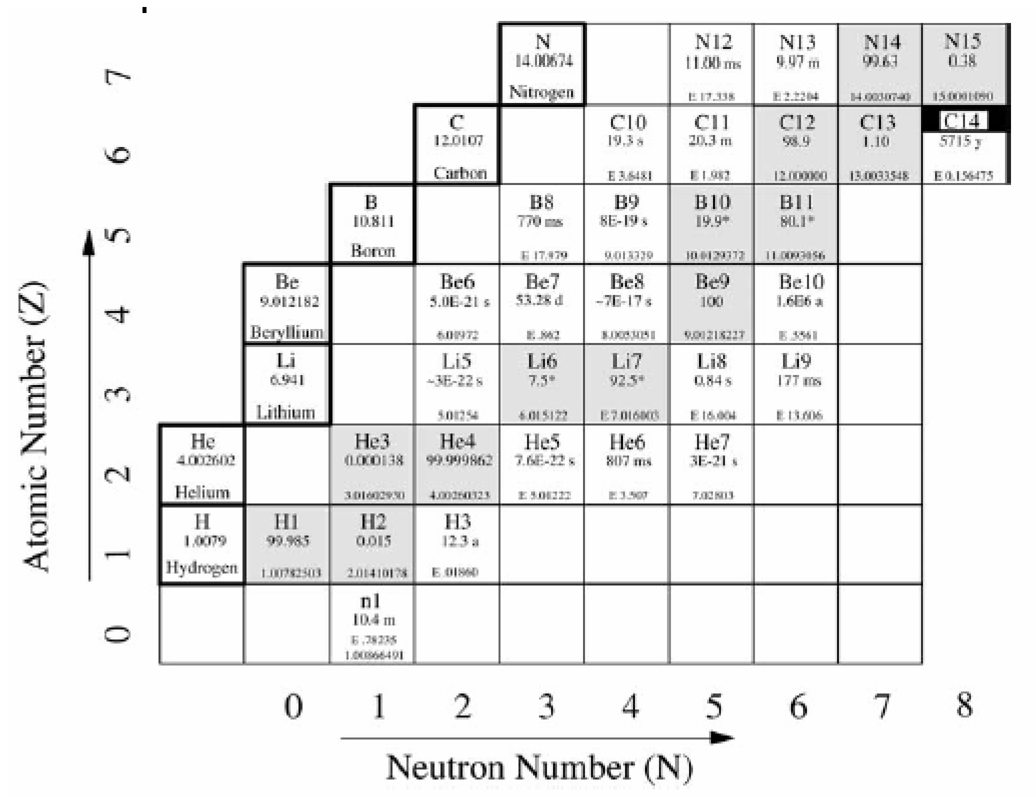


Figure 1.4 Part of nuclear chart (Source: Chart of the nuclides, Nuclides and Isotopes, 15th edn, General Electric Co., 1996.)

1.4. Nuclear Size and Nucleon Distribution

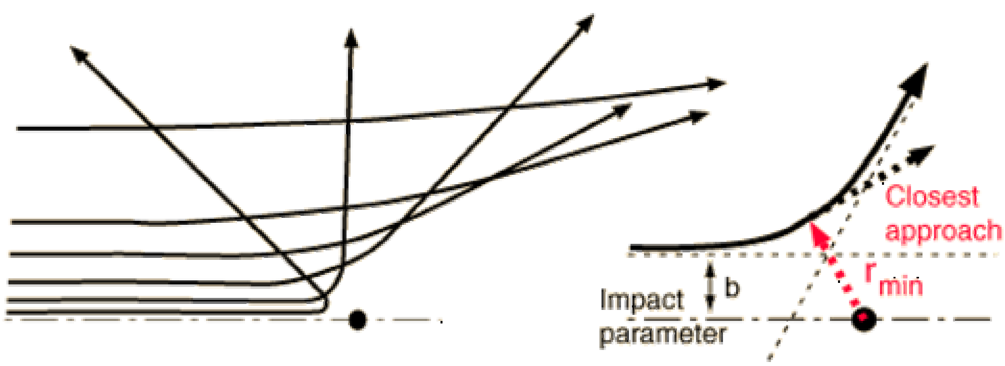
The radius of a nucleus is not a precisely defined quantity; the nucleus is not like a solid sphere of abrupt boundary. The straightforward approach to study the size and shape of nuclei is to shoot probing particles at them and measure the effects produced. There is, however, one well-known limitation in this, i.e. the wavelength of the probing particles must be of the order of the size of the nuclei being studied or smaller. Since ordinary light, for example, has a wavelength of about 10^{-7} m, which is many orders of magnitude larger than the nuclear size, it is not suitable. Light of very short wavelength, i.e., gamma rays, is also unsuitable because electrons surround nuclei and electromagnetic waves interact more strongly with these electrons than with the nucleus. It is therefore better to employ particles such as electrons, protons, neutrons, and alpha particles as probes. Neutrons and protons have the advantage that their wavelength is sufficiently short for energies of about 20 MeV, whereas for electrons over 100 MeV of energy is required, which is more difficult to obtain. However, electrons have the advantage that their interaction with the nucleus is very well known (it is the familiar electromagnetic interaction), so the most accurate results have been obtained with electrons as probes.

But before we investigate electron scattering, let us review the Rutherford's alpha scattering experiment as a means of estimating the nuclear size.

> **NOTE**
>
> In terms of dimension, the nucleus is a very tiny part of the atom; the atomic radius is about 10,000 times the nuclear radius. An approximate analogy to the relation between the nucleus and an atom could be like a coin in a football field.

Estimation of the Nuclear Size based on Rutherford's alpha scattering

Rutherford's theory of the scattering of alpha particles gives some idea about the size of the atomic nucleus. Suppose the gold nuclei in the Rutherford's scattering experiment are spherical and have radius R. The scattering of the alpha particle by the central repulsive Coulomb force leads to a hyperbolic trajectory. An alpha particle trajectory can be specified by its ***impact parameter, b***; this is the distance between the trajectory of the incident alpha particle (if it were undeflected) and the parallel line passing through the center of the gold nucleus, i.e., the perpendicular distance between any two such parallel lines (See figure 1.5 below).



(a) (b)

Figure 1.5. (a) Possible alpha particles scattering angles at different impact parameters, (b) Relation between impact parameter and distance of closest approach.

From the scattering angle and momentum, one can calculate the impact parameter and closest approach to the target nucleus.

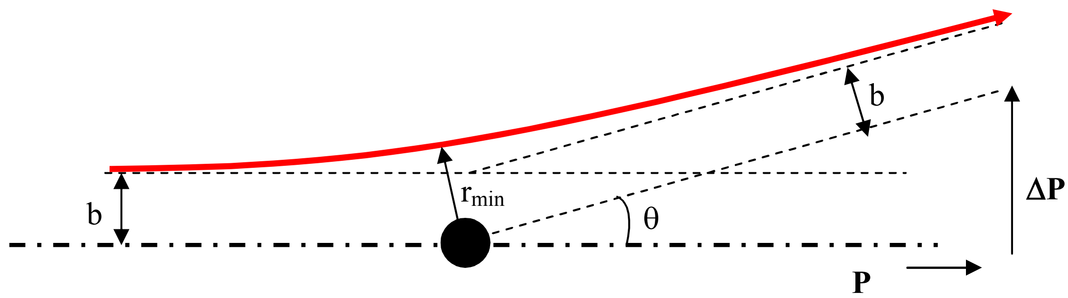


Figure 1.6. Change in angular momentum P after scattering through angle θ.

At an arbitrary position $r >> R$, the deflection is small because the alpha particle experiences less Coulombic repulsive force;

$$\bar{F} = \frac{2Ze^2}{4\pi\varepsilon_0 r^3}\bar{r} \qquad 1.7$$

Where 2e and Ze are the charges on alpha and the nucleus, respectively. (Note that the exact expression should consider the motion of the nucleus by introducing the reduced mass of the alpha – nucleus system).

But the repulsion and the scattering angle increase as the trajectory of the alpha particles gets closer to the periphery of the nucleus, and they are maximum for $r \sim R$, i.e. when the alpha particle just grazes the nucleus edge:

F slows down the alpha particle as it approaches the nucleus and accelerates it as it recedes away from the nucleus. Therefore, F is maximum at the point of closest approach and it is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{r_{\min}^{\ 2}} \qquad 1.8$$

If the alpha particle has a velocity v, the time within which it experiences this force is

$$\Delta t = \frac{r_{\min}}{v} \qquad 1.9$$

The momentum change ΔP produced (perpendicular to the direction of incidence) is obtained in terms of Newton's law and impulse:

$$\Delta P = F\Delta t = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{r_{\min}{}^2}\frac{r_{\min}}{v} \qquad 1.10$$

The scattering angle θ is given by; $\theta \approx Sin\theta = \frac{\Delta P}{P}$

Therefore, $$\theta = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{r_{\min}}\frac{1}{v}\frac{1}{mv} \qquad 1.11$$

Or $$r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{mv^2}\frac{1}{\theta} \qquad 1.12$$

Although Rutherford occasionally observed large scattering angles (θ), the maximum or most frequently observed $\theta \sim 1$ radian. Therefore, r_{min} corresponding to maximum deflection is obtained by putting $\theta = 1$ in equation 1.12:

$$r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{mv^2} \qquad 1.13$$

Putting $Z = 79$ for gold, $m = 6 \times 10^{-27}$ kg for alpha particle, $v = 10^7$ m s^{-1} and $e = 1.6 \times 10^{-19}$ C, means that $r_{min} \sim 4.2 \times 10^{-14}$ m. Since R (the nuclear radius) is less than r_{min}, it is obvious that R will be of the order of 10^{-15} to 10^{-14} m. The above estimation is based on the assumption that Coulomb's law of electrostatic repulsion still holds good at such short distances. However, the nuclear radius determined from this consideration is bound to be defective; since the Coulombic force has far greater range compared to the dimension of the nucleus.

Alternative estimate of nuclear radius

When we talk of the nuclear radius, the nucleus is assumed to have spherical shape. This is expected because of the short range character of the nuclear force. However, small departure from sphericity exist, inferred from existence of electric quadrupole moment, which should be zero for spherical nuclei.

Assuming that the nuclear charge is uniformly distributed, i.e. nuclear charge density ρ is approximately constant. Since nuclear mass is almost linearly proportional to the mass number A, this means that:

$$\rho \approx A/V = constant$$

i.e, $V = \frac{4}{3}\pi R^3 \propto A$

or, $R \propto A^{\frac{1}{3}}$ so that $R = r_o A^{\frac{1}{3}}$, where r_o is a constant, known as the nuclear radius parameter.

Electron Scattering Experiment for Estimating the Nuclear Size

The estimates of nuclear radius based on Rutherford scattering were not very accurate. This is expected considering that Rutherford scattering was based on long range and central force between alpha and the scattering nucleus, whereas the dominant force in the nucleus is the short range non central nuclear force.

Scattering of high energy electrons by nuclei constitutes the most direct method of measuring the charge radius of the nucleus and the nature of the nuclear charge distribution. This is because nuclear forces do not act on electrons. Only the Coulomb attractive force acts on them. If the de Broglie wavelength of the electrons is small compared to the nuclear radius, the electron scattering experiment can reveal many details of the nuclear charge distribution.

According to de Broglie's theory of wave-corpuscular dualism, the wavelength of a realistic electron of rest mass m_o, having the total energy $E > m_o c^2$ is given by:

$$\lambda = \frac{ch}{e\left\{V(V + 2m_o c^2 / e)\right\}^{\frac{1}{2}}} \qquad 1.14$$

Where e is the electronic charge and eV is equal to the kinetic energy of the electron. For electrons of kinetic energy = 200 MeV, the corresponding wavelength is about 6 x 10^{-5} Angstrom. This is considerably smaller than the radius of most nuclei, hence electrons of a few hundred MeV energy can reveal considerable details about the nuclear charge distribution.

The experiment consists of bombarding a thin target of the material under study with high-energy electrons, e.g. from an accelerator, and observing the probability of various angular deflections (θ). For each θ, the ratio of the number of scattered electrons (recorded by the detector) to the number of electrons in the incident beam is calculated. Practically all of the electrons in the incident beam are undeflected, very few electrons are deflected. Typical experimental results are shown in figure 1.7.

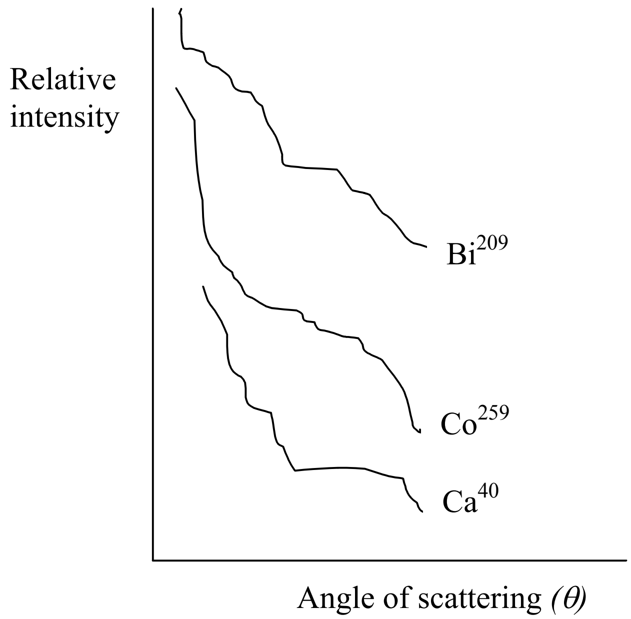


Figure 1.7 Angular distribution of electrons scattered from various nuclei

One can guess a density distribution *ρ(r),* the probability of various angular deflection can be calculated and compared with the experimental results. If they do not fit, another *ρ(r)* can be tried until a fit is obtained. The experiments have been performed for many nuclei and at several incident energies. All the results can be approximately described by a form of charge distribution known as Fermi distribution:

$$\rho(r) = \frac{\rho_o}{1 + \exp\left\{(r - R_{\frac{1}{2}})/a\right\}} \qquad 1.15$$

Where ρ_o is the nucleon density near the center of the nucleus, $R_{1/2}$ is the half-value radius, i.e., radius at which the density has decreased by a factor of 2, i.e. $\rho(r = R_{1/2}) = \rho_o/2$ and 'a' is a measure of how rapidly ρ falls towards zero. $R_{1/2}$ and a are adjusted to get the best fit with experimental data. Figure 1.8 shows two typical nucleon density distributions obtained by high-energy electron scattering by ^{40}Ca and ^{209}Ca. It is obtained from the fits of 1.15 to experimental data.

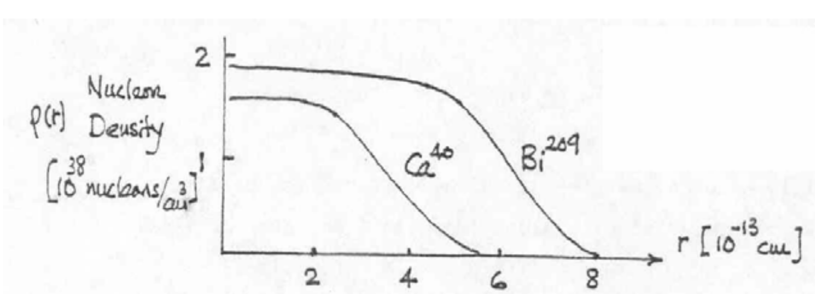


Figure 1.8 Nucleon density in two nuclei ^{40}Ca and ^{209}Ca

A sketch of the distribution, given in figure 1.9, shows clearly the core and boundary components of the nucleon density. A new length unit is called Fermi (F), 1 F = 10^{-13} cm.

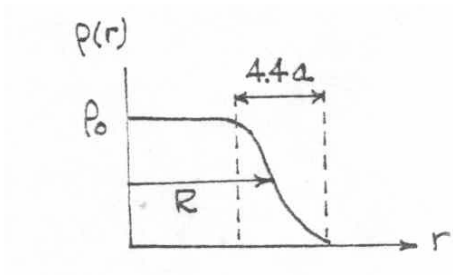


Figure 1.9. Schematic of the nuclear density distribution, *R* is a measure of the nuclear radius, and the width of the boundary region is given by 4.4a.

SUMMARY

In this lecture we have:

- Learnt nuclear terminologies such as ***nucleon, mass number, atomic mass, isotopic mass, nuclear binding energy, atomic mass unit, etc.***
- Stated the general properties of stable nuclides
- Described the Rutherford's experimental setup and the electron scattering methods for determining the density distribution of the nucleus and hence the radius of the nucleus.
- Shown how to convert from atomic mass unit to kg and vice versa
- Explained the meanings of binding energy, binding energy per nucleon, separation energy, and described how to calculate these quantities for a specified nucleus.

EXERCISE 1

1. Distinguish between atomic mass and isotopic mass.
2. Distinguish between mass excess and mass defect.
3. The atomic masses of $^{11}B_5$, $^{12}C_6$, $^{14}N_7$ and $^{16}O_8$ are 11.009305, 12.0, 14.003074, and 15.994915 a.m.u, respectively. Calculate their binding energies and list them in order of their stability . Mass of hydrogen atom is 1.007825 and mass of neutron is 1.008665 a.m.u.
4. Atomic masses of Tritium ($^3H_1$) and Helium ($^3He_2$) are 3.016050 and 3.016030 a.m.u. respectively. Find their binding energies, and explain the origin of the difference in their binding energies.
5. Gallium occurs with two natural isotopes, 69Ga (60.2% abundant) and 71Ga (39.8%), having atomic weights 68.93 and 70.92. What is the atomic weight of the element?
6. Calculate the total binding energy of the alpha particle $^4He_2$.
7. What is the mass of a 6Li atom in grams?
8. Calculate the average binding energy per nucleon for the Nuclide $^{40}K_{19}$.
9. Show that 1 a.m.u. = 1.49×10^{-10} J.

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Lecture 2: Nuclear Force

Introduction

In this Lecture we consider the characteristics of the force that hold the nucleus together, and relate it to the other forms of the forces of nature that we have come across in other areas of physics.

Objectives

At the end of this lecture you should be able to

1. State the properties of nuclear force
2. Distinguish between nuclear force and the other types of forces that you are familiar with
3. Explain the meson theory of nuclear force

2.1. Characteristics of Nuclear force

These are forces operating between nucleons inside the nucleus. They are characterized by the following properties:

(a) Within a certain distance between the nucleons, nuclear forces are attractive. This is also indicated by the high binding together of nucleons and the stability of nuclei. Nuclear attraction is a great deal stronger than the electrostatic repulsion between protons

(b) They are short-range forces, i.e. they drop off to a negligible value after a short distance, of the order of 10^{-15}m

(c) They are charge independent, i.e. the interactions between nucleons are independent of whether one or both Nucleons are independent of whether one or both nucleons have electric charge or not. In other words, n-n, n-p and p-p interactions are almost indistinguishable. The charge independence of nuclear forces was established from experiments on scattering of protons by deuterons and of neutrons by protons.

(d) Nuclear forces are non central forces, or tensor i.e. their direction partly depend on the spin orientation of the Nucleons, which may be parallel or anti parallel. (the strength of this non central force or tensor force, depends not only on the separation between the interacting prior of particles but also on the ample between the <u>spins</u> of the particles)

Mirror nuclei and the charge independence of nuclear forces

Mirror Nuclei are those in which number of protons in one is equal to the number of neutrons in the other, e.g. $^3He_2$ and $^3H_1$ (or two Nuclei for which Nucleons transformed into the other by exchanging all neutrons for protons and vice versa)

The difference in binding energy between the mirror nuclei is used to confirm the charge independence of Nuclear forces. For example the binding energies of ^3He and $^3H_1$ are 7.72 MeV and 8.49 MeV, respectively. The difference (0.77 MeV) is attributed to coulomb repulsion between the two protons in helium. Where 0.77 MeV is found to be the potential energy (U) due to coulomb repulsion between the protons i.e. $U = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r} = \dfrac{e^2}{4\pi\varepsilon_0 r}$

For U = 0.77 MeV, the distance r, between the protons is about 1.9 x 10^{-15}m, which is of the same order as the range of nuclear forces (2.2 x 10^{-15}m).

(e) Nuclear forces saturate in a similar way to that of chemical bonds between valence electrons in the atom. The saturation of the Nuclear forces means that a nucleon can only form bonds with a certain definite number of its neighbours and no more, even though they may be within the effective range of Nuclear forces. This another explanation for Nuclear stability/instability of Nuclei, in that beyond the number of bonded neighbours, the extra Nucleons experiment the repulsive force.

2.2. Meson theory of Nuclear Force

An 'exchange character' is that which is transferred between (or exchanged by) two interacting bodies. For example the exchange electromagnetic interaction is through the exchange of E.M. field quanta (i.e. photons) and gravitational attraction is exchange of gravitational attraction is an exchange of gravitational-field quanta (i.e. gravitons). Graviton is still a theoretical concept, it has not yet been found in nature, but research is going on! Inter-nucleon forces have also been successful explained on the basis of the hypothesis that they have an 'exchange character'.

In 1935, Yukawa (Japanese Physicist) explained the interaction of nucleons as an exchange of a special nuclear-field quanta called pi-mesons or pions. Pions have since been found experimentally. They are elementary particles with rest mass of about 250 times that of an electron (Note, however, that there are other types of mesons).

Properties of π meson:

- Mass of π meson ~ 250 times mass of electron (i.e. Mass of π meson ~ 140 MeV)
- Occurs in three forms, i.e. negatively charged, positively charged and neutral (with zero electric charge)
- The immediate and simple consequence of the fact that the field particle for the nuclear force, unlike for the E.M. and gravitational forces, has a finite mass is that nuclear force has a short range.

According to Yukawa's meson theory, when two nucleons are near each other, the meson travels from one nucleon towards the nucleon which absorbs it. During the exchange, the energy of the nucleon which gives up a meson is decreased, and that increased of the nucleon which absorbs the meson theory.

Exchange of π meson (pion) between two nucleons can be, symbolically, described as follows;

(i) Exchange of π^+ between proton (p) and neutron (n):

$$p + n \rightarrow n' + \pi^+ + n \rightarrow n' + p'$$

The explanation is as follows:

The initial proton becomes a neutron, by losing a positive pion (π^+); the initial neutron absorbs the positive pion (π^+) and becomes a proton. The initial/original nucleons have thus exchanged their coordinates.

Similarly:

(ii) Exchange of π^- between proton and neutron

$$n + p \rightarrow p' + \pi^- + p \rightarrow p' + n'$$

(iii) Exchange of π° between proton and proton

$$p + p \rightarrow p' + \pi^+ + p \rightarrow p' + p'$$

(iv) Exchange of π° between neutron and neutron

$$n + n \rightarrow n' + \pi^\circ + n \rightarrow n' + n'$$

Determination of the range of nuclear forces

Note that when a pion is sent from one nucleon to another, the creation of the pion momentarily violates the conservation of energy by an amount $\Delta E = M_\pi c^2$, where M_π is mass of the pion, and c is the velocity of light (i.e., assuming the pion moves at the speed of light). Such a violation of energy conservation cannot last longer than a time Δt, which from uncertainty principle is given by:

$$\Delta t = \frac{\hbar}{\Delta E} = \frac{\hbar}{M_\pi c^2} \qquad 2.1$$

Even with the assumption that the pion travels with a velocity of light (***c***), the farthest distance it can travel in this time is given by:

$$x = c\Delta t = \frac{\hbar}{M_\pi c} \qquad 2.2$$

This will be the order of the range of the nuclear force. Numerically, it works out to be ~ 1.4 F.

Example

Suppose a pion of mass 2.3 X 10^{-28} kg is exchanged between two nucleons during a nuclear interaction. Assume the pion travels at the speed of light and use equation 2.2 to calculate the range of the nuclear force of interaction between the nucleons.

Solution

$$x = c\Delta t = \frac{\hbar}{M_\pi c} = \frac{1.0545X10^{-34}\,J.s}{2.3X10^{-28}\,kgX3.0X10^{8}\,ms^{-1}} = 1.53X10^{-15}\,m$$

SUMMARY

In this lecture we have:

- Stated the properties of nuclear force
- Distinguished between nuclear force and the other types of forces;
- Explained the meson theory of nuclear force;
- Used the meson theory to estimate the nuclear radius.

EXERCISE 2

1. What are mirror nuclei? Give two examples of mirror nuclei pair.
2. Explain the charge independent nature of nuclear forces. Calculate the binding energies for $^3H_1$ and $^3He_2$. Hence or otherwise show quantitatively that nuclear forces are charge independent (Atomic masses in a.m.u. are 3.016050 and 3.016030 for $^3H_1$ and $^3He_2$, respectively; assume distance between any two nucleons in the nucleus to be 2.0 X 10^{-15} m.
3. Show that for most nuclei, binding energy (BE) is directly proportional to mass number (A). Explain how the direct proportionality between BE and A is a confirmation that nuclear forces saturate.

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Lecture 3: Nuclear Models

Introduction

We have recognized the complex nature of the nucleus. It is a many-body system and it is difficult to give full theoretical treatment (solution) of the interactions among the constituents. As a result of this difficulty, nuclear models were introduced, which account in a semi-quantitative fashion for many of the nuclear properties. What nuclear physicists try to do – within the constraints imposed by the many-body problem – is to understand the structure of nuclei in terms of their constituent particles, the dynamics of nuclei in terms of the motions of these particles, and the fundamental interactions among particles that govern these motions. Experimentally, they study these concepts through nuclear spectroscopy and the analysis of nuclear reactions of many kinds. Theoretically, they construct simplifying mathematical models to make the many-body problem tractable. These nuclear models are of different kinds. *Independent-particle models* allow the motion of a single nucleon to be examined in terms of a steady, average force field produced by all the other nucleons. The best-known independent-particle model is the *shell model*, so-called because it entails the construction of "shells" of nucleons analogous to those of the electrons in the theory of atomic structure. At the other extreme, *collective models* view the nucleons in a nucleus as moving in concert (collectively) in ways that may be simple or complex – just as the molecules in a flowing liquid may move smoothly or turbulently. In fact, the best-known collective model, the *liquid-drop model*, is based on analogies with the behavior of an ordinary drop of liquid. The above descriptions are necessarily oversimplified. The actual models in question, as well as related ones, are very sophisticated, and their success in explaining most of what we know about nuclear structure and dynamics is remarkable. In this lecture we will review three of these models and discuss the way in which they are related to the 'true' description of the nucleus.

Objectives

At the end of this lecture you should be able to

1. Give three different examples of Nuclear models
2. Discuss the Successes and failures of the Liquid drop model of the nucleus
3. Discuss the main features of the Fermi Gas Model
4. Define Magic Numbers and give examples of Nuclei with magic numbers
5. Describe the background to the to the Nuclear Shell model theory
6. Describe the procedure for determining the energy levels and the ground state angular momentum and parity of specific nuclei

3.1. The Liquid Drop Model

The liquid drop model was proposed first in 1936 by Frenkel and later elaborated by Bohr and Weizsacker. It is based on the outer analogy between the atomic nucleus and a charged liquid drop.

Similarities between the properties of a Nucleus and those of a drop of liquid.

1. Constant density which is independent of size;
2. Constant binding energy per nucleon of nucleus corresponding to latent heat of vaporization of liquid
3. The constituent molecules of a liquid drop operate over a short range like the nuclear forces
4. The intermolecular force in the liquid drop saturates just as does the nuclear force between nucleons.

Difference

The nucleus is charged and obeys the laws of quantum mechanics. In this, it differs from a liquid drop.

Calculation of binding energy on the basis of the Liquid Drop Model

We will treat the nucleus as an assemblage of interacting particles similar in some ways to a drop of liquid, but we will introduce (i) presence of Coulomb forces; (ii) effects of Pauli exclusion principle; (iii) other effects due to the complex structure of the nucleus, i.e. quantum principle. We can therefore treat the binding energy of a nucleus as a combination of many terms, i.e., $E = E_V - E_S - E_C - E_{Sym} - E_P$ where E_V, E_S, E_C, E_{Sym}, E_P are the volume, surface, Coulomb, symmetry, and pairing terms, respectively.

(i) Volume term (E_V)

The volume term is equivalent to binding energy of a liquid drop, i.e. energy required to evaporate a liquid drop (called heat of evaporation). This energy is directly proportional to the volume of the liquid drop. Similarly, the volume term of the nuclear binding energy is proportional to the volume of nucleus which is also proportional to the mass number A. Therefore

$$E_V = c_V A \qquad 3.1$$

where c_V is a proportionality constant.

(ii) Surface term (E_S)

The statement in (i) above, i.e., $E \propto A$ is under the assumption that every nucleon (or liquid molecule) is surrounded from all sides by neighbours, i.e., they all experience equal attraction. This is not true, those on the surface interact with less nucleons compared to those close to the center. Therefore it can be said that the presence of surface reduces the binding energy from what it would have been if the nucleus were to have no surface. This surface energy term is to the surface area, i.e. $E_S \alpha R^2$ since surface area is proportional to square of radius. Nuclear radius is proportional to $A^{1/3}$, therefore surface area $\propto A^{2/3}$, i.e., $E_S = c_S A^{\frac{2}{3}}$, where c_S is a proportionality constant.

The net binding energy, taking the surface effect into consideration, is:

$$E = c_V A - c_S A^{2/3} \qquad 3.2.$$

(iii) Coulomb term (E_C)

Some work is required to overcome the repulsive Coulombic forces in order to bind the nucleus. This columbic energy term is proportional to square of the atomic number (*Z*), i.e., $E_C = c_C Z(Z-1)$ 3.3

(iv) Symmetry or asymmetry term (E_{sym})

The inequality of numbers of protons and neutrons in the nucleus gives rise to a decrease in the binding energy. The exclusion principle makes it more expensive in energy for a nucleus to have more of one type of nucleon than the other. It explains the difference in stability between nuclei containing unequal numbers of protons and neutrons. Nuclear stability is directly related to energy, and the most stable system is one having the lowest energy, i.e., most stable nuclei are found with highest binding energy per nucleon and they contain equal numbers of protons and neutrons. The difference N – Z is called neutron excess. The deficit in the binding energy resulting from the neutron excess is proportional to the neutron excess $(N-Z)$ and to the neutron excess ratio $\frac{(N-Z)}{A}$ i.e., $E_{Sym} \alpha (N-Z)$ and $E_{Sym} \alpha \frac{(N-Z)}{A}$. This means that:

$$E_{Sym} = c_{Sym} \frac{(N-Z)(N-Z)}{A} = c_{Sym} \frac{(A-2Z)^2}{A} \qquad 3.4$$

(v) Pairing term (E_P)

Allows for the fact that the interactions between nucleons depend on their relative spin orientation. It is maximum for nuclei containing an even number of protons and neutrons (i.e. even-even or e-e nuclei), and minimum for nuclei containing odd numbers of protons and neutrons, i.e. odd-odd or o-o nuclei. The empirical relation for this component of the binding energy is given by Fermi as:

$$E_P = +c_P A^{-3/4}$$
$$E_P = -c_P A^{-3/4} \qquad 3.5$$
$$E_P = 0$$

Where + is for e-e nuclei; - is for o-o nuclei and 0 is for o-e or e-o nuclei.

The total binding energy can therefore be written as the combination of the five terms:

$$E(total) = E_V - E_S - E_C - E_{Sym} + E_P \qquad 3.6$$

Therefore the liquid model of the Nucleus can be used to calculate its binding energy.

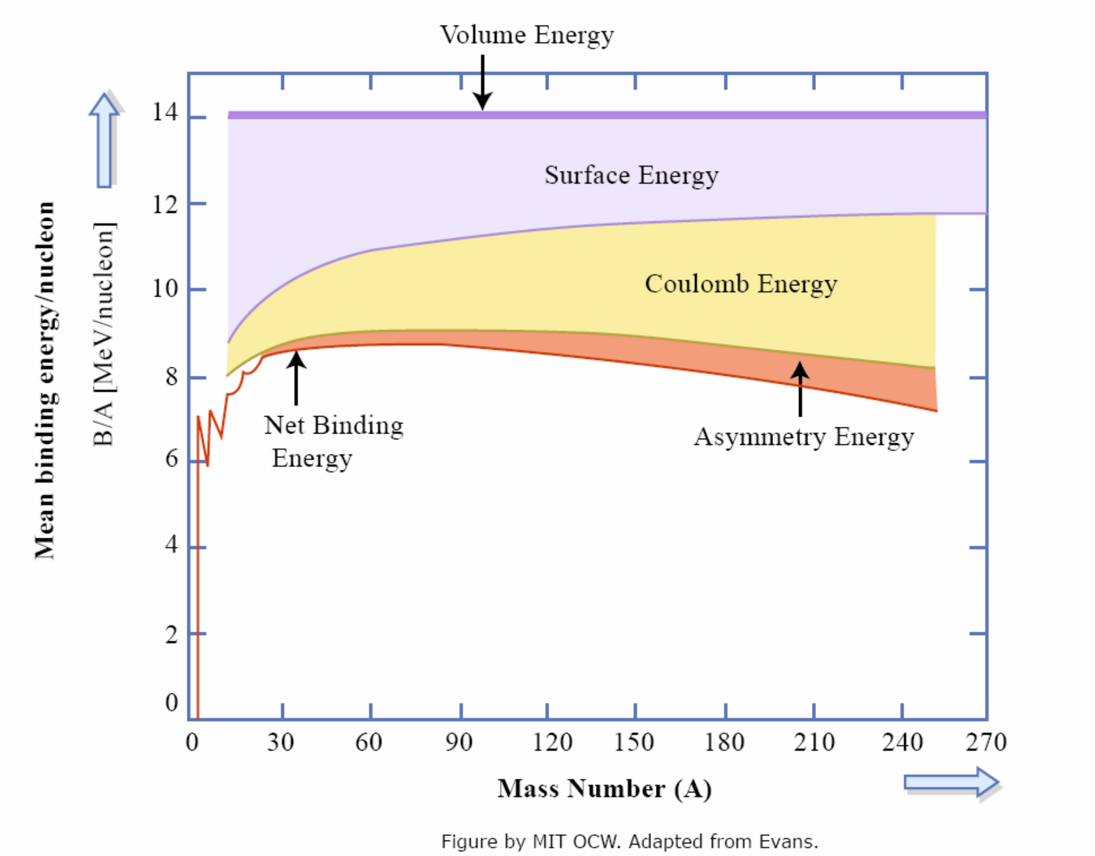

Figure 3.1. Relative contributions to the binding energy per nucleon showing the importance of the various terms in the semi-empirical Weizsacker formula [from Evans]

Substituting for $E(total) = \{ZM_H + (A-Z)M_n - M\}c^2$, E_V, E_S, E_C, E_{Sym}, and E_P into 3.6, and solving for mass M, we obtain the Weizsacker semi-empirical mass formula:

$$M(Z,A) = ZM_H + (A-Z)M_n - c_V A + c_S A^{\frac{2}{3}} + c_C Z^2 A^{-\frac{1}{3}} + c_{sym}(A-2Z)^2 A^{-1} \mp c_p A^{-\frac{3}{4}}$$

3.7

Or $M(Z,A) = \mu A + \beta Z + \gamma Z^2 - (\pm \partial)$ 3.8

Where $\mu = M_n - (c_V - c_{sym} - \frac{c_S}{A^{\frac{1}{3}}})$

$$\beta = -4c_{sym} - (M_N - M_H)$$

$$\gamma = (4\frac{c_p}{A} + \frac{c_C}{A^{\frac{1}{3}}}) = 4\frac{c_{sym}}{A}\left(1 + \frac{A^{\frac{2}{3}}}{4c_{sym}/c_C}\right) \qquad 3.9$$

$$\partial = \frac{c_p}{A^{\frac{3}{4}}}$$

One possible set of the numerical values of the coefficients is as follow: c_V = 14 MeV; c_S = 13 MeV; c_C = 0.6 MeV; c_{Sym} = 19 MeV; and c_P = ±33.5 MeV or 0.

Applications of the Weizsacher's semi-empirical Mass Formula

- Estimation of masses of nuclei (The five constants of the semi-empirical mass formula when substituted in the mass formula, should give atomic masses or approximately masses of nuclei);
- Estimation of Nuclear radius;
- Explaining/predicting the behaviour of isobars in beta decay

Prediction of stability of beta-emitting Isobar

For constant *A*, *M(Z,A)* is a parabolic function of *Z*. Therefore the plot of *M(Z,A)* versus *Z* should be a parabola with the minimum *M* corresponding to Z_s for the most stable isobar in the isobaric family.

Note that $\left(\frac{\partial M}{\partial Z}\right)_{A=Cons\tan t} = \beta + 2\gamma Z_s = 0$ 3.10

This implies that $Z(stable) = Z_s = -\beta / 2\gamma$ 3.11

<u>For Odd A ($\partial = 0$)</u>

The plot of M(Z,A) versus Z gives only one parabola. The mass of the most stable isobar is then given (substituting $Z = Z_s$, and $\partial = 0$ into 3.8) by:

$$M(Z_s, A) = \mu A - 2\gamma Z_s Z_s + \gamma Z_s^2$$

i.e., $M(Z_s, A) = \mu A - \gamma Z_s^2$ 3.12

The difference in masses of the isobars with odd A is obtained by subtracting equation 3.12 from equation 3.8:

$$M(Z,A) - M(Z_s,A) = \beta Z + \gamma Z^2 + \gamma Z_s{}^2 = -2\gamma Z_s Z + \gamma Z^2 + \gamma Z_s{}^2 = \gamma(Z - Z_s)^2 \qquad 3.13$$

Most of the unstable nuclides are natural beta-emitters. Hence we can predict the stability of emitting isobars from the mass formula.

Using the Q-value problem

The energy released or absorbed during transitions or decays can be obtained using the Q – value problem:

$Q_{\beta-} = M(Z,A) - M(Z+1,A)$ and substituting for the difference in masses in terms of Z and Z_s, we obtain:

$$Q_{\beta-} = \gamma\left[(Z - Z_s)^2 - (Z + 1 - Z_s)^2\right] = 2\gamma(Z_s - Z - \frac{1}{2}) \qquad 3.14$$

Similarly, $Q_{\beta+} = 2\gamma(Z - Z_s - \frac{1}{2})$ 3.15

Example

Question:

For the family of isobars with A = 91, estimate (i) Nuclear charge of the most stable isobar, (ii) the energy released Q^- and Q^{+}, for transitions to the most stable isobar.

Solution:

(i) The nuclear charge (or Z number) is given by: $Z_s = \frac{-\beta}{2\gamma}$

But $\beta = -4\alpha_{sym} - (M_n - M_p) \approx -(76 + 0.8)MeV \approx -77MeV$ and $\gamma = 0.96MeV$ for A = 91. Therefore the Z number of the most stable isobar is $Z_s = -\frac{(-77)}{0.96x2} = 40.104$. This corresponds to A = 91 and Z = 40, which correspond to ${}^{91}_{40}Zr$, and it can be seen how excellent the estimate based on the liquid drop model is. Other examples for A = 135 is described in figure 3.2.

<u>For even A ($\partial \neq 0$)</u>

Mass of the most stable isobar is $M(Z_s,A) = \mu A - \gamma Z_s{}^2 \mp \partial$. Therefore there are two possible most stable isobars; one for $+\partial$ (even - seven nuclei) and the other for $-\partial$ (for odd-odd nuclei). The plot of $M(Z, A)$ vs Z results in two parabolas displaced in binding energy by 2∂ (see figure 3.2 b).

The difference in masses is given by :

$M(Z,A) - M(Z_s,A) = \gamma(Z - Z_S)^2 + 2\partial$ (for odd Z)

$M(Z,A) - M(Z_s,A) = \gamma(Z - Z_S)^2 + 0$ (for even *Z*).

The Q value relations are:

$$Q_{\beta^-} = M(Z,A) - M(Z+1,A) = 2\gamma(+Z_s - Z - ½) + 2\partial \text{ for odd Z,} \quad 3.16$$

$$Q_{\beta^{+-}} = M(Z,A) - M(Z-1,A) = 2\gamma(Z - Z_s - ½) - 2\partial \text{ for even Z.} \quad 3.17$$

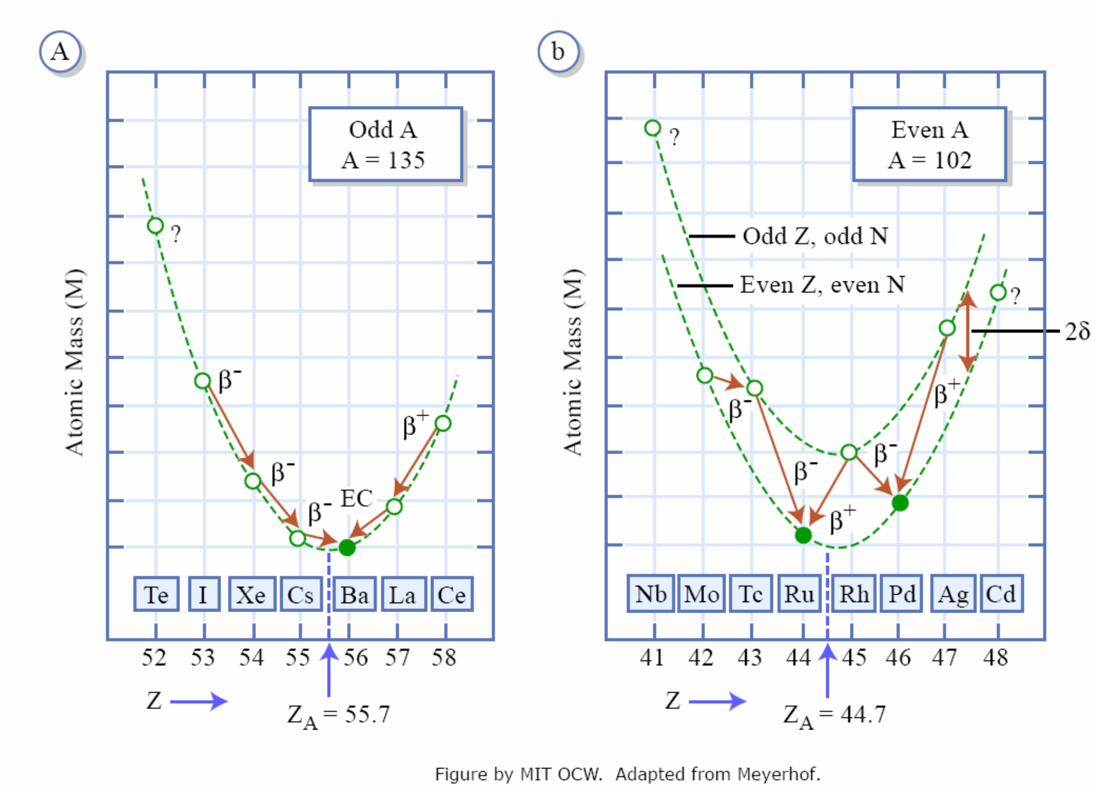

Figure 3.2 Mass parabolas for odd and even isobars. Stable and radioactive nuclides are denoted by closed and open circles respectively [from Meyerhof]

Spontaneous fission

For heavier nuclei (beyond ^{56}Fe) the binding energy decreases as the mass increases. A nucleus with Z > 40 can therefore, in principle, split into two lighter nuclei. Thankfully, the potential barrier is generally so large that such reactions are extremely unlikely.

The lightest nuclides where the probability of spontaneous fission is significant are certain uranium isotopes.

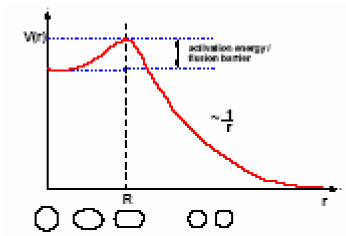

Figure 3.3. Schematic of the potential barrier against spontaneous fission

The height of the barrier for fission determines the probability for spontaneous fission.

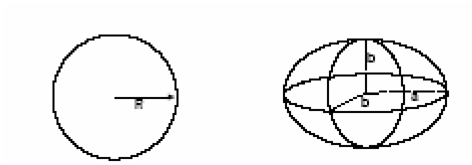

Figure 3.4. Deformation of spherical heavy nucleus

Let's consider the deformation of a heavy nucleus from spherical shape to an ellipsoid with constant volume and axes

$a = R(1+\epsilon)$ and $b = R(1-\epsilon/2)$. 3.18

The surface, and therefore the surface energy, will increase. At the same time the Coulomb energy decreases. The surface energy E_S and the Coulomb energy E_C can be shown to behave like

$$E_s = a_s A^{2/3}\left(1+\frac{2}{5}\epsilon^2+\ldots\right)$$
$$E_c = a_c Z^2 A^{-1/3}\left(1-\frac{1}{5}\epsilon^2+\ldots\right) \quad 3.19$$

Hence a deformation changes the total energy by

$$\Delta E = \frac{\epsilon^2}{5}\left(2a_s A^{2/3} - a_c Z^2 A^{-1/3}\right) \quad 3.20$$

The fission barrier disappears for

$$\frac{Z^2}{A} \geq \frac{2a_s}{a_c} \approx 48 \quad \text{i.e.} \quad Z > 114 \text{ and } A > 270 \quad 3.21$$

Which is roughly in the same ball park as ^{238}U, but not exactly the same. This shows us the limitations of our simple liquid drop model and we'll now move on to more elaborate models.

3.2. Fermi Gas Model

Assumptions

- The potential that an individual nucleon feels is the superposition of the potentials of the other nucleons. This potential has the shape of a sphere of radius, $R = R_o A^{1/3}$ fm, equivalent to a 3-D square potential well with radius R.
- Nucleons move freely (like gas) inside the nucleus, i.e. inside the sphere of radius R
- Nucleons fill energy levels in well up to the 'Fermi energy' E_F.
- Potential wells for protons and neutrons can be different.
    - If the Fermi energy were different for protons and neutrons, the nucleus would undergo β-decay into an energetically more favourable state.
    - Generally stable heavy nuclei have a surplus of neutrons.
    - Therefore the well for the neutron gas has to be deeper than for the proton gas.
    - Protons are therefore on average less strongly bound than neutrons (Coulomb repulsion).
- 2 protons / 2 neutrons per energy level, since spins can be up or down. The number of possible states available to a nucleon inside a volume V and a momentum region dp is

$$dn = \frac{4\pi p^2 dp}{(2\pi\hbar)^3} \cdot V \qquad 3.22$$

- In the nuclear ground state all states up to a maximum momentum, the Fermi momentum p_F, will be occupied. Integration leads us to the following number of states n. Since every state can contain two fermions, the number of protons Z and neutrons N are also given:

$$n = \frac{V(p_F)^3}{6\pi^2\hbar^3} \qquad Z = \frac{V(p_F^p)^3}{3\pi^2\hbar^3} \qquad N = \frac{V(p_F^n)^3}{3\pi^2\hbar^3} \qquad 3.23$$

- The nuclear volume V is given by

.

$$V = \frac{4}{3}\pi dR^3 = \frac{4}{3}\pi R_0^3 A \qquad 3.24$$

(experimental value from electron scattering: R_o = 1.21 fm)

- Assuming that the proton and neutron wells have the same radius, and assuming that N = Z = A/2, we find the Fermi momentum

$$p_F = p_F^n = p_F^p = \frac{\hbar}{R_0}\left(\frac{9\pi}{8}\right)^{1/3} \approx 250\text{MeV}/c$$

3.25

- The nucleons apparently move freely inside the nucleus with large momenta. This is in agreement with experimental data from electron scattering.
- The energy of the highest occupied state, the Fermi energy E_F, is

$$E_F = \frac{p_F^2}{2M} \approx 33\text{MeV}$$

3.26

where M is the nucleon mass

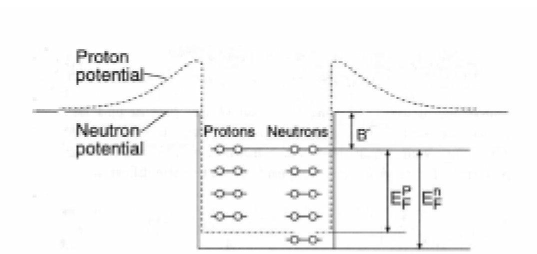

Figure 3.5. Neutron and proton potential wells

- The difference between the Fermi energy and the top of the potential well is the binding energy B'= 7-8 MeV/nucleon that we already know from our treatment of the liquid drop model. The depth of the potential well is to a good extend independent of the mass number.

- The depth of the potential well V_o is to a good extend independent of the mass number A:

$$V_0 = E_F + B' \approx 40 \text{ MeV}$$

3.27

- The average kinetic energy per nucleon is:

$$\langle E_{\text{kin}} \rangle = \frac{\int_0^{p_F} E_{\text{kin}} p^2 dp}{\int_0^{p_F} p^2 dp} = \frac{3}{5} \cdot \frac{p_F^2}{2M} \approx 20 \text{ MeV}$$

3.28

- The total kinetic energy of the nucleus is therefore

$$E_{\text{kin}}(N,Z) = N\langle E_n \rangle + Z\langle E_p \rangle = \frac{3}{10M}\left(N \cdot (p_F^n)^2 + Z \cdot (P_F^p)^2\right)$$

3.29

- Using the expressions for the numbers of proton and neutron states derived earlier, and assuming that the radii of proton and neutron well are the same, this can be re-expressed as

$$E_{\mathrm{kin}}(N,Z) = \frac{3}{10M}\frac{\hbar^2}{R_0^2}\left(\frac{9\pi}{4}\right)^{2/3}\frac{N^{5/3}+Z^{5/3}}{A^{2/3}} \qquad 3.30$$

- For fixed $A = N + Z$ the term $N^{5/3} + Z^{5/3}$ has a minimum for $N = Z$; therefore also the average kinetic energy has a minimum there. Hence the binding energy shrinks for $N \neq Z$.

- If one expands the expression $E_{kin}(N, Z)$ for in the difference $N - Z$ one obtains

$$E_{\mathrm{kin}}(N,Z) = \frac{3}{10M}\frac{\hbar^2}{R_0^2}\left(\frac{9\pi}{4}\right)^{2/3}\left(A+\frac{5}{9}\frac{(N-Z)^2}{A}+...\right) \qquad 3.31$$

which gives the functional dependence on the neutron surplus.

- The first term contributes to the volume term in the mass formula. The second term corresponds to the asymmetry term in the mass formula.

- To reproduce the asymmetry term to reasonable accuracy the change in the potential for $N \neq Z$ must be taken into account.

- Not a strict derivation, but it shows that the asymmetry term in the mass formula is plausible.

Summary of Liquid Drop and Fermi gas models

- We have seen that the liquid drop model allows a reasonably good descriptions of the binding energy. It also gives a qualitative explanation for spontaneous fission.
- The Fermi gas model, assuming a simple 3D square well potential (different for protons and neutrons) explained the terms in the semi-empirical mass formula that were not derived from the liquid drop picture.
- We've also seen that nucleons can move freely inside the nucleus. This agrees with the idea that they experience an overall effective potential created by the sum of the other nucleons.
- We will now have a look at further experimental evidence, that the Fermi gas model cannot explain and then see how we can improve the model. This will lead us to the Shell Model.

3.3. The Shell Model

The liquid model emphasized the properties of the entire nucleus but not those of the individual nucleons. In the successful atomic model, emphasis is on motion of electrons in the central (coulomb) field provided by the nucleus.

Precursors to the shell model:
These are the events that preceded the shell model theory and contributed to its promotion. They include:

(i) success of the atomic shell model theory;

(ii) recognition of the existence of some nuclei (with magic numbers) , which are associated with special characteristics of stability. This was reminiscent of the stability properties exhibited by the atoms of noble gases.

There are similarities between the electronic structure of atoms and nuclear structure. Atomic electrons are arranged in orbits (energy states) subject to the laws of quantum mechanics. The distribution of electrons in these states follows the Pauli exclusion principle. Atomic electrons can be excited up to normally unoccupied states, or they can be removed completely from the atom. From such phenomena one can deduce the structure of atoms. In nuclei there are two groups of like particles, protons and neutrons. Each group is separately distributed over certain energy states subject also to the Pauli exclusion principle. Nuclei have excited states, and nucleons can be added to or removed from a nucleus.

Electrons and nucleons have intrinsic angular momenta called intrinsic spins. The total angular momentum of a system of interacting particles reflects the details of the forces between particles. For example, from the coupling of electron angular momentum in atoms we infer an interaction between the spin and the orbital motion of an electron in the field of the nucleus (the spin-orbit coupling). In nuclei there is also a coupling between the orbital motion of a nucleon and its intrinsic spin (but of different origin). In addition, nuclear forces between two nucleons depend strongly on the relative orientation of their spins.

The structure of nuclei is more complex than that of atoms. In an atom the nucleus provides a common center of attraction for all the electrons and inter-electronic forces generally play a small role. The predominant force (Coulomb) is well understood.

Nuclei, on the other hand, have no center of attraction; the nucleons are held together by their mutual interactions which are much more complicated than Coulomb interactions.
All atomic electrons are alike, whereas there are two kinds of nucleons. This allows a richer variety of structures. Notice that there are ~ 100 types of atoms, but more than 1000 different nuclides. Neither atomic nor nuclear structures can be understood without quantum mechanics.

Experimental Basis of nuclear shell structure

There exists considerable experimental evidence pointing to the shell-like structure of nuclei, each nucleus being an assembly of nucleons. Each shell can be filled with a given number of nucleons of each kind. These numbers are called magic numbers; they are **2, 8, 20, 28, 50, 82, and 126**. (For the as yet undiscovered superheavy nuclei the magic numbers are expected to be N = 184, 196, (272), 318, and Z = 114, (126), 164 [Marmier and Sheldon, p. 1262].)

Nuclei whose N and Z numbers are magic numbers are particularly stable these are the so-called ***'Doubly magic'*** nuclei, e.g., ${}^{4}_{2}He_{2}, {}^{16}_{8}O_{8}, {}^{40}_{20}Ca_{20}, {}^{48}_{20}Ca_{28}, {}^{208}_{82}Pb_{126}$

Characteristics of nuclei with magic numbers

(a) Energies emitted by alpha particles emitted from *Rn* peak at N = 128 for parent, i.e. N = 126 for daughter nucleus;
(b) Neutron capture cross sections show a sharp decrease (of 2 orders of magnitude) near N = 50, 80, 126;
(c) Changes in the nuclear charge radius are particularly significant at 20, 28, 50, 82 and 126.

Nuclei with magic number of neutrons or protons, or both, are found to be particularly stable, as can be seen from the following data.

(i) Figure 3.6. shows the abundance of stable isotones (same N) is particularly large for nuclei with magic neutron numbers.

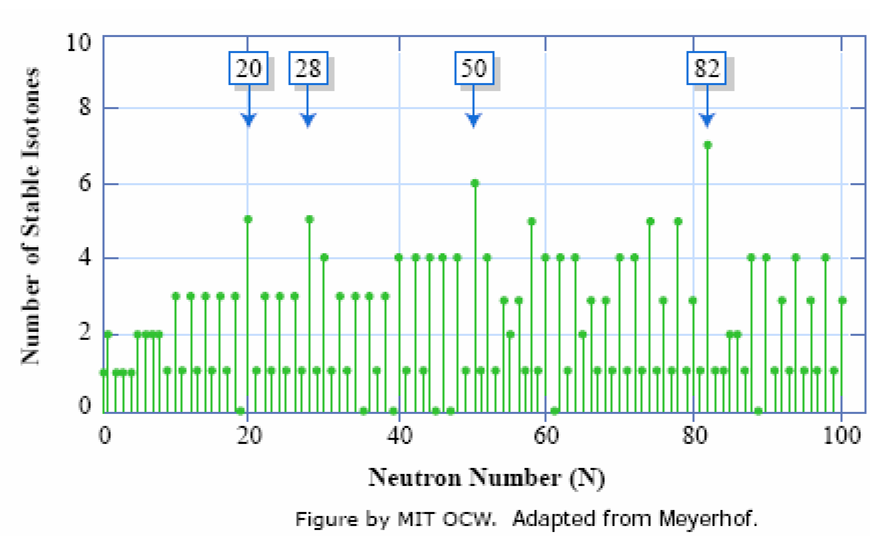

Figure 3.6. Histogram of stable isotopes showing nuclides with neutron numbers 20, 28, 50, and 82 are more abundant by 5 to 7 times than those with non-magic neutron numbers [from Meyerhof]

(ii) Figure 3.7. shows that the neutron separation energy S_n is particularly low for nuclei with one more neutron than the magic numbers, where

$$S_n = [M(A-1, Z) + M_n - M(A, Z)]c^2 \quad 3.32$$

This means that nuclei with magic neutron numbers are more tightly bound.

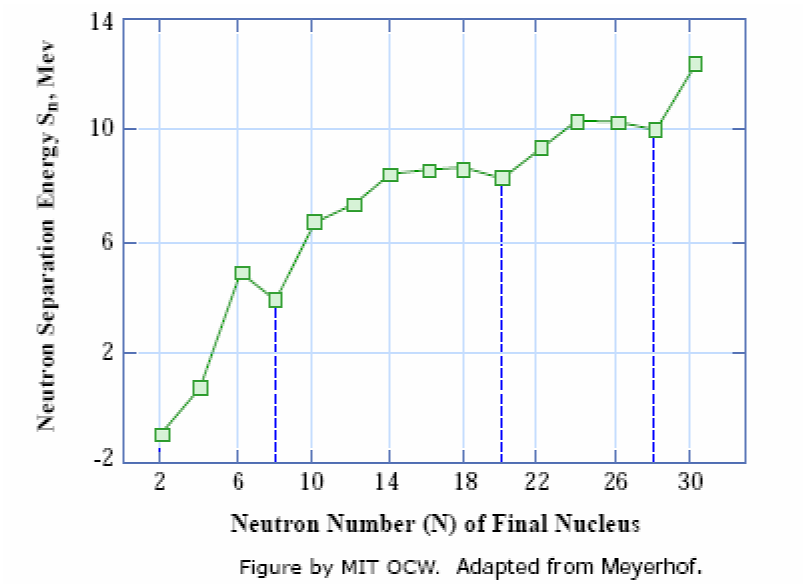

Figure 3.7. Variation of neutron separation energy with neutron number of the final nucleus M(A,Z) [from Meyerhof].

(iii) The first excited states of even-even nuclei have higher than usual energies at the magic numbers, indicating that the magic nuclei are more tightly bound (see Figure 3.8).

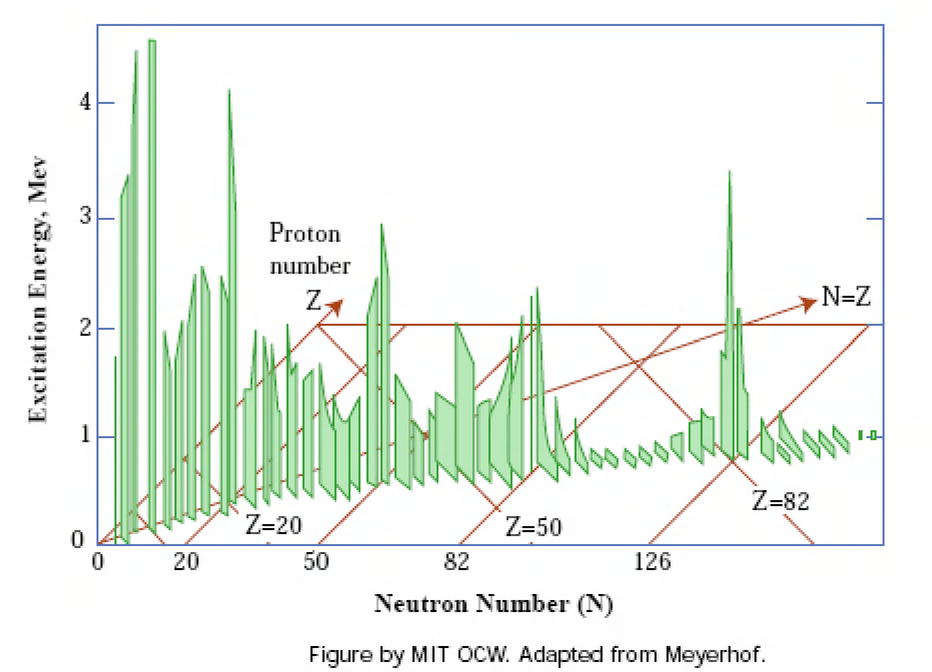

Figure 3.8. First excited state energies of even-even nuclei [from Meyerhof]

(iv) The neutron capture cross sections for magic nuclei are small, indicating a wider spacing of the energy levels just beyond a closed shell, as shown in Figure 3.9.

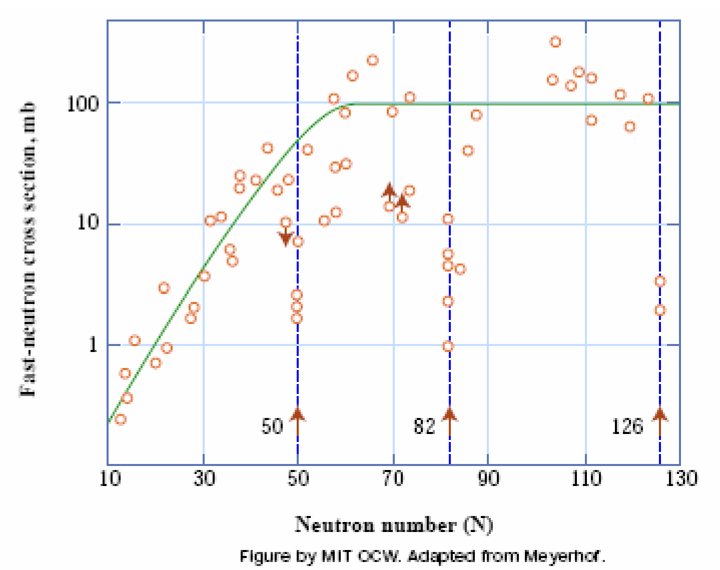

Figure 3.9. Cross section for capture at 1 MeV [from Meyerhof].

The Closed Shells

The first task in the construction of the shell model is the explanation of the magic numbers. In the independent single-particle (shell) model it is assumed that the nucleons move independently in a common (overall) nuclear field whose potential is determined by the average motion of all the other (A-1) nucleons. The (central) potential is assumed to be proportional to the nuclear density distribution which in turn is approximately the same as the charge distribution.

The spherical nuclei, the charge distribution can be represented to a first approximation by the Fermi distribution. Therefore it is appropriate to start the investigation of the single - particle levels by using a potential that has the form of an attractive Fermi distribution.

The Schrödinger equation for such a potential can not be solved in closed form. The realistic potential is consequently replaced by approximate ones that can be treated easily, either a square well or a harmonic oscillator potential.

Simple Shell Model

The basic assumption of the shell model is that the effects of internuclear interactions can be represented by a single-particle potential. One might think that with very high density and strong forces, the nucleons would be colliding all the time and therefore cannot maintain a single-particle orbit. But, because of Pauli exclusion the nucleons are restricted to only a limited number of allowed orbits. A typical shell-model potential is

$$V(r) = -\frac{V_o}{1+\exp[(r-R)/a]} \qquad 3.33$$

where typical values for the parameters are Vo ~ 57 Mev, R ~ $1.25A^{1/3}$ F, a ~ 0.65 F. In addition one can consider corrections to the well depth arising from (i) symmetry energy from an unequal number of neutrons and protons, with a neutron being able to interact with a proton in more ways than n-n or p-p (therefore n-p force is stronger than n-n and p-p), and (ii) Coulomb repulsion. For a given spherically symmetric potential V(r), one can examine the bound-state energy levels that can be calculated from radial wave equation for a particular orbital angular momentum ℓ ,

$$-\frac{\hbar}{2m}\frac{d^2u_l}{dr^2}+\left[\frac{l(l+1)\hbar^2}{2mr^2}+V(r)\right]u_l(r)=Eu_l(r) \qquad 3.34$$

Figure 3.10 shows the energy levels of the nucleons for an infinite spherical well and a harmonic oscillator potential, $V(r) = m\omega^2 r^2/2$. While no simple formulas can be given for the former, for the latter one has the expression

$$E_v = \hbar\omega(v+3/2) = \hbar\omega(n_x+n_y+n_z+3/2) \qquad 3.35$$

Where $v = 0,1,2,..,.., and, n_x, n_y, n_z = 0,1,2,..,..,$ are quantum numbers. One should notice the degeneracy in the oscillator energy levels. The quantum number v can be divided into *radial* quantum number n (1, 2, …) and *orbital* quantum numbers ℓ (0, 1, …) as shown in Figure 3.10. One can see from these results that a central force potential is able to account for the first three magic numbers, 2, 8, 20, but not the remaining four, 28, 50, 82, 126. This situation does not change when more rounded potential forms are used. The implication is that something very fundamental about the single-particle interaction picture is missing in the description.

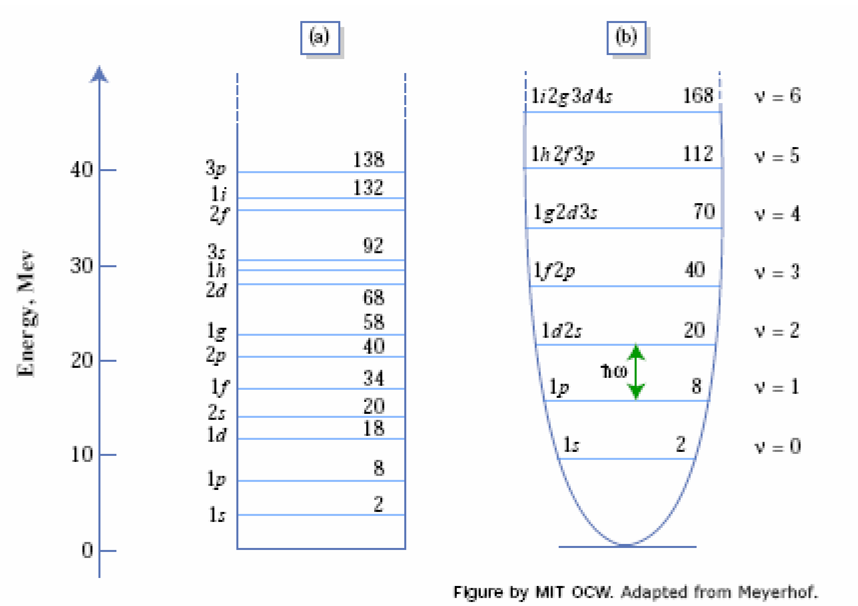

Figure 3.10. Energy levels of nucleons in (a) infinite spherical well (range R = 8F) and (b) a parabolic potential well. In the spectroscopic notation (n, ℓ), n refers to the number of times the orbital angular momentum state ℓ has appeared. Also shown at certain levels are the cumulative number of nucleons that can be put into all the levels up to the indicated level [from Meyerhof].

Shell Model with Spin-Orbit Coupling

It remains for M. G. Mayer and independently Haxel, Jensen, and Suess to show (1949) that an essential missing piece is an attractive interaction between the orbital angular momentum and the intrinsic spin angular momentum of the nucleon. To take into account this interaction we add a term to the Hamiltonian H,

$$H = \frac{p^2}{2m} + V(r) + V_{so}(r)\underline{s} \cdot \underline{L} \qquad 3.36$$

where V_{so} is another central potential (known to be attractive). This modification means that the interaction is no longer spherically symmetric; the Hamiltonian now depends on the relative orientation of the spin and orbital angular momenta. It is beyond the scope of this class to go into the bound-state calculations for this Hamiltonian. In order to understand the meaning of the results of such calculations (eigenvalues and eigenfunctions) we need to digress somewhat to discuss the addition of two angular momentum operators.

The presence of the spin-orbit coupling term in (3.36) means that we will have a different set of eigenfunctions and eigenvalues for the new description. What are these new quantities relative to the eigenfunctions and eigenvalues we had for the problem without the spin-orbit coupling interaction? We first observe that in labeling the energy levels in Fig. 9.5 we had already taken into account the fact that the nucleon has an orbital angular momentum (it is in a state with a specified ℓ), and that it has an intrinsic spin of ½ (in unit of $\hbar$). For this reason the number of nucleons that we can put into each level has been counted correctly. For example, in the 1s ground state one can put two nucleons, for zero orbital angular momentum and two spin orientations (up and down).

The student can verify that for a state of given ℓ , the number of nucleons that can go into that state is 2(2 ℓ +1). This comes about because the eigenfunctions we are using to describe the system is a representation that *diagonalizes* the square of the orbital angular momentum operator L^2, its z-component, L_z, the square of the intrinsic spin angular momentum operator S^2, and its z-component S_z. Let us use the following notation to label these eigenfunctions (or representation),

$$\left|l, m_l, s, m_s\right\rangle \equiv Y_l^{m_l} \chi_s^{m_s} \qquad 3.37$$

Where $Y_l^{m_l}$ is the spherical harmonic. It is the eigenfunction of the square of the orbital angular momentum operator L^2 (it is also the eigenfunction of L_z). The function $\chi_s^{m_s}$ is the spin eigenfunction with the expected properties,

$$S^2 \chi_s^{m_s} = s(s+1)\hbar^2 \chi_s^{m_s} ,......s = 1/2 \qquad 3.38$$

$$S_z \chi_s^{m_s} = m_z \hbar \chi_s^{m_s} ,......-s \le m_s \le s \qquad 3.39$$

The properties of χ_s with respect to operations by S^2 and S_z completely mirror the properties of $Y_\ell^{m_l}$ with respect to L^2 and L_z. Going back to our representation (3.37) we see that the eigenfunction is a "ket" with indices which are the good quantum numbers for the problem, namely, the orbital angular momentum and its projection (sometimes called the magnetic quantum number m, but here we use a subscript to denote that it goes with the orbital angular momentum), the spin (which has the fixed value of ½) and its projection (which can be +1/2 or -1/2).

The representation given in (3.37) is no longer a good representation when the spin-orbit coupling term is added to the Hamiltonian. It turns out that the good representation is just a linear combination of the old representation. It is sufficient for our purpose to just know this, without going into the details of how to construct the linear combination. To understand the properties of the new representation we now discuss angular momentum addition.

The two angular momenta we want to add are obviously the orbital angular momentum operator L and the intrinsic spin angular momentum operator S, since they are the only angular momentum operators in our problem. Why do we want to add them? The reason lies in (3.36). Notice that if we define the total angular momentum as

$$\underline{j} = \underline{S} + \underline{L} \qquad 3.40$$

We can then write

$$\underline{S} \cdot \underline{L} = (j^2 - S^2 - L^2)/2 \qquad 3.41$$

so the problem of diagonalizing (3.36) is the same as diagonalizing j^2, S^2, and L^2. This is then the basis for choosing our new representation. In analogy to (3.37) we will denote the new eigenfunctions by $\left| jm_j ls \right\rangle$, which has the properties

$$j^2 \left| jm_j ls \right\rangle = j(j+1)\hbar^2 \left| jm_j ls \right\rangle, \ldots\ldots\ldots |l - s| \le j \le l + s \qquad 3.42$$

$$j_z \left| jm_j ls \right\rangle = m_j \hbar \left| jm_j ls \right\rangle, \ldots\ldots\ldots - j \le m_j \le j \qquad 3.43$$

$$L^2 \left| jm_j ls \right\rangle = l(l+1)\hbar^2 \left| jm_j ls \right\rangle, \ldots\ldots\ldots l = 0,1,2,\ldots,\ldots\ldots \qquad 3.44$$

$$S^2 \left| jm_j ls \right\rangle = s(s+1)\hbar^2 \left| jm_j ls \right\rangle, \ldots\ldots\ldots s = \tfrac{1}{2} \qquad 3.45$$

In (3.42) we indicate the values that j can take for given ℓ and s (=1/2 in our discussion), the lower (upper) limit corresponds to when S and L are antiparallel (parallel) as shown in the sketch.

Returning now to the energy levels of the nucleons in the shell model with spin-orbit coupling we can understand the conventional spectroscopic notation where the value of j is shown as a subscript.

This is then the notation in which the shell-model energy levels are displayed in Figure 3.11.

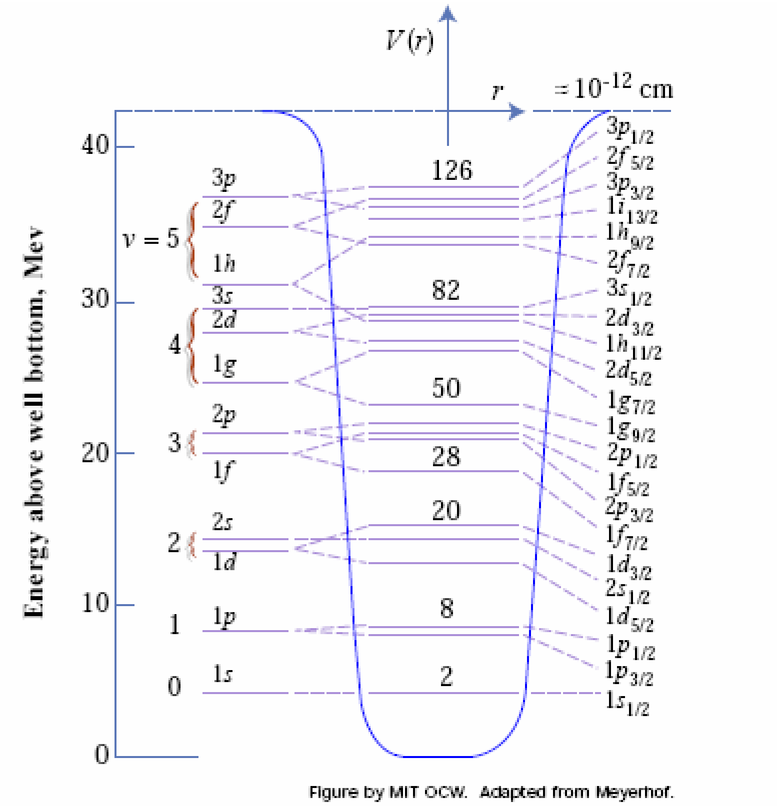

Figure 3.11. Energy levels of nuclei in a smoothly varying potential well with a strong spin-orbit coupling term [from Meyerhof].

For a given (n,ℓ, j) level, the nucleon occupation number is 2j+1. It would appear that having 2j+1 identical nucleons occupying the same level would violate the Pauli exclusion principle. But this is not the case since each nucleon would have a distinct value of m_j (this is why there are 2j+1 values of m_j for a given j).

We see in Figure 3.11 the shell model with spin-orbit coupling gives a set of energy levels having breaks at the seven magic numbers. This is considered a major triumph of the model, for which Mayer and Jensen were awarded the Noble prize in physics. For our purpose we will use the results of the shell model to predict the ground-state spin and parity of nuclei. Before going into this discussion we leave the student with the following comments.

1. The shell model is most useful when applied to closed-shell or near closed-shell nuclei.
2. Away from closed-shell nuclei collective models taking into account the rotation and vibration of the nucleus are more appropriate.
3. Simple versions of the shell model do not take into account pairing forces, the effects of which are to make two like-nucleons combine to give zero orbital angula momentum.

4. Shell model does not treat distortion effects (deformed nuclei) due to the attraction between one or more outer nucleons and the closed-shell core. When the nuclear core is not spherical, it can exhibit "rotational" spectrum.

Prediction of Ground-State Spin and Parity

There are three general rules for using the shell model to predict the total angular momentum (spin) and parity of a nucleus in the ground state. These do not always work, especially away from the major shell breaks.

1. Angular momentum of odd-A nuclei is determined by the angular momentum of the last nucleon in the species (neutron or proton) that is odd.
2. Even-even nuclei have zero ground-state spin, because the net angular momentum associated with even N and even Z is zero, and even parity.
3. In odd-odd nuclei the last neutron couples to the last proton with their intrinsic spins in parallel orientation.

To illustrate how these rules work, we consider an example for each case. Consider the odd-A nuclide Be^{9} which has 4 protons and 5 neutrons. With the last nucleon being the fifth neutron, we see in Fig. 9.6 that this nucleon goes into the state $1p_{3/2}$ (ℓ =1, j=3/2). Thus we would predict the spin and parity of this nuclide to be $3/2^{-}$. For an even-even nuclide we can take A^{36}, with 18 protons and neutrons, or Ca^{40}, with 20 protons and neutrons. For both cases we would predict spin and parity of 0^{+}. For an odd-odd nuclide we take Cl^{38}, which has 17 protons and 21 neutrons. In Figure 3.11 we see that the 17th proton goes into the state $1d_{3/2}$ (ℓ =2, j=3/2), while the 21st neutron goes into the state $1f_{7/2}$ (ℓ =3, j=7/2). From the ℓ and j values we know that for the last proton the orbital and spin angular momenta are pointing in opposite direction (because j is equal to ℓ -1/2). For the last neutron the two momenta are pointing in the same direction (j = ℓ +1/2). Now the rule tells us that the two spin momenta are parallel, therefore the orbital angular momentum of the odd proton is pointing in the opposite direction from the orbital angular momentum of the odd neutron, with the latter in the same direction as the two spins. Adding up the four angular momenta, we have +3+1/2+1/2-2 = 2. Thus the total angular momentum (nuclear spin) is 2. What about the parity? The parity of the nuclide is the product of the two parities, one for the last proton and the other for the last neutron. Recall that the parity of a state is determined by the orbital angular momentum quantum number ℓ , $\pi = (-1)^{\ell}$. So with the proton in a state with ℓ = 2, its parity is even, while the neutron in a state with ℓ = 3 has odd parity. The parity of the nucleus is therefore odd. Our prediction for Cl^{38} is then 2^{-}. The student can verify, using for example the Nuclide Chart, the foregoing predictions are in agreement with experiment.

Potential Wells for Neutrons and Protons

We summarize the qualitative features of the potential wells for neutrons and protons. If we exclude the Coulomb interaction for the moment, then the well for a proton is known to be deeper than that for a neutron. The reason is that in a given nucleus usually there are more neutrons than protons, especially for the heavy nuclei, and the n-p interactions can occur in more ways than either the n-n or p-p interactions on account of the Pauli exclusion principle. The difference in well depth ΔV_s is called the symmetry energy; it is approximately given by

$$\Delta V_s = \pm 27 \frac{(N-Z)}{A} MeV \qquad 3.46$$

where the (+) and (-) signs are for protons and neutrons respectively. If we now consider the Coulomb repulsion between protons, its effect is to raise the potential for a proton. In other words, the Coulomb effect is a positive contribution to the nuclear potential which is larger at the center than at the surface. Combining the symmetry and the Coulomb effects we have a sketch of the potential for a neutron and a proton as indicated in Figure 3.12.

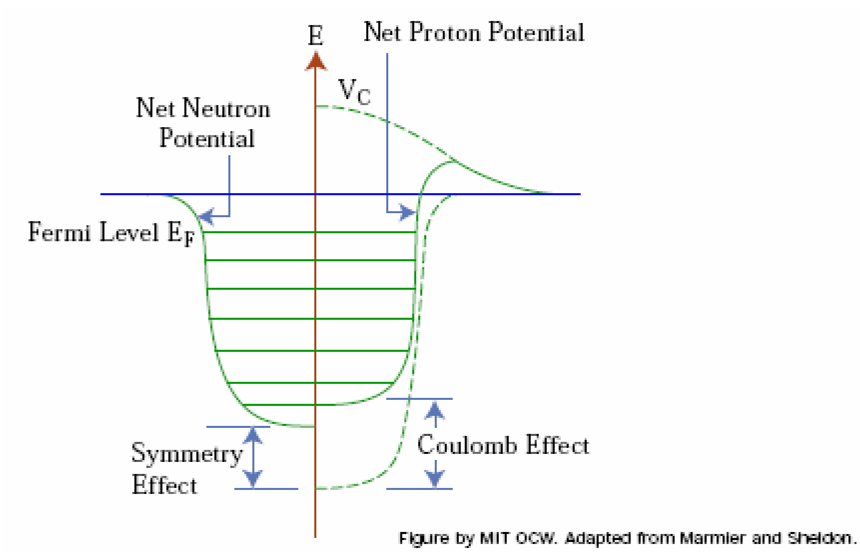

Figure 3.12. Schematic showing the effects of symmetry and Coulomb interactions on the potential for a neutron and a proton [from Marmier and Sheldon].

One can also estimate the well depth in each case using the Fermi Gas model. One assumes the nucleons of a fixed kind behave like a fully degenerate gas of fermions (degeneracy here means that the states are filled continuously starting from the lowest energy state and there are no unoccupied states below the occupied ones), so that the number of states occupied is equal to the number of nucleons in the particular nucleus. This calculation is carried out separately for neutrons and protons. The highest energy state that is occupied is called the *Fermi level*, and the magnitude of the difference between this state and the ground state is called the *Fermi energy* E_F. It turns out that E_F is proportional to $n^{2/3}$, where n is the number of nucleons of a given kind, therefore E_F (neutron) > E_F (proton). The sum of E_F and the separation energy of the last nucleon provides an estimate of the well depth. (The separation energy for a neutron or proton is about 8 Mev for many nuclei.) Based on these considerations one obtains the results shown in Figure 3.13.

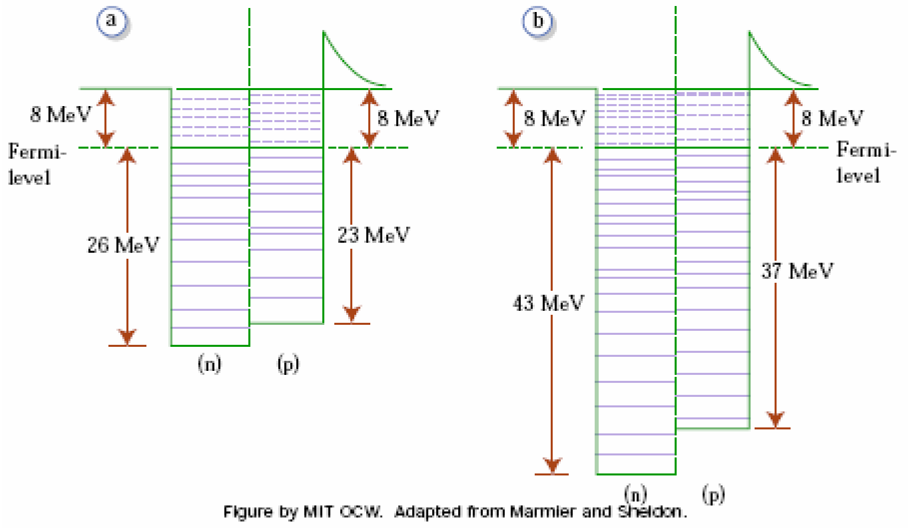

Figure 3.13. Nuclear potential wells for neutrons and protons according to the Fermi-gas model, assuming the mean binding energy per nucleon to be 8 MeV, the mean relative nucleon admixture to be N/A ~ 1/1.8m Z/A ~1/2.2, and a range of 1.4 F (a) and 1.1 F (b) [from Marmier and Sheldon]

We have so far considered only a spherically symmetric nuclear potential well. We know there is in addition a centrifugal contribution of the form $\ell(\ell+1)\hbar^2/2mr^2$ and a spin-orbit contribution. As a result of the former the well becomes narrower and shallower for the higher orbital angular momentum states. Since the spin-orbit coupling is attractive, its effect depends on whether S is parallel or anti-parallel to L. The effects are illustrated in Figures 3.14 and 3.15. Notice that for $\ell = 0$ both are absent.

We conclude this lecture with the remark that in addition to the bound states in the nuclear potential well there exist also virtual states (levels) which are positive energy states in which the wave function is large within the potential well. This can happen if the deBroglie wavelength is such that approximately standing waves are formed within the well. (Correspondingly, the reflection coefficient at the edge of the potential is large.) A virtual level is therefore not a bound state; on the other hand, there is a non-negligible probability that inside the nucleus a nucleon can be found in such a state. See Figure 3.16.

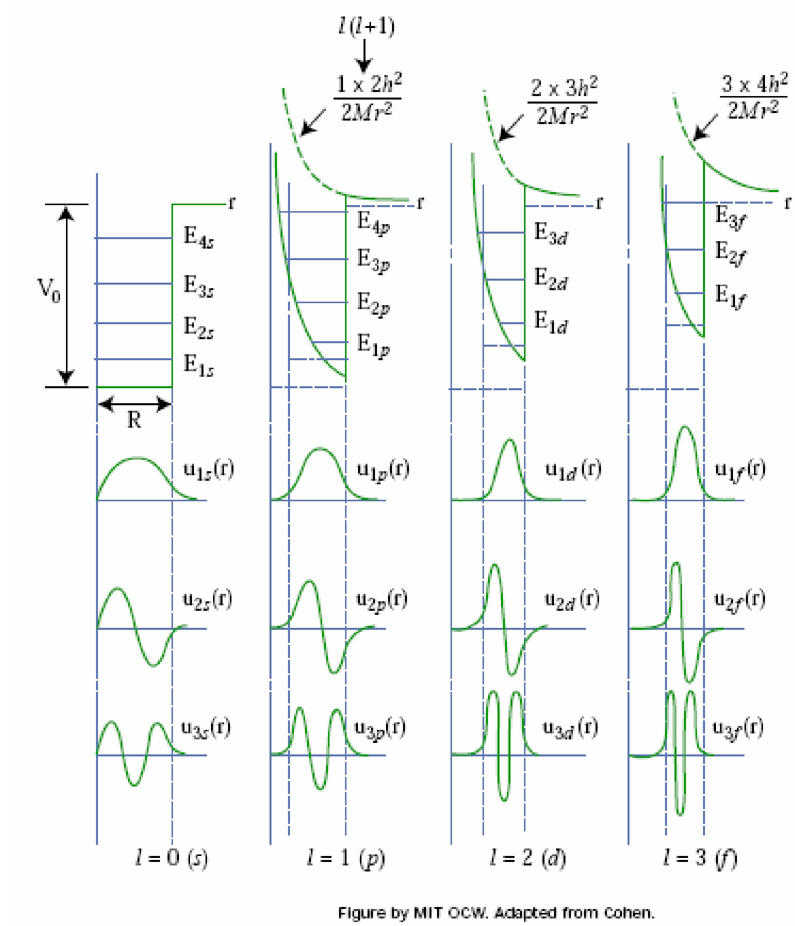

Figure 3.14. Energy levels and wave functions for a square well for l = 0, 1, 2, and 3 [from Cohen].

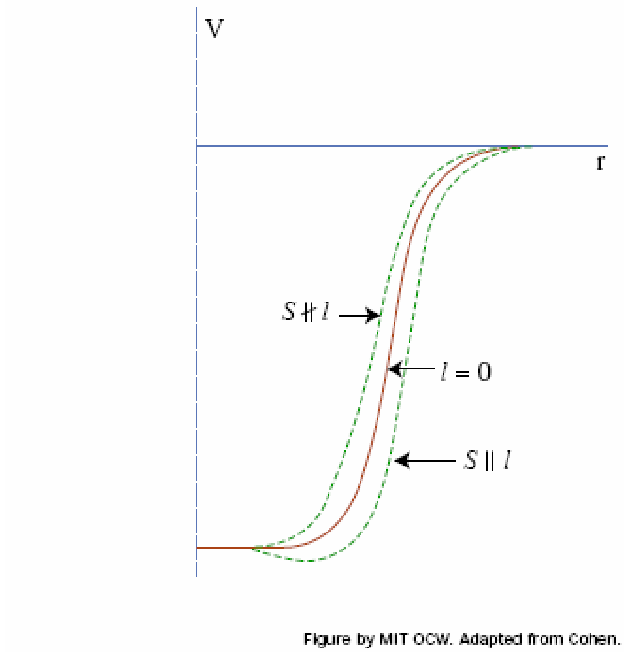

Figure 3.15. The effect of spin-orbit interaction on the shell-model potential [from Cohen]

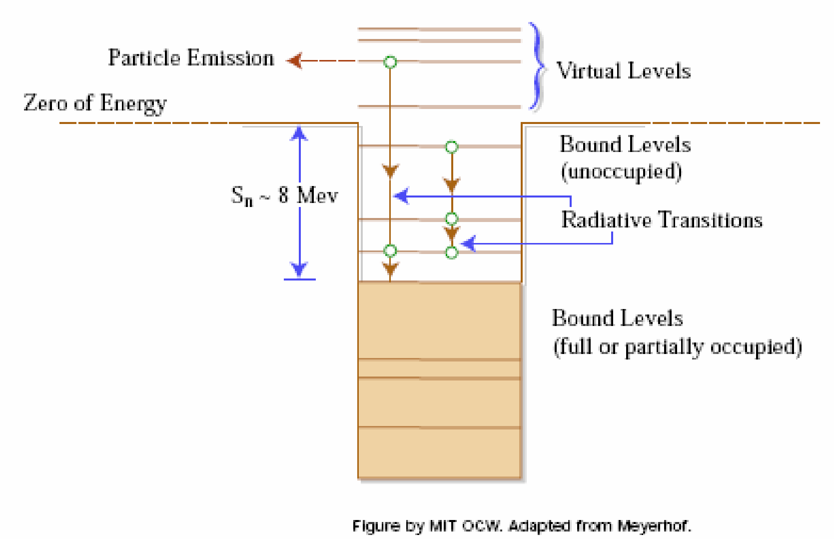

Figure 3.16. Schematic representation of nuclear levels [from Meyerhof].

SUMMARY

- We have seen that the liquid drop model allows a reasonably good descriptions of the binding energy. It also gives a qualitative explanation for spontaneous fission.
- The Fermi gas model, assuming a simple 3D square well potential (different for protons and neutrons) explained the terms in the semi-empirical mass formula that were not derived from the liquid drop picture.
- We've also seen that nucleons can move freely inside the nucleus. This agrees with the idea that they experience an overall effective potential created by the sum of the other nucleons.
- We have also looked at further experimental evidence, that the Fermi gas model cannot explain and we have invoked the shell model to explain them. The Shell Model has explained the existence of magic number nuclei as evidence of the existence of nuclear shell structures..

EXERCISE 3

4. List the analogies between liquid drop and a nucleus.
5. What are the main assumptions on which the Fermi gas model is based?
6. What are magic numbers? Give three examples of 'doubly-magic' nuclei.
7. What are the predictions of the nuclear shell model on the ground state values angular momentum and parity of nuclei?
8. Using the shell model predictions, determine the total angular momenta and parities for the ground states of $^{11}B_5$ and $^{12}C_6$.

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Lecture 4: Radioactivity

Introduction

In previous lectures, we have pointed out that while some nuclei are particularly stable many more are not. A nucleus does not remain indefinitely in an unstable state. Unstable nuclei have the property of radioactivity, i.e. the property to transform from one nuclear state to another in a process that involves emission of radiation. In this lecture we will examine the phenomenon called radioactivity and the general characteristics of the different types of radioactive decay, as well as establish the relationship between radioactive nuclei and their decay products.

Objectives

At the end of this lecture you should be able to

1. Explain what we mean by radioactivity
2. Differentiate between stable and unstable/radioactive nuclei
3. Define Activity, Half-life, Mean life, and decay constant
4. Give examples of spontaneous and induced radioactivity
5. Derive the decay law

4.1. Decay of Unstable Nuclei

A nucleus in an excited state is unstable because it can always undergo a transition (decay) to a lower-energy state of the *same nucleus*. Such a transition will be accompanied by the emission of gamma radiation. A nucleus in either an excited or ground state also can undergo a transition to a lower-energy state of *another nucleus*. This decay is accomplished by the emission of a particle such as an alpha, electron or positron, with or without subsequent gamma emission. A nucleus which undergoes a transition *spontaneously*, that is, without being supplied with additional energy as in bombardment, is said to be radioactive. It is found experimentally that naturally occurring radioactive nuclides emit one or more of the three types of radiations, $\alpha-$ particles, $\beta-$ particles, and $\gamma-$ rays. Measurements of the energy of the nuclear radiation provide the most direct information on the energy-level structure of nuclides.

Radioactivity is the transformation of one nuclear specie into another accompanied with the emission of radiation. It is a property of some unstable nuclei. It is not a process. The transformation can also be referred to as radioactive decay or disintegration.

Radiation
It is a mode of energy transfer through space or materials. Radiation can be ionizing or non-ionizing. Ionizing radiation is very energetic and so its able to separate electrons from the atoms of materials. They could be particle radiation, e.g. beta particle, alpha particle, neutron, etc., or electromagnetic, e.g. gamma and X-rays.

Naturally occurring and Artificially induced radioactivity
Radioactivity can occur naturally (spontaneously) or it can be artificially induced. As a rule, natural radioactivity is displayed by the heavy nuclei at the end of the periodic table, usually beyond lead, e.g. ^{238}U, ^{232}Th, etc. But there are some naturally light nuclei, such as ^{87}Rb, ^{40}K, ^{14}C, ^{3}H, etc.

Radioactive radionuclides are produced by bombardment of stable nuclei with:

(i) Alpha bombardment, e.g.;

$$^{27}Al + {}^{4}He \Rightarrow {}^{30}Si + {}^{1}H$$

(ii) Neutron bombardment, as in the nuclear reactors, e.g.;

$$^{14}N + {}^{1}n \Rightarrow {}^{14}C + {}^{1}H$$
$$^{31}P + {}^{1}n \Rightarrow {}^{32}P + \gamma$$

Importance of some of these artificially produced radio-isotopes will be discussed later.

The nucleus that undergoes decay is called PARENT, the intermediate products are called DAUGHTERS and the final stable elements are called END PRODUCTS.

NOTE

What is radioactivity?

Radioactivity is **a property** of unstable nuclei, **rather than a process**. It is the transformation of one nuclear specie into another accompanied with emission of ionizing radiation

NOTE

What is radiation?
Radiation is a form of energy transfer through space or material medium. Ionizing radiation can be particle or electromagnetic wave. Example of particle radiation include alpha, beta, neutron, neutrino, and antineutrino, while examples of the electromagnetic radiation include x-ray and gamma-ray.

4.2. The decay law

The number of parent nuclei decreases with time due to radioactive decay. If there are N untransformed nuclei or atoms at time *t*, and N-ΔN untransformed nuclei at time $t+\Delta t$. The number of nuclei that transform within time Δt is therefore directly proportional to the number of available nuclei (N) and also to the time interval Δt , i.e.:

$$\Delta N(t) \alpha N(t) \text{ and } \Delta N(t) \alpha \Delta t \qquad 4.1$$

Where α is the proportionality sign. Equation(1) can be re-written as:

$$\Delta N(t) \alpha N(t) \Delta t \text{ or } \Delta N(t) = -\lambda N(t) \Delta t \qquad 4.2$$

Where λ (called decay constant) is the proportionality constant, and it is the probability of decay per unit time. The negative sign indicates that the change $\Delta N(t)$ is a decrease as *t* increases.

Activity

This the rate of decay or disintegration $\frac{dN(t)}{dt}$ and it is given by:

$$\frac{dN(t)}{dt} = \lim_{\Delta t \to 0} \frac{\Delta N(t)}{\Delta t} = -\lambda N(t) \qquad 4.3$$

The S.I unit of activity is Bequerel (Bq). 1 Bq = 1 disintegration per second. The old unit is Curie (Ci). 1 Ci = 3.7 x 10^{10} disintegrations per second.

$$\int_{N_0}^{N} \frac{dN}{N} = \int_{0}^{t} -\lambda dt$$

$$\ln(N)\Big|_{N_0}^{N} = -\lambda t\Big|_{0}^{t}$$

Where N_0 is the initial number of parent nuclei (i.e. at time $t = 0$). The equation reduces to

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t \text{ or } \frac{N}{N_0} = e^{-\lambda t}$$

Hence

$$N(t) = N_0 e^{-\lambda t} \qquad 4.4$$

Equation 4.4 is called the exponential decay law.

Half –life ($T_{1/2}$)

Decay or disintegration is a statistical or probabilistic process, i.e., it is impossible to tell when a particular nucleus/atom will decay. It can however be stated that after a certain time interval a certain fraction or percentage of the atoms/nuclei originally present in the material would have decayed. The time it takes for half of the atoms originally present to decay is called half life. Thus when the number of available radioactive atoms, i.e. $N(t)$ is equal to $N_0/2$, $t = T_{1/2}$. Substituting these into the decay law, we obtain a relation between $T_{1/2}$ and λ:

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{\frac{1}{2}}}$$

i.e. $\ln(\frac{1}{2}) = -\lambda T_{\frac{1}{2}}$ or $\ln 2 = \lambda T_{\frac{1}{2}}$

hence $T_{\frac{1}{2}} = \dfrac{0.693}{\lambda}$ 4.5

Note that λ has dimension of reciprocal of time. Also, the definition of $T_{1/2}$ does not imply that all the nuclei in the sample material will decay in a time equal to two $T_{1/2}$. Rather, after two $T_{1/2}$, the number of untransformed or undecayed nuclei will be half of $N_0/2$, i.e. $N_0/4$.

Mean life (τ)

In a sample of radioactive materials, the particles live for different times, some longer than others. Therefore the actual life time of any particular atom or nucleus can have any value between 0 and ∞. The average life of a large number of atoms/nuclei is, referred to as mean life (τ) of the sample, can therefore be defined as the average time it takes before a typical atom decays. It can also be described as the average of all possible life times, i.e.

$$\tau = \frac{Total - life - times - of - all - the - atoms}{number - of - atoms} = \frac{\int_0^{N_0} t dN}{\int_0^{N_0} dN} \qquad 4.6$$

It can be shown that

$$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} = 1.44 T_{1/2} \qquad 4.7$$

Exercise
Derive equation (4.7).

Solution

$$\tau = \frac{\int_0^{N_0} t dN}{\int_0^{N_0} dN} = \frac{\int_0^{N_0} t dN}{N_0} \qquad 4.8$$

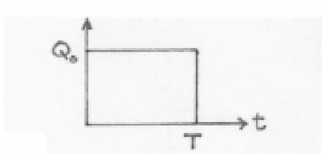
Recall that $dN = -\lambda N dt$ and $N(t) = N_0 e^{-\lambda t}$. This implies that for $N(t) = 0$, $t = \infty$; and for $N(t) = N_0$, $t = 0$. Therefore (4.8) becomes:

$$\tau = \frac{-N_0 \int_\infty^0 \lambda e^{-\lambda t} t dt}{N_0} = \int_0^\infty e^{-\lambda t} \lambda t dt = \frac{1}{\lambda}\int_0^\infty e^{-\lambda t}(\lambda t) d(\lambda t)$$ and using the integration-by-parts relation, i.e. $\int u dv = uv - \int v du$, where $\lambda t = u$ and $e^{-\lambda t} d(\lambda t) = dv$. Equation (4.7) is obtained after applying this relation as well as the limits of integration.

4.3. Decay Series

Radioisotope Production by Bombardment

There are two general ways of producing radioisotopes, activation by particle or radiation bombardment such as in a nuclear reactor or an accelerator, and the decay of a radioactive series. Both methods can be discussed in terms of a differential equation that governs the number of radioisotopes at time t, N(t). This is a first-order linear differential equation with constant coefficients, to which the solution can be readily obtained. Although there are different situations to which one can apply this equation, the analysis is fundamentally quite straightforward. We will treat the activation problem first. Let Q(t), the rate of production of the radioisotope, have the form shown in the sketch below.



This means the production takes place at a constant Q_o for a time interval (0, T), after which production ceases. During production, t < T, the equation governing N(t) is

$$\frac{dN(t)}{dt} = Q_o - \lambda N(t) \qquad 4.9$$

Because we have an external source term, the equation is seen to be inhomogeneous. The solution to (4.9) with the initial condition that there is no radioisotope prior to production, N(t = 0) = 0, is

$$N(t) = \frac{Q_o}{\lambda}(1 - e^{-\lambda t}),...,t < T \qquad 4.10$$

For t>T, the governing equation is (4.9) without the source term. The solution is

$$N(t) = \frac{Q_o}{\lambda}(1 - e^{-\lambda T})e^{-\lambda(t-T)} \qquad 4.11$$

A sketch of the solutions (4.10) and (4.11) is shown in Figure 4.1. One sees a build up of N(t) during production which approaches the asymptotic value of Q_o/λ , and after production is stopped N(t) undergoes an exponential decay, so that if λT >>1,

$$N(t) \approx \frac{Q_o}{\lambda} e^{-\lambda(t-T)} \qquad 4.12$$

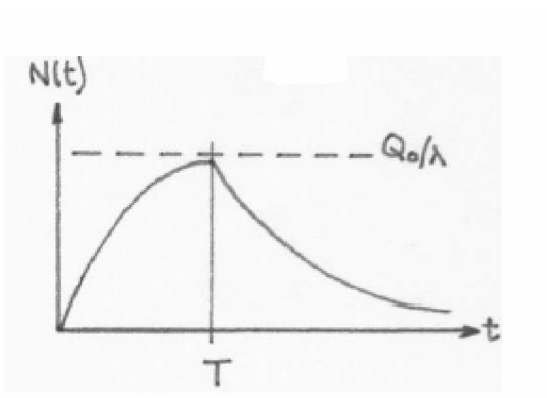


Figure 4.1. Time variation of number of radioisotope atoms produced at a constant rate Q_o for a time interval of T after which the system is left to decay

Radioisotope Production in Series Decay

Radioisotopes also are produced as the product(s) of a series of sequential decays. Consider the case of a three-member chain;

$$N_1 \xrightarrow{\lambda_1} N_2 \xrightarrow{\lambda_2} N_3 \text{ (stable)}$$

where λ_1 and λ_2 are the decay constants of the parent (N_1) and the daughter (N_2) respectively. The governing equations are:

$$\frac{dN_1(t)}{dt} = -\lambda_1 N_1(t) \qquad 4.13$$

$$\frac{dN_2(t)}{dt} = \lambda_1 N_1(t) - \lambda_2 N_2(t) \qquad 4.14$$

$$\frac{dN_3(t)}{dt} = \lambda_2 N_2(t) \qquad 4.15$$

For the initial conditions we assume there are $N_{1,0}$ nuclides of species 1 and no nuclides of species 2 and 3. The solutions to (4.13) – (4.15) are

$$N_1(t) = N_{1,0} e^{-\lambda_1 t} \qquad 4.16$$

$$N_2(t) = N_{1,0} \frac{\lambda_1}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}) \qquad 4.17$$

$$N_3(t) = N_{1,0} \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\left(\frac{1 - e^{-\lambda_1 t}}{\lambda_1} - \frac{1 - e^{-\lambda_2 t}}{\lambda_2}\right) \qquad 4.18$$

Equations (12.16) through (12.18) are known as the Bateman equations. One can use them to analyze situations when the decay constants λ_1 and λ_2 take on different relative values. We consider two such scenarios, the case where the parent is short-lived, $\lambda_1 >> \lambda_2$ and the opposite case where the parent is long- lived, $\lambda_2 >> \lambda_1$.

One should notice from (4.13) – (4.15) that the sum of these three differential equations is zero. This means that $N_1(t) + N_2(t) + N_3(t)$ = constant for any t. We also know from our initial conditions that this constant must be $N_{1,0}$. One can use this information to find $N_3(t)$ given $N_1(t)$ and $N_2(t)$, or use this as a check that the solutions given by (4.16) – (4.18) are indeed correct.

Series Decay with Short-Lived Parent

In this case one expects the parent to decay quickly and the daughter to build up quickly. The daughter then decays more slowly which means that the grand daughter will build up slowly, eventually approaching the initial number of the parent. Figure 4.2 shows schematically the behavior of the three isotopes. The initial values of $N_2(t)$ and $N_3(t)$ can be readily deduced from an examination of (4.17) and (4.18).

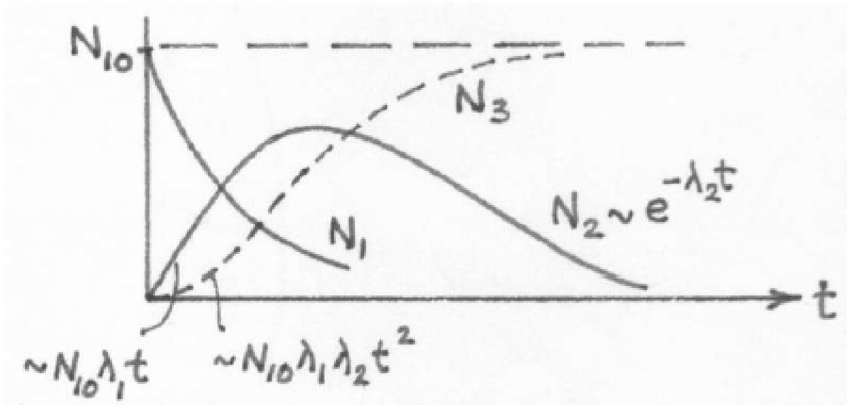


Figure 4.2. Time variation of a three member decay chain for the case $\lambda_1 >> \lambda_2$.

Series Decay with Long-Lived Parent

When $\lambda_1 << \lambda_2$, we expect the parent to decay slowly so the daughter and grand daughter will build up slowly. Since the daughter decays quickly the long-time behavior of the daughter follows that of the parent. Figure 4.3 shows the general behavior of the radioisotopes.

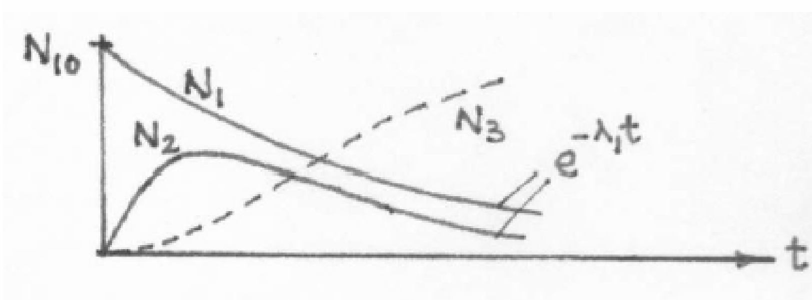


Figure 4.3. Time variation of a three-member chain with a long-lived parent

In this case we find $N_2(t) \approx N_{1,0}\dfrac{\lambda_1}{\lambda_2}e^{-\lambda t}$ or $\lambda_2 N_2(t) \approx \lambda_1 N_1(t)$. The condition of approximately equal activities is called *secular equilibrium*. Generalizing this to an arbitrary chain, we can say for the series

$N_1 \to N_2 \to N_3 \to$........

If $\lambda_2 >> \lambda_1, \lambda_3 >> \lambda_1$,....... then $\lambda_1 N_1 \approx \lambda_2 N_2 \approx \lambda_3 N_3 \approx$......

This condition can be used to estimate the half life of a very long-lived radioisotope. An example is U^{238} whose half life is so long that it is difficult to determine by directly measuring its decay. However, it is known that $U^{238} \rightarrow Th^{234} \rightarrow \ldots \rightarrow Ra^{226} \rightarrow \ldots$, and in uranium mineral the ratio of $N(U^{238})/N(Ra^{226}) = 2.8 \times 10^{6}$ has been measured, with $t_{1/2}(Ra^{226}) = 1620$ yr. Using these data we can write

$$\frac{N(U^{238})}{t_{\frac{1}{2}}(U^{238})} = \frac{N(Ra^{226})}{t_{\frac{1}{2}}(Ra^{226})} \text{ or } t_{1/2}(U^{238}) = 2.8 \times 10^{6} \times 1620 = 4.5 \times 10^{9} \text{ yr.}$$

In so doing we assume that all the intermediate decay constants are larger than that of U^{238}. It turns out that this is indeed true, and that the above estimate is a good result. For an extensive treatment of radioactive series decay, the student should consult the atomic nucleus by Evans.

SUMMARY

In this lecture we have:

1 Explained what we mean by radioactivity;
2 Differentiate between stable and unstable/radioactive nuclei;
3 Defined some terms including Activity, Half-life, Mean life, and decay constant;
4 Given examples of spontaneous and induced radioactivity;
5 Derived the decay law;
6 Explain the relationship between radioactive parents and progenies in decay series.

EXERCISE 4

1. The activity of a radioisotope is found to decrease by 30% in 1 wk. What are the values of its (**a)** decay constant, **(b)** half-life **(c)** mean life?

2. What percentage of the original activity of a radionuclide remains after **(a)** 5 half-lives **(b)** 10 half-lives?

3. The isotope ^{132}I decays by *β*– emission into stable ^{132}Xe with a half-life of 2.3 h **(a)** How long will it take for 78% of the original ^{132}I atoms to decay? **(b)** How long it will take for a sample of ^{132}I to lose 95% of its activity?

4. A radioactive sample consists of a mixture of ^{35}S and ^{32}P. Initially, 5% of the activity is due to the ^{35}S and 95% to the ^{32}P. At what subsequent time will the activities of the two nuclides in the sample be equal?

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Lecture 5: Applications of Nuclear Physics

Introduction

In this lecture we will consider some of the ways in which the nucleus affects our lives and the world around us. Its most important role is an indirect one as the center of the atom; its electric charge determines the number of electrons it attracts to form atoms, which in turn determines all chemical behaviour and physical properties of materials. However, there are several areas in which the structure of the nucleus, its decay properties, and the reactions it undergoes have a direct and controlling influence. These include the production of energy in stars, the origin of the elements (nucleosynthesis), the development of energy sources to power our technological civilization, and a wide variety of uses for radioactivity. These form the subject of this chapter.

Objectives

At the end of this lecture you should be able to

1. List some of the major applications of Nuclear Physics
2. Describe at least one application of nuclear physics in each of the following areas; medicine, power production, agriculture, industry
3. Give some examples of the dangers posed to the world by negative applications of nuclear physics

5.1. Tracer Techniques

Radio-isotopes are widely used as tracers. The radioactivity of a nuclide is not affected by the chemical and physical properties of the matrix containing it, e.g., the radioactivity in ^{24}Na is not destroyed by forming a compound such as $^{24}NaCl$ (salt), nor is it destroyed by dissolving it in water or adding it to cooking soup. Therefore the presence of radioactive isotopes in compounds can be traced as they pass through matter. Such (tracer) methods are used in industry, agriculture, medicine, environmental studies, etc.

Uses in industry

Beta emitters are used as thickness gauges in the manufacture of paper, plastics, linoleum, etc. The beta emitter is placed below the material and a detector, e.g., a Geiger-Muller detector is placed above the material. The flux or intensity of the radiation from the source is attenuated as it passes though the material, and the thicker the material the more the attenuation. Therefore, the intensity of radiation reaching the detector is inversely proportional to the thickness of the material. The detector can be calibrated such that its count-rate is read in thickness directly.

Radiation sources are also used as level indicators in automatic control of manufacturing process, e.g., to check the filling of toothpaste tubes, cement bags, powder soap packets, etc.

Uses in agriculture

Phosphorous is an essential element in any fertilizer. Uptake of phosphorous by growing plants from soil or manure can be studied by labeling the fertilizer with the radioactive isotope ^{32}P and its uptake is followed through the root system to the foliage by means of a radiation décor (e.g. Geiger Muller tube). This method has been used to find out whether a plant require root or foliage feeding.

Carbon-14 is also used to study the kinetics of plant photosynthesis. It has been shown that, by growing plants in controlled atmosphere containing $^{14}CO_2$, it is possible to understand thoroughly the complicated biochemical reactions involved in photosynthesis.

5.2. Diagnostic and Therapeutic Applications (uses in Medicine)

Na-24, as soluble $^{24}NaCl$, is used to study the flow dynamics of the body. If radioactive sodium is injected at one end of the body it can be detected within a few seconds at the other extreme end. The flow of blood can therefore be followed and any constrictions in blood vessels can be readily detected. ^{24}Na is short-lived, transforming to stable ^{24}Mg.

I-131 is known to accumulate in the thyroid gland and in the brain. It is a γ-emitter and therefore it is useful in locating deep-seated disorders such as brain tumors and malignant thyroid tumors.

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Lecture 6: Introduction to Radiation Protection and Dosimetry

Introduction

The importance of nuclear radiation in our everyday life has exposed the human and the non-human species of the environment to various levels of the radiation. It is therefore important to be aware of the likely effects of nuclear radiation on matter (Biological matter), and the need for regulation and control in applications of nuclear physics. Radiation dosimetry is a subject dedicated to the quantification of radiation interaction with matter – particularly in the living matters. The aim of radiation dosimetry is to protect living matter from damaging effects of radiation, i.e. Radiation Protection. This will be the subject of this lecture, but we will start with the quantifications of radiation in matter.

Objectives

At the end of this lecture you should be able to

1. Define dosimetric quantities, such as dose, dose rate, exposure, absorbed dose, dose equivalent, effective dose, etc.
2. Give examples of external and internal exposures to radiation
3. Differentiate between deterministic and stochastic effects of radiation
4. List the basic elements of radiation protection

6.1. Definitions of Dosimetric Quantities

Exposure (X)

Exposure is defined for gamma and X rays in terms of the amount of ionization they produce in air. The unit of exposure is called the roentgen (R) and was introduced at the Radiological Congress in Stockholm in 1928. It was originally defined as that amount of gamma or X radiation that produces in air 1 esu of charge of either sign per 0.001293 g of air. (This mass of air occupies 1 cm^3 at standard temperature and pressure.) The charge involved in the definition of the roentgen includes both the ions produced directly by the incident photons as well as ions produced by all secondary electrons. Since 1962, exposure has been defined by the International Commission on Radiation Units and Measurements (ICRU) as the quotient $\Delta Q/\Delta m$, where ΔQ is the sum of all charges of one sign produced in air when all the electrons liberated by photons in a mass Δm of air are completely stopped in air. The unit roentgen is now defined as

$$1\ \text{R} = 2.58\times10^{-4}\ \text{Ckg}^{-1}. \qquad 6.1$$

The concept of exposure applies only to electromagnetic radiation; the charge and mass used in its definition, as well as in the definition of the roentgen, refer only to air.

Example
Show that 1 esu cm^{-3} in air at STP is equivalent to the definition (6.1) of 1 R of exposure.

Solution
Since the density of air at STP is 0.001293 g cm^{-3} and 1 esu = 3.34×10^{-10} C, we have
1 esu/cm^3 = (3.34×10^{-10} C)/ (0.001293 g × 10^{-3} kg g^{-1}) = 2.58×10^{-4} Ckg^{-1}.

NOTE

Exposure is a measure of the ability of photons to cause ionization in air. It is defined as the sum of the charge of one sign (+ or -) produced by photon irradiation per unit mass of air. The traditional unit of exposure is the Roentgen (R). No new unit for exposure, although Coulomb/kg is sometime applicable. The use of exposure has since been de-emphasized.

Absorbed dose (D)
The concept of exposure and the definition of the roentgen provide a practical, measurable standard for electromagnetic radiation in air. However, additional concepts are needed to apply to other kinds of radiation and to other materials, particularly tissue. The primary physical quantity used in dosimetry is the absorbed dose. It is defined as the energy absorbed per unit mass from any kind of ionizing radiation in any target. The unit of absorbed dose, J kg^{-1}, is called the gray (Gy). The older unit, the rad, is defined as 100 erg g^{-1}. It follows that

$$1\ \text{Gy} \equiv 1\ \text{J kg}^{-1} = 10^7\ \text{erg} / 10^3\ \text{g}^1 = 10^4\ \text{erg g}^{-1} = 100\ \text{rad}. \qquad 6.2$$

The absorbed dose is often referred to simply as the dose. It is treated as a point function, having a value at every position in an irradiated object. One can compute the absorbed dose in air when the exposure is 1 R. Photons produce secondary electrons in air, for which the average energy needed to make an ion pair is W = 34 eV per ion pair = 34 JC^{-1}. Using a more precise W value) one finds

$$1\ \text{R} = 2.58 \times 10^{-4}\ \text{C/kg} \times 33.97\ \text{J/C} = 8.76 \times 10^{-3}\ \text{Jkg}^{-1}. \qquad 6.3$$

Thus, an exposure of 1 R gives a dose in air of 8.76×10^{-3} Gy (= 0.876 rad).
Calculations also show that a radiation exposure of 1 R would produce a dose of 9.5×10^{-3} Gy (= 0.95 rad) in soft tissue. This unit is called the rep ("roentgen equivalent-physical") and was used in early radiation-protection work as a measure of the change produced in living tissue by radiation. The rep is no longer employed.

Absorbed dose in a medium can also be given in terms of the number of ion pairs (ionization) caused:

$$Dose = \frac{Nxw}{mass} \qquad 6.4$$

Where N = no of ion pairs formed and w = energy required to form 1 ion pair.

Example

A point source of radiation causes ionization at a point in air. Calculate the dose (in Gy) at the point in air where the ionization is 2.4 ion pairs per cm^3. Take the energy required to produce one ion pair as 34 eV and density of air to be 1.29 x 10^{-3} of dry air kg m^{-3}.

Solution

Equation 6.4 can be rewritten as:

$$Dose = \frac{Nxw}{mass} = \frac{(N/volume)w}{mass/volume} = \frac{(2.4x10^6)(34x1.6x10^{-19})}{(1.29x10^{-3})} J/kg$$

Dose Equivalent (H)

It has long been recognized that the absorbed dose needed to achieve a given level of biological damage (e.g., 50% cell killing) is often different for different kinds of radiation. Radiation with a high linear energy transfer (LET) is generally more damaging to a biological system per unit dose than radiation with a low LET.

To allow for the different biological effectiveness of different kinds of radiation, the International Commission on Radiological Protection (ICRP), introduced the concept of dose equivalent for radiation-protection purposes. The dose equivalent H is defined as the product of the absorbed dose D and a dimensionless quality factor Q, which depends on LET:

$$H= QD. \qquad 6.5$$

In principle, other multiplicative modifying factors can be included along with Q to allow for additional considerations (e.g., dose fractionation), but these are not ordinarily used. Until the 1990 recommendations made in ICRP Publication 60, the dependence of Q on LET was defined as given in Table 6.1. Since then, the ICRP has defined Q in accordance with Table 6.2. In the context of quality factor, LET is the unrestricted stopping power, $L\infty$. For incident charged particles, it is the LET of the radiation in water, expressed in keV per μm of travel. For neutrons, photons, and other uncharged radiation, LET refers to that which the secondary charged particles they generate would have in water. Like absorbed dose, dose equivalent is a point function. When dose is expressed in Gy, the (SI) unit of dose equivalent is the sievert

(Sv). With the dose in rad, the older unit of dose equivalent is the rem ("roentgen-equivalent man"). Since 1 Gy = 100 rad, 1 Sv = 100 rem.

Table 6.1 Dependence of Quality factor Q on LET of radiation as formerly recommended by ICRP (Turner, 2007)

LET (keV μm^{-1} in water)	Q
3.5 or less	1
3.5 – 7.0	1-2
7.0 – 23	2-5
23 – 53	5-10
53 -175	10-20
Gamma rays, X-rays, electrons, positrons of any LET	1

Table 6.2 Dependence of Quality factor Q on LET as currently recommended by ICRP (Turner, 2007)

LRT, L(keV μm^{-1} in water)	Q
<10	1
10-100	0.32L – 2.2
>100	$300/\sqrt{L}$

Dose equivalent has been used extensively in protection programs as the quantity in terms of which radiation limits are specified for the exposure of individuals. Dose equivalents from different types of radiation are simply additive.

Example
A worker receives a whole-body dose of 0.10 mGy from 2-MeV neutrons. Estimate the dose equivalent, based on Table 6.1.

Solution
Most of the absorbed dose is due to the elastic scattering of the neutrons by the hydrogen in tissue. To make a rough estimate of the quality factor, we first find Q for a 1-MeV proton — the average recoil energy for 2-MeV neutrons. The stopping power for a 1-MeV proton in water is 270 $MeVcm^{-1}$ = 27 keV μm^{-1}. Under the current recommendations of the ICRP, Q is defined according to Table 6.2. However, the older recommendations, which include Table 6.1, are still in effect.

We see from Table 6.1 that an estimate of $Q \sim 6$ should be reasonable for the recoil protons. The recoil O, C, and N nuclei have considerably higher LET values, but do not contribute as much to the dose as H. (LET is proportional to the square of a particle's charge.) Without going into more detail, we take the overall quality factor, $Q \sim 12$, to be twice that for the recoil protons alone. Therefore, the estimated dose equivalent is $H \sim 12 \times 0.10 = 1.2$ mSv. [The value $Q = 10$ is obtained from detailed calculations] We note that Table 6.2 implies a comparable value, $Q = 6.4$, for the protons.

By the early 1990s, the ICRP had replaced the use of LET-dependent quality factors by *radiation weighting factors*, *w*, specified for radiation of a given type and energy. The quantity on the left-hand side of the new equation, $H = wD$, is then called the *equivalent dose.* In some regulations the older terminology, dose equivalent and quality factor, is still employed. However, the latter has come to be specified by radiation type and energy, rather than LET.

Equivalent Dose
The equivalent dose, HT,R, in a tissue or organ T due to radiation R, is defined as the product of the average absorbed dose, DT,R, in T from R and a dimensionless radiation weighting factor, wR, for each radiation:

$$H_{T,R} = w_R D_{T,R}. \qquad 6.6$$

The values of w_R specified by the ICRP are shown in Table 6.3. When the radiation consists of components with different w_R, then the equivalent dose in T is given by summing all contributions:

$$H_T = \sum_R w_R D_{T,R} \qquad 6.7$$

With $D_{T,R}$ expressed in Gy (1 Gy = 1 Jkg^{-1}), $H_{T,R}$ and H_T are in Sv (1 Sv = 1 Jkg^{-1}).

NOTE
The equivalent dose replaces the dose equivalent for a tissue or organ, defined earlier. The two are conceptually different. Whereas dose equivalent in an organ is defined as a point function in terms of the absorbed dose weighted by a quality factor everywhere, equivalent dose in the organ is given simply by the *average* absorbed dose weighted by the factor w_R.

Table 6.3 Radiation weighting factors, w_R by ICRP

Radiation	w_R
- X and gamma rays, electrons, positrons, and muons	1
- Neutrons, energy <10 keV	5
10 keV to 100 keV	10
>100 keV to 2 MeV	20
> 2 MeV to 20 MeV	10
> 20 MeV	5
- Protons, other than recoil protons and energy > 2 MeV	5
- Alpha particles, fission fragments, and nonrelativistic heavy nuclei	20

For radiation types and energies not included in Table 6.3, the ICRP give a prescription for calculating an approximate value of w_R as an average quality factor, $\overline{Q}$. For this purpose, the quality factor Q is defined in terms of the linear energy transfer L by means of Table 6.2, given earlier in the text. One computes the dose–average value of Q at a depth of 10 mm in the standard tissue sphere of diameter 30 cm specified by the ICRU. Specifically, at the prescribed depth, one calculates

$$w_R \cong \overline{Q} = \frac{1}{D}\int_0^\infty Q(L)D(L)dL\,, \qquad 6.8$$

where $D(L)\mathrm{d}L$ is the absorbed dose at linear energy transfer (LET) between L and L + dL.

Effective Dose (E)

Since different tissues of the body respond differently to radiation, the probability for stochastic effects that result from a given equivalent dose will generally depend upon the particular tissue or organ irradiated. To take such differences into account, the ICRP assigned dimensionless tissue weighting factors w_T, shown in Table 6.4, which add to unity when summed over all tissues T. The equivalent dose H_T in a given tissue, weighted by w_T, gives a quantity that is intended to correlate with the overall detriment to an individual, independently of T. The detriment includes the different mortality and morbidity risks for cancers, severe genetic effects, and the associated length of life lost. Table 6.4 implies, for example, that an equivalent dose of 1 mSv to the lung entails the same overall detriment for stochastic effects as an equivalent dose to the thyroid of (0.12/0.05)×(1 mSv) = 2.4 mSv.

Table 6.4 Tissue weighting factors, w_T

Tissue or Organ	w_T
Gonads	0.20
Bone marrow	0.12
Colon	0.12
Lung	0.12
Stomach	0.12
Bladder	0.05
Breast	0.05
Liver	0.05
Esophagus	0.05
Thyroid	0.05
Skin	0.01
Bone surface	0.01
Remainder*	0.05

* Note: The data refer to a reference population of equal numbers of both sexes and a wide range of ages. In the definition of effect dose, they apply to workers, to the whole population, and to either sex. The w_T are based on rounded values of the organ's contribution to the total detriment.

The risk for all stochastic effects for an irradiated individual is represented by the effective dose, E, defined as the sum of the weighted equivalent doses over all tissues:

$$E = \sum_T w_T H_T \qquad 6.9$$

Like H_T, E is expressed in Sv. The risk for all stochastic effects is dependent only on the value of the effective dose, whether or not the body is irradiated uniformly. In the case of uniform, whole-body irradiation, H_T is the same throughout the body. Then, since the tissue weighting factors sum to unity,

$$E = \sum_T w_T H_T = H_T \sum_T w_T = H_T \qquad 6.10$$

the value of the equivalent dose everywhere. The effective dose replaces the earlier effective dose equivalent. The latter quantity was defined the same way as E in Equation 6.9, with H_T being the organ or tissue dose equivalent.

It should be understood that the procedures embodied in Equation 6.9 have been set up for use in radiological protection. As the note to Table 6.4 specifies, the values of w_T are simplified and rounded for a reference population of equal numbers of males and females over a wide range of ages. They "should not be used to obtain specific estimates of potential health effects for a given individual."

Committed Equivalent Dose

When a radionuclide is taken into the body, it can become distributed in various tissues and organs and irradiate them for some time. For the single intake of a radionuclide at time t_0, the committed equivalent dose over a subsequent time τ in an organ or tissue T is defined as

$$H_T(\tau) = \int_{t_0}^{t_0+\tau} \dot{H}_T \, dt \qquad 6.11$$

where $\dot{H}_T$ is the equivalent-dose rate in T at time t. Unless otherwise indicated, an integration time τ = 50 y after intake is implied for occupational use and 70 y for members of the public.

Committed Effective Dose

By extension, the committed effective dose $E(\tau)$ following the intake of a radionuclide is the weighted sum of the committed equivalent doses in the various tissues T:

$$E(\tau) = \sum_T w_T H_T(\tau) \qquad 6.12$$

The effective half-life of a radionuclide in a tissue is determined by its radiological half-life and its metabolic turnover rate. For radionuclides with effective half-lives of no more than a few months, the committed quantities, Eqs. (6.11) and (6.12), are practically realized within one year after intake. If a radionuclide is retained in the body for a long time, then the annual equivalent and effective doses it delivers will be considerably less than the committed quantities.

The committed effective dose replaces the earlier committed effective dose equivalent. The latter is defined like Eq. (6.12), with *H*T representing the committed dose equivalent in the organ or tissue T.

Collective Quantities

The quantities just defined relate to the exposure of an individual person. The ICRP has defined other dosimetric quantities that apply to the exposure of groups or populations to radiation. The *collective equivalent dose* and the *collective effective dose* are obtained by multiplying the average value of these quantities in a population or group by the number of persons therein. The collective quantities are then expressed in the unit, "person-sievert," and can be associated with the total consequences of a given exposure of the population or group.

The Commission additionally defines collective dose commitments as the integrals over infinite time of the average individual $\dot{H}_T$ and $\dot{E}$ due to a specified event, either for a critical population group or for the world population.

Other Dosimetric Concepts and Quantities

Kerma

A quantity related to dose for indirectly ionizing radiation (photons and neutrons) is the initial kinetic energy of all charged particles liberated by the radiation per unit mass. This quantity, which has the dimensions of absorbed dose, is called the *kerma* (Kinetic Energy Released per unit mass). By definition, kerma includes energy that may subsequently appear as bremsstrahlung and it also includes Auger-electron energies. The absorbed dose generally builds up behind a surface irradiated by a beam of neutral particles to a depth comparable with the range of the secondary charged particles generated. The kerma, on the other hand, decreases steadily because of the attenuation of the primary radiation with increasing depth. The two are identical as long as all of the initial kinetic energy of the recoil charged particles can be considered as being absorbed locally at the interaction site. Specifically, kerma and absorbed dose at a point in an irradiated target are equal when charged-particle equilibrium exists there and bremsstrahlung losses are negligible. It is often of interest to consider kerma or kerma rate for a specific material at a point in free space or in another medium. The specific substance itself need not actually be present. Given the photon or neutron fluence and energy spectra at that point, one can calculate the kerma for an imagined small amount of the material placed there. It is thus convenient to describe a given radiation field in terms of the kerma in some relevant, or reference, material. For example, one can specify the air kerma at a point in a water phantom or the tissue kerma in air. Additional information on kerma can be found in the references listed at the end of the lecture.

6.2 Biological Effects of Radiation

The process of ionization charges atoms and molecules. In cells, some of the initial changes may have both short and long-term consequences. Cellular damages as a result of irradiation (ionization) may lead to cell's death, its impairment to reproduce, or its modification. These outcomes have profoundly differently implications for the organism as a whole.

It is generally assumed that biological effects on the cell result from both direct and indirect action of radiation. Direct effects are produced by the initial action of the radiation itself and indirect effects are caused by the later chemical action of free radicals and other radiation products. An example of a direct effect is a strand break in DNA caused by an ionization in the molecule itself. An example of an indirect effect is a strand break that results when an OH radical attacks a DNA sugar at a later time (between about 10^{-12} s and about 10^{-9} s).

Depending on the dose, kind of radiation, and observed endpoint, the biological effects of radiation can differ widely. Some occur relatively rapidly while others may take years to become evident. Table 6.3 (13.1) includes a summary of the time scale for some important biological effects caused by ionizing radiation. Probably by about 10^{-3} s, radicals produced by a charged-particle track in a biological system have all reacted. Some biochemical processes are altered almost immediately, in less than about 1 s. Cell division can be affected in a matter of minutes. In higher organisms, the time at which cellular killing becomes expressed as a clinical syndrome is related to the rate of cell renewal. Following a large, acute, whole-body dose of radiation, hematopoietic death of an individual might occur in about a month. A higher dose could result in earlier death (1 to 2 wk) from damage to the gastrointestinal tract. At still higher doses, in the range of 100 Gy, damage to membranes and to blood vessels in the brain leads to the cerebrovascular syndrome and death within a day or two. Other kinds of damage, such as lung fibrosis, for example, may take several months to develop. Cataracts and cancer occur years after exposure to radiation. Genetic effects, by definition, are first seen in the next or subsequent generations of an exposed individual.

Classification of Radiation effects

The biological effects of radiation can be divided into two general categories, stochastic and deterministic, or nonstochastic.

Stochastic effects

As the name implies, stochastic effects are those that occur in a statistical manner. Cancer is one example. If a large population is exposed to a significant amount of a carcinogen, such as radiation, then an elevated incidence of cancer can be expected. Although we might be able to predict the magnitude of the increased incidence, we cannot say which particular individuals in the population will contract the disease and which will not. Also, since there is a certain natural incidence of cancer without specific exposure to radiation, we will not be completely certain whether a given case was induced by or would have occurred without the exposure. In addition, although the expected incidence of cancer increases with dose, the severity of the disease in a stricken individual is not a function of dose.

Stochastic effects of radiation have been demonstrated in man and in other organisms only at relatively high doses, where the observed incidence of an effect is not likely due to a statistical fluctuation in the normal level of occurrence. At low doses, one cannot say with certainty what the risk is to an individual. As a practical hypothesis, one usually assumes that any amount of radiation, no matter how small, entails some risk. However, there is no agreement among experts on just how risk varies as a function of dose at low doses.

Deterministic effects

In contrast, deterministic effects are those that show a clear causal relationship between dose and effect in a given individual. Usually there is a threshold below which no effect is observed, and the severity increases with dose. Skin reddening is an example of a deterministic effect of radiation. Other examples include blistering, loss of skin surface, induction of opacities in the lens and visual impairment (cataract); inflammation of organs, death; mental retardation in case of exposure in uterus.

6.3. Basics of Radiation Protection

Man benefits greatly from the use of X rays, radioisotopes, and fissionable materials in medicine, industry, research, and power generation. However, the realization of these gains entails the routine exposure of persons to radiation in the procurement and normal use of sources as well as exposures from accidents that might occur. Since any radiation exposure presumably involves some risk to the individuals involved, the levels of exposures allowed should be worth the result that is achieved.

In principle, therefore, the overall objective of radiation protection is to balance the risks and benefits from activities that involve radiation. If the standards are too lax, the risks may be unacceptably large; if the standards are too stringent, the activities may be prohibitively expensive or impractical, to the overall detriment to society.

The balancing of risks and benefits in radiation protection cannot be carried out in an exact manner. The risks from radiation are not precisely known, particularly at the low levels of allowed exposures, and the benefits are usually not easily measurable and often involve matters that are personal value judgments. Because of the existence of legal radiation-protection standards, in use everywhere, their acceptance rests with society as a whole rather than with particular individuals or groups. Even if the risks from low-level radiation were established quantitatively on a firm scientific basis, the setting of limits would still represent a social judgment in deciding how great a risk to allow. The setting of highway speed limits is an example of such a societal decision—one for which extensive quantitative data are available at the levels of risk actually permitted and accepted.

The specific objectives of radiation protection are:

(1) to prevent the occurrence of clinically significant radiation-induced deterministic effects by adhering to dose limits that are below the apparent threshold levels and
(2) to limit the risk of stochastic effects, cancer and genetic effects, to a reasonable level in relation to societal needs, values, benefits gained and economic factors.

Elements of Radiation-Protection Programs

Different uses of ionizing radiation warrant the consideration of different exposure guidelines. Medical X rays, for example, are generally under the control of the physician, who makes a medical judgment as to their being warranted. Specific radiation-protection standards, such as those recommended by the International Commission on Radiological Protection (ICRP), have been traditionally applied to the "peaceful uses of atomic energy," the theory being that these activities justify the exposure limits being specified. In contrast, different exposure criteria might be appropriate for military or national-defense purposes or for space exploration, where the risks involved and the objectives are of an entirely different nature than those for other uses of radiation.

The maximum levels of exposure permitted are deemed acceptable in view of the benefits to mankind, as judged by various authorities and agencies who, in the end, have the legal responsibility for radiation safety. Since, in principle, the benefits justify the exposures, the limits apply to an individual worker or member of the public independently of any medical, dental, or background radiation exposure he or she might receive.

Different permissible exposure criteria are usually applied to different groups of persons. Certain levels are permitted for persons who work with radiation. These guidelines are referred to as "occupational" or "on-site" radiation-protection standards. Other levels, often one-tenth of the allowable occupational values, apply to members of the general public. These are referred to as "non-occupational" or "offsite" guides. Several philosophical distinctions can be drawn in setting occupational and nonoccupational standards. In routine operations, radiation workers are exposed in ways that they and their employers have some control over. The workers are also compensated for their jobs and are free to seek other employment. Members of the public, in contrast, are exposed involuntarily to the gaseous and liquid effluents that are permitted to escape from a site where radioactive materials are handled. In addition, off-site exposures usually involve a larger number of persons as well as individuals in special categories of concern, such as children and pregnant women. (Special provisions are also made for occupational radiation exposure of women of child-bearing age.)

On a worldwide scale, the potential genetic effects of radiation have been addressed in setting radiation standards. Exposure of a large fraction of the world's population to even a small amount of radiation represents a genetic risk to mankind that can be passed on indefinitely to succeeding generations. In contrast, the somatic risks are confined to the persons actually exposed.

An essential facet of the application of maximum permissible exposure levels to radiation-protection practices is the ALARA (as low as reasonably achievable) philosophy. The ALARA concept gives primary importance to the principle that exposures should always be kept as low as practicable. The maximum permissible levels are not to be considered as "acceptable," but, instead, they represent the levels that should not be exceeded.

Another consideration in setting radiation-protection standards is the degree of control or specificity that the criteria may require. The ICRP has generally made recommendations for the limits for individual workers or members of other groups in a certain length of time, for example, a year or three months. Without requiring the specific means to achieve this end, the recommendations allow maximum flexibility in their application. Many federal and international agencies, however, have very specific regulations that must be met in complying with the ICRP limits.

Occupational Limits
The ICRP recommends occupational annual effective-dose limit of 50 mSv. However, its cumulative limit is different, being simply 100 mSv in any consecutive 5-y period.

For preventing deterministic effects, the ICRP recommends the following annual occupational equivalent-dose limits: 150 mSv for the crystalline lens of the eye and 500 mSv for localized areas of the skin, the hands, and feet. The limits for deterministic effects apply irrespective of whether one or several areas or tissues are exposed.

Nonoccupational Limits
Historically, limits for nonoccupational exposures have been one-tenth those for occupational exposures. That practice continues. The following recommendations for the exposure of an individual to man-made sources (natural background and medical exposures are not to be included) apply:

For continuous (or frequent) exposure, it is recommended that the annual effective dose not exceed 1 mSv. Furthermore, a maximum annual effective dose limit of 5 mSv is recommended to provide for infrequent annual exposures.

For deterministic effects, the recommendations in ICRP Publication 60 are: An individual annual effective dose limit of 1 mSv is also set for nonoccupational exposures. There is a proviso that a higher annual limit may be applied, if the annual average over 5 y does not exceed 1 mSv.

Principle of External Radiation Protection

We now describe procedures for limiting the dose received from radiation sources outside the human body. There are other procedures for limiting dose received from radionuclides that can enter the body.

Distance, Time, and Shielding
In principle, one’s dose in the vicinity of an external radiation source can be reduced by increasing the distance from the source, by minimizing the time of exposure, and by the use of shielding. Distance is often employed simply and effectively. For example, tongs are used to handle radioactive sources in order to minimize the dose to the hands as well as the rest of the body. Limiting the duration of an exposure significantly is not always feasible, because a certain amount of time is usually required to perform a given task. Sometimes, though, practice runs beforehand without the source can reduce exposure times when an actual job is carried out.

While distance and time factors can be employed advantageously in external radiation protection, shielding provides a more reliable way of limiting personnel exposure by limiting the dose rate. In principle, shielding alone can be used to reduce dose rates to desired levels. In practice, however, the amount of shielding employed will depend on a balancing of practical necessities such as cost and the benefit expected.

EXERCISE 6

1. Define the following dosimetric quantities: Exposure, absorbed dose, Dose equivalent, equivalent dose, and effective dose.

2. What are the specific objectives of radiation protection?

3. A worker receives a lung dose of 6 mGy from alpha radiation from an internally deposited radionuclide plus a 20-mGy uniform, whole-body dose from external gamma radiation.
(a) What is the equivalent dose to the lung?
(b) What is the his or her effective dose?

4. Calculate the effective dose for an individual who has received the following exposures: 1 mGy alpha to the lung; 2 mGy thermal neutrons, whole body; 5 mGy gamma, whole body; 200 mGy beta to the thyroid.

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Lecture 7: Introduction to Elementary Particle

Introduction

So far in our discussions we have identified electrons, photons, protons and neutrons as the fundamental constituents of the atom. But recent studies, using nuclear accelerators, show that there are about a hundred so called elementary particles. In Particle physics, also called High-Energy Physics, we study the interactions between elementary constituents of the nucleus and the rules governing their behaviours. These particles are actively being studied presently, and the effort is leading to a deeper understanding of nature. A systematic treatment of particle interactions is beyond the level of this study material. In this lecture we will limit our discussions to the grouping of particles into different families and identifying the major characteristics that distinguish one family from the others. The material in this lecture is adopted from Nuclear Physics by Nuclear Physics Panel (NPP, 1986).

Objectives

At the end of this lecture you should be able to

1. Classify elementary particles into Leptons, Hadrons and list the distinguishing characteristics
2. List the six basic kinds of quacks;
3. Explain the existence of nuclear forces as a vestige of two forces: (1) 'color' force between quacks and (2) exchange force between hadrons;
4. Give at least three examples of particles and their corresponding antiparticles

7.1. THE ELEMENTARY PARTICLES – Historical review

The experimental study of elementary-particle physics—also known by the inexact name high-energy physics—diverged from that of nuclear physics around 1950, when developing accelerator technology made it relatively easy to search for other—and ultimately more basic—"elementary" particles apart from the hitherto well known proton and neutron. An enormous variety of sub-nuclear particles has by now been discovered and characterized, some of which are truly elementary (as far as we can tell in 1984), but most of which are not.

Along with the discovery of these particles came major theoretical advances, such as the electroweak synthesis, and mathematical theories attempting to classify and explain the seemingly arbitrary proliferation of particles (several hundred by now) as accelerator energies becomes higher. Chief among these theories, because of their great power and generality, are the *quantum field theories* of the fundamental interactions. All such theories are relativistic, i.e., they incorporate *relativity* into a quantum- mechanical framework suitable to the problem at hand. They thus represent the deepest level of understanding of which we are currently capable.

7.2 Classes of elementary particles

The nucleus as we now perceive it does *not* consist of just protons and neutrons, which are not even elementary particles to begin with. To understand the atomic nucleus properly, therefore, we must take into account all the other particles that exist there under various conditions, as well as the compositions of the nucleons and of these other particles.

Physicists now believe that there are three classes of elementary particles— *leptons, quarks*, and *elementary vector bosons*—and that every particle, elementary or not, has a corresponding *antiparticle*. Here we must make a short digression into the subject of *antimatter*. An antiparticle differs from its ordinary particle only in having some opposite elementary properties, such as electric charge. Thus, the antiparticle of the electron is the positively charged *positron;* the antinucleons are the negatively charged *antiproton* and the neutral *antineutron*. The antiparticle of an antiparticle is the original particle; some neutral particles, such as the photon, are considered to be their own antiparticles. In general, when a particle and its corresponding antiparticle meet, they can annihilate each other (vanish completely) in a burst of pure energy, in accord with the Einstein *mass-energy equivalence* formula, $E = mc^2$. Antiparticles are routinely observed and used in many kinds of nuclear-and particle-physics experiments, so they are by no means hypothetical. In the ensuing discussions of the various classes of particles, it should be remembered that for every particle mentioned there is also an antiparticle.

NOTE
The three classes of elementary particles are:
(i) Leptons
(ii) Quarks
(iii) Elementary vector bosons

7.2.1. Leptons

Leptons are weakly interacting particles, i.e., they experience the weak interaction but not the strong interaction; they are considered to be pointlike, structureless entities. The most familiar lepton is the *electron*, a very light particle (about 1/1800 the mass of a nucleon) with unit negative charge; it therefore also experiences the electromagnetic interaction. The *muon* is identical to the electron, as far as we know, except for being about 200 times heavier. The tau particle, or *tauon*, is a recently discovered lepton that is also identical to the electron except for being about 3500 times heavier (making it almost twice as heavy as a nucleon). The very existence of these "heavy electrons" and "very heavy electrons" is a major puzzle for physicists.

Associated with each of the three charged leptons is a lepton called a *neutrino*: thus there is an electron neutrino, a muon neutrino, and a tauon neutrino. Neutrinos are electrically neutral and therefore do not experience the electromagnetic interaction. They have generally been assumed to have zero rest mass and must therefore move at the speed of light, according to relativity, but the question of their mass is currently controversial. If the electron neutrino, in particular, does have any mass, it is very slight indeed. The possible existence of such a mass, however, has great cosmological significance: because there are so *many* neutrinos in the universe, left over from the big bang, their combined mass might exert a gravitational effect great enough to slow down and perhaps halt the present outward expansion of the universe.

Neutrinos and antineutrinos are commonly produced in the radioactive process called *beta decay* (a weak-interaction process). Here a neutron in a nucleus emits an electron (often called a *beta particle*) and an antineutrino, becoming a proton in the process. Similarly, a proton in a nucleus may beta- decay to emit a positron and a neutrino, becoming a neutron in the process. Neutrinos and antineutrinos thus play an important role in nuclear physics. Unfortunately, they are extremely difficult to detect, because in addition to being neutral, they have the capability of passing through immense distances of solid matter without being stopped. With extremely large detectors and much patience, however, it is possible to observe small numbers of them.

We have now seen that there are three pairs, or families, of charged and neutral weakly interacting leptons, for a total of six; there are therefore also six antileptons. Let us next look at the quarks, of which there are also three pairs— but there the similarity ends.

7.2.2 Quarks

Quarks are particles that interact both strongly and weakly. They were postulated theoretically in 1964 in an effort to unscramble the profusion of known particles, but experimental confirmation of their existence was relatively slow in coming. This difficulty was due to the quarks' most striking single characteristic: they apparently cannot be produced as free particles under any ordinary conditions. They seem instead always to exist as bound combinations of three quarks, three antiquarks, or a quark-antiquark pair.

Thus, although they are believed to be truly elementary particles, they can be studied—so far—only within the confines of composite particles (which are themselves often inside a nucleus). This apparent inability of quarks, under ordinary conditions, to escape from their bound state is called *quark confinement*. There are six basic kinds of quarks, classified in three pairs, or families; their names are *up* and *down, strange* and *charm*, and *top* and *bottom*. Only the top quark has not yet been shown to exist, but preliminary evidence for it was reported in the summer of 1984. The six varieties named above are called the quark *flavors*, and each flavor is believed to exist in any of three possible states called *colors*. (None of these names have any connection with their usual meanings in everyday life; they are all fanciful and arbitrary.) Flavor is a property similar to that which distinguishes the three families of leptons electron, muon, and tauon), whereas color is a property more analogous to electric charge.

Another odd property of quarks is that they have fractional electric charge; unlike all other charged particles, which have an integral value of charge, quarks have a charge of either -1/3 or + 2/3. Because free quarks have never been observed, these fractional charges have never been observed either—only inferred. They are consistent, however, with everything we know about quarks and the composite particles they constitute. These relatively large composite particles are the *hadrons*, all of which experience the strong interaction as well as the weak interaction. Although all quarks are charged, not all hadrons are charged; some are neutral, owing to cancellation of quark charges. There are two distinctly different classes of hadrons: baryons and mesons. *Baryons*—which represent by far the largest single category of subnuclear particles—consist of three quarks (antibaryons consist of three antiquarks) bound together inside what is referred to as a *bag*. This is just a simple model (not a real explanation) to account for the not yet understood phenomenon of quark confinement: the quarks are assumed to be "trapped" in the bag and cannot get out.

Now, finally, we can say what nucleons really are: they are baryons, and they consist of up (*u*) and down (*d*) quarks. Protons have the quark structure *uud*, and neutrons have the quark structure *udd*. A larger class of baryons is that of the *hyperons*, unstable particles whose distinguishing characteristic is *strangeness*, i.e., they all contain at least one strange (*s*) quark. In addition, there are dozens of *baryon resonances*, which are massive, extremely unstable baryons with lifetimes so short (about 10^{-23} second) that they are not considered to be true particles.

What are nucleons?

They are baryons, and they consist of up (*u*) and down (*d*) quarks.
Protons have the quark structure *uud*, and neutrons have the quark structure *udd*.

The other class of hadrons is the *mesons*, of which there are also many kinds. These are unstable particles consisting of a quark-antiquark pair, to which the bag model can also be applied. Like the baryons, all mesons experience the strong and weak interactions, and the charged ones also experience the electromagnetic interaction. The most commonly encountered mesons are pi mesons (*pions*) and *K* mesons (*kaons*); the latter are strange (in the quark sense) particles. All hadrons are subject to the strong force. But the strong force, as it turns out, is merely a vestige of the much stronger force that governs the interactions among the quarks themselves: the *color force*. The two forces are actually the same force being manifested in different ways, at different levels of strength.

These two manifestations of the force that holds nuclei together are of great importance, because they underlie two distinctly different levels of understanding of nuclear phenomena, beyond the simple view that encompasses only nucleons as constituents of the nucleus. The strong force is related to the presence of large numbers of mesons (especially pions) in the nucleus, and many concepts of nuclear physics cannot be understood unless the nucleus is viewed as consisting of baryons and mesons. The color force, on the other hand, is related to the presence of particles called *gluons* inside the baryons and mesons themselves; this represents a different and much deeper view of nuclear phenomena—one that is not nearly so well understood, from either theoretical arguments or experimental evidence. Gluons belong to the third class of elementary particles, the elementary vector bosons, which we will examine shortly, after a brief introduction to the concept of spin. In addition to their mass and charge, all subatomic particles (including nuclei themselves) possess an intrinsic quality called *spin*, which can be viewed naively in terms of an object spinning about an axis. The values of spin that particles can have are *quantized*: that is, they are restricted to integral values (0, 1, 2,....) or half-integral values (1/2, 3/2, 5/2,...) of a basic quantum-mechanical unit of measure. All particles that have integral values of spin are called *bosons*, and all particles that have half-integral values are *fermions*. Thus, all particles, regardless of what else they may be called, are also either bosons or fermions. Following the sequence of particles that we have discussed thus far, the classification is as follows: all leptons are fermions; all quarks are fermions; hadrons are divided—all baryons are fermions, but all mesons are bosons. In broad terms, fermions are the building-block particles that comprise nuclei and atoms, and bosons are the particles that mediate the fundamental interactions.

The significance of the fermion-boson classification lies in a quantum-mechanical law called the *Pauli exclusion principle*, which is obeyed by fermions but not by bosons. The exclusion principle states that in any system of particles, such as a nucleus, no two fermions are allowed to coexist in the identical quantum state (i.e., they cannot have identical values of every physical property). This means that all the protons and all the neutrons in a nucleus must be in different quantum states, which places restrictions on the kinds of motions that they are able to experience. No such restrictions apply to mesons, however, because they are bosons. This situation has profound consequences in the study of nuclear physics.

Most of the bosons to be discussed in the next section are elementary particles—unlike mesons—and are called *vector bosons* (because they have spin 1).

Nuclear force manifests in two ways:

There are two manifestations of nuclear force: the ***exchange force*** and the ***colour force***. The two forces are actually the same force being manifested in different ways, at different levels of strength.

7.2.3 Elementary Vector Bosons

Earlier it was mentioned that the fundamental interactions are mediated by the exchange of certain particles between the interacting particles. These *exchange particles* are the elementary vector bosons (and some mesons, as mentioned below), whose existence is predicted by the quantum field theories of the respective interactions. For example, the theory of the electromagnetic interaction, called *quantum electrodynamics* (QED), predicts the photon to be the carrier of the electromagnetic force. A photon acting as an exchange particle is an example of a *virtual particle*, a general term used for particles whose ephemeral existence serves no purpose other than to mediate a force between two material particles: in a sense, the virtual particles moving from one material particle to the other *are* the force between them (see Figure 7.1).

The virtual particle appears spontaneously near one of the particles and disappears near the other particle. This is a purely quantum-mechanical effect allowed by a fundamental law of nature called the *Heisenberg uncertainty principle*. According to this principle, a virtual particle is allowed to exist for a time that is inversely proportional to its mass as a material particle. (Under certain conditions, a virtual particle can become a material particle.) The allowed lifetime of a virtual particle determines the maximum distance that it can travel and, therefore, the maximum range of the force that it mediates. Hence, the greater the mass of the material particle, the shorter the distance it can travel as a virtual particle, and vice versa. Photons have zero mass, so the range of the electromagnetic force is infinite.

By contrast with QED, the theory of the weak interaction (the electroweak theory, actually) predicts the existence of three different carriers of the weak force, all of them extremely massive: about 90 to 100 times the mass of a nucleon. These elementary particles are the W^+, W^-, and Z^0 bosons, collectively called the *intermediate vector bosons*. Their discovery in 1983 dramatically confirmed the validity of the electroweak theory. Because of their great mass, these particles are restricted by the uncertainty principle to lifetimes so short that they can travel only about 10^{-28} m before disappearing. This explains the extremely short range of the weak force. The strong force exists in two guises, as we have seen. Here the fundamental quantum field theory, called *quantum chromodynamics* (QCD), predicts the existence of no less than eight vector bosons—the *gluons* —to mediate the color force between quarks. Experimental evidence for the gluons has been obtained. Gluons are massless, like photons, but because of quark confinement, the range of the color force does not extend beyond the confines of the hadrons (the quark bags).

In its second, vestigial guise, the strong force is experienced by hadrons (baryons and mesons) and is mediated by mesons—by pions at the largest distances. Here we have a type of particle, the meson (which is a boson, but not an elementary one and not necessarily of the vector kind), that can act as its own exchange particle, i.e., material mesons can interact through the exchange of virtual mesons. (This is not a unique case, however, because the gluons, which themselves possess an intrinsic color, are also self-interacting particles.) The range of the strong force—very short, yet much longer than that of the weak force—is explained by the mesons' moderate masses, which are typically less than that of a nucleon and very much less than that of an intermediate vector boson. What is most significant for nuclear physics is that the nucleons interact via the exchange of virtual mesons, so the nucleus is believed always to contain swarms of these particles among its nucleons.

Thus the traditional picture of the nucleus as consisting simply of protons and neutrons has given way to a more complex picture in which the strong nucleon-nucleon interactions must be viewed in terms of *meson-exchange* effects. And even this view is just an approach to the deeper understanding of nuclear structure and dynamics that can come about only through detailed considerations of the *quark-gluon* nature of the nucleons and mesons themselves. Ultimately, the nucleus must be explainable in terms of a very complex many-body system of interacting quarks and gluons. The experimental and theoretical challenges posed by this goal are enormous, but so are the potential rewards in terms of our understanding of the nature of nuclear matter.

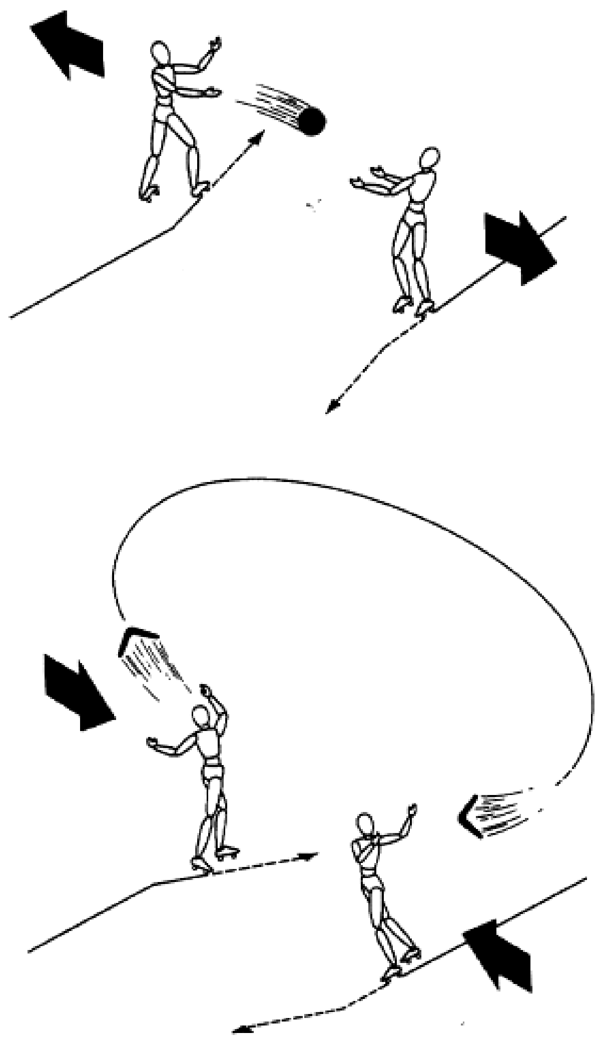

Figure7.1 The way in which force is transmitted from one particle to another can be visualized (crudely) through the example of two roller skaters playing different games of catch as they pass each other. Throwing and catching a ball tends to push the skaters apart, but using a boomerang tends to push them together. (After D. Wilkinson, in *The Nature of Matter*, J. H. Mulvey, ed., Oxford University Press, Oxford, 1981.)

7.3. CONSERVATION LAWS AND SYMMETRIES

The total amounts of certain quantities in the universe, such as electric charge, appear to be immutable. Physicists say that these quantities are conserved, and they express this idea in the form of a *conservation law*.

The law of the conservation of charge, for example, states that the total charge of the universe is a constant—or, simply, "charge is conserved." This means that no process occurring in any isolated system can cause a *net* change in its charge. Individual charges may be created or destroyed, but the algebraic sum of all such changes in charge must be zero, thus conserving the original charge, whatever it might have been.

Another important quantity that is conserved is mass-energy. Before Einstein, it was thought that mass and energy were always conserved separately, but we now know that this is not strictly true: mass and energy are interconvertible, so it is their *sum* that is conserved. Mass, in the form of elementary or composite particles, can be created out of pure energy, or it can be destroyed (annihilated) to yield pure energy; both of these processes are commonplace in nuclear and particle physics. This example illustrates the important point that although any conserved quantity may change its form, the conservation law is not invalidated. Energy itself, for instance, can exist in many different forms—chemical, electrical, mechanical, and nuclear, for example—all of which are interconvertible in one way or another without any net gain or loss, provided one accounts for any mass-energy conversion effects. Such effects are significant only in subatomic processes and are, in fact, the basis of nuclear energy.

Two other conserved quantities, linear momentum and angular momentum, are related to the linear and rotational motions, respectively, of any object. Conservation laws for these quantities and the others mentioned above apply to all processes, at every level of the structure of matter. However, there are also conservation laws that have meaning only at the subatomic level of nuclei and particles. One such law is the conservation of baryon number, which states that baryons can be created or destroyed only as baryon-antibaryon pairs.

All baryons have baryon number + 1, and all antibaryons have baryon number -1; these numbers cancel each other in the same way that opposite electric charges cancel. Thus, a given *allowed* process may create or destroy a number of baryons, but it must also create or destroy the same number of antibaryons, thereby conserving baryon number. Processes that violate this law are assumed to be *forbidden*—none has ever been observed to occur.

There is no conservation law for meson number, so mesons, as well as other bosons, can proliferate without such restrictions. A law of nature that predicts which processes are allowed and which are forbidden—with virtual certainty and great generality, and without having to take into account the detailed mechanism by which the processes might occur— represents a tool of immeasurable value in the physicist's effort to understand the subtleties and complexities of the universe. Conservation laws are therefore often regarded as the most fundamental of the laws of nature. Like all such laws, however, they are only as good as the experimental evidence that supports them. Even a single proved example of a violation of a conservation law is enough to invalidate the law—for that class of processes, at least—and to undermine its theoretical foundation. We will see that violations of certain conservation laws do occur, but first let us examine another important aspect of conservation laws: their connections with the symmetries of nature. *Symmetry* of physical form is so common in everything we see around us—and in our own bodies—that we take it for granted as a basic (though clearly not universal) feature of the natural world. For example, the fundamental symmetry of space and time with respect to the linear motions and rotations of objects leads directly to the laws of the Conservation of linear and angular momentum. Similarly, the mathematical foundations of the quantum field theories imply certain symmetries of nature that are manifest as various conservation laws in the subatomic domain. One such symmetry, called *parity*, has to do with the way in which physical laws should behave if every particle in the system in question were converted to its mirror image in all three spatial senses (i.e., if right were exchanged for left, front for back, and up for down). Conservation of parity would require that any kind of experiment conducted on any kind of system should produce identical results when performed on the kind of mirror-image system described above. For many years, it. was believed that parity was an exact (universal) symmetry of nature. In 1956, however, it was discovered by nuclear and particle physicists that this is not so; parity is not conserved in weak interactions, such as beta decay. However, it is conserved, as far as we know, in all the other fundamental interactions and thus represents a simplifying principle of great value in constructing mathematical theories of nature.

A similar, albeit isolated, example of symmetry violation has been found for the equally fundamental and useful principle called *time-reversal invariance*, which is analogous to parity except that it entails a mirror imaging with respect to the direction of time rather than to the orientation of particles in space. This symmetry has been found to be violated in the decays of the neutral kaon. No other instances of the breakdown of time-reversal invariance are known—yet—but physicists are searching carefully for other cases in the hope of gaining a better insight into the underlying reason for this astonishing flaw in an otherwise perfect symmetry of nature.

The implications of such discoveries extend far beyond nuclear or particle physics; they are connected to basic questions of cosmology, such as the ways in which the primordial symmetry that is believed to have existed among the fundamental interactions at the instant of the big bang was then "broken" to yield the dramatically different interactions as we know them now. The efforts of theoretical physicists to construct *Grand Unified Theories* of the fundamental interactions, in which these interactions are seen merely as different manifestations of a single unifying force of nature, depend strongly on experimental observations pertaining to symmetries, conservation laws, and their violations.

A most important observation in this regard would be any evidence of a violation of the conservation of baryon number, which may not be a universal law after all. Certain of the proposed Grand Unified Theories predict, in fact, that such a violation should occur, in the form of spontaneous proton decay— not in the sense of a radioactive beta decay, in which a proton would be converted to a neutron (thus conserving baryon number) but rather as an outright disappearance of a baryon (the proton) as such. Extensive searches have been mounted to find evidence for proton decay, so far without success.

Also of great importance would be any violation of the conservation of lepton number. This law, which is also obeyed in all currently known cases, is analogous to the conservation of baryon number, but with an added twist: lepton number (+ 1 for leptons, -1 for antileptons) appears to be conserved not only for leptons as a class but also for each of the three families of leptons individually (the electron, muon, and tauon, with their respective neutrinos). Any violation of lepton-number conservation would mean that neutrinos are not, in fact, massless and that they can *oscillate* (change from one family to another) during their flight through space. Exactly these properties are also predicted by certain of the proposed Grand Unified Theories, and this provides the impetus for searching for them in various types of nuclear processes. Such searches for violations of conservation laws represent an important current frontier of nuclear physics as well as of particle physics.

EXERCISE 7

1. What are quacks? Explain the difference between the various types of quacks.

2. Differentiate among quacks, leptons, and elementary vector bosons.

3. What is color force?

4. Explain how the nuclear force between nucleons is a vestige of both color force between quacks and exchange force between hadrons

5. Mention and describe any two examples of symmetry violation.

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