PHS 105

Course Outline (Mechanics Synopsis)

Linear Motion: (Motion in a straight)

* Measurement, Standard, Unit and Errors
* Displacement, Average Velocity
* Instantaneous Velocity
* Acceleration
* Acceleration of falling Bodies and Gravity

Motion in a Circle: (Circular Motion)

* Centripetal Acceleration
* Centripetal Force
* Inertia Force in Rotation (Moment of Inertia)
* Centrifugal Force

Simple Harmonic Motion

* Periodic Motion (Periodic time, Frequency and Amplitude)
* Dynamics of Simple Harmonic Motion
* Resonance
* Damped and Force Oscillations

Gravitation

* Newton' Law of Universal Gravitation
* Satellites and Weightlessness
* Kepler's Laws

Statics and Hydrostatics

Statics

* Mass, Force and Weight
* Forces in Equilibrium

* Resolution of Forces
* Moment of Forces
* Principles of Moment
* Couple
* General Conditions of Equilibrium

Hydrostatics (Fluid at Rest)

* Fluid, Pressure
* Transmission of Fluid Pressure
* Density, Relative Density, Specific Weight and Specific Gravity
* Pressure in a liquid due to its own weight and pressure measurements

Elasticity

* Stress, Strain
* Young's Modules, Bulk and Sheer Moduli

Friction, Viscosity and Surface Tension

* Sliding and Static Friction
* Viscosity (Laminar and Turbulent flows)
* Surface Tension and Capillarity

Text Books

* Applied Mechanics ---- Hannah & Hillier
* Physics (principles with applications) ------ Douglas C. Giancoli
* General Physics ----- Sternhein and Kana
* Any other physics text books covering these synopses

Mechanics: Study of Motions of objects and of the forces that affect their motions.

Linear Motion

Physics like many other sciences is largely based on quantitative measurements. A quantitative discussion of motion requires measurements of time and distance, so that we can consider the standards, units and errors involved in physical measurement.

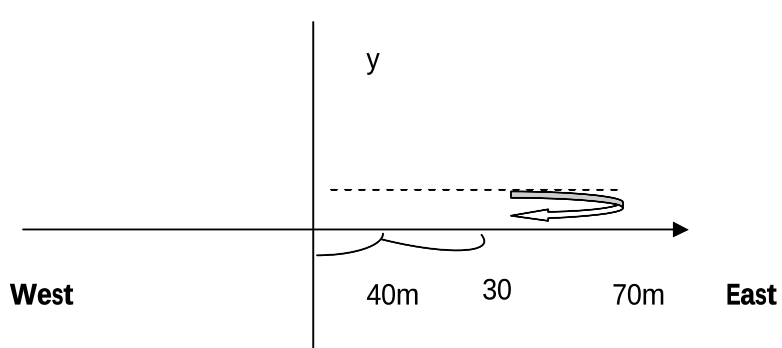
Measurements: Quantitative physical measurements must be expressed by numerical comparison of some agreed set of standards. All measuring devices are calibrated directly in terms of primary standards of L**ength, Time and Mass** as established by international scientific community. All physical quantities can be expressed in terms of some combinations of these three fundamental dimensions, which we denote as **L, T and M** respectively.

Internationally accepted set of metrics units called the **"Systeme Internationale** (S.I) units are: **Metre, Kilogram and Second**, termed as basic units i.e., **"M. K.S" System;** older units are **C.G.S. units.**

Errors: Measurements and predictions are both subject to errors.
Measurement errors are of two types: Random and systematic.

Both errors are present in all experiments and can be reduced by taking the average of many measurements (Random)

Displacement: Change in position of an object or distance between two points in a specified direction. To distinguish between distance and displacement, take for instance, a person walking 70m east and then turn around to walk 30m west.



The total distance walked is 100m but displacement is 40.0m I.e. 40m from the starting point

Velocity and Speed

Velocity: Signifies both magnitude (numerical value) and direction, which makes velocity a vector quantity. Speed: Signifies only magnitude.

Note: Average velocity is defined in terms of displacement rather than total distance travelled.

Hence, average velocity = $\frac{\text{Displacement}}{\text{Time Elapsed}}$

Unit = m/s.

Let $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$

Δx = Displacement and Δt = change in time or elapsed time

$\therefore$ Average velocity $\tilde{V} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$

Instantaneous Velocity: Velocity at any instance of time, defined as average velocity over an indefinitely short time interval.

$V_{nst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$, unit is m/s.

Acceleration = $\frac{\text{Change of velocity}}{\text{Time elapsed;}}$ the unit is m/s^2

Hence, average acceleration, $\bar{a} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$

Similarly, Instantaneous acceleration is:

$a_{inst} = \frac{\lim}{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

unit: m/s^2

Uniformly Accelerated Motion

This occurs when acceleration remains constant over time. But when acceleration changes, the change is sufficiently small such that we assume it to be constant; thus situation is treated as ***uniformly accelerated motion.*** In this case, instantaneous and average accelerations are equal.

Suppose the initial time $t_1 = 0$, then

$T = t_2$ (time elapsed)

Let the initial position x_1 and initial velocity v_1 be represented as x_0 and v_0

Hence, $\tilde{V}$ (average velocity) = $\frac{\varkappa - \varkappa 0}{t} = \frac{\varkappa - \varkappa 0}{t}$

So, also,

$$a = \frac{v - v0}{t}$$

$$At = v - v_0$$

$$V = V_0 + at \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (1)$$

Recall $\tilde{V} = \frac{\varkappa 2 - \varkappa 1}{t2 - t1}$ and that $\varkappa_1 = \varkappa_0, \varkappa_2 = \varkappa$

When $t_1 = 0$

$$\tilde{V} = \frac{\varkappa - \varkappa 0}{t}$$

$$\therefore \varkappa = \tilde{V}t + \varkappa_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (2)$$

Since velocity increases at a uniform rate, the average velocity $\tilde{V}$, will be midway between the initial and final velocities, hence

$\tilde{V} = \frac{v0 + v}{2}$, now substituting this in equation (2),

We obtain $\varkappa = \varkappa_0 + (\frac{v0 + v}{2})\,t = \varkappa_0 + (\frac{v0 + v + at}{2})\,t$

$$X = \varkappa_0 + v_0 t + \tfrac{1}{2} at^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

To obtain the velocity v at time t in term of v_0, a, $\varkappa$, and $\varkappa_0$, we begins as

$$\varkappa = \varkappa_0 + \tilde{V}t = \varkappa_0 + (\frac{v + v0}{2})\,t$$

Recall from equation (1) that

$$t = \frac{v + v0}{a} .$$

$$\therefore \varkappa = \varkappa_0 + \left(\frac{v+v0}{2}\right)\left(\frac{v-v0}{a}\right)$$

$$= \varkappa_0 + \frac{v^2 - v_0^2}{2a}$$

$$V^2 = v_0^2 + 2a(\varkappa - \varkappa_0) \quad \ldots\ldots\ldots\ldots (4)$$

Hence, kinematic equations for constant acceleration are:

$V = v_0 + at$	$V = v_0 + a\Delta t$
$\varkappa = \varkappa_0 v_0 + \frac{1}{2} at^2$	$\Delta\varkappa = v_0\Delta t + \frac{1}{2} a(\Delta t)^2$
$V^2 = v_0^2 + 2a(\varkappa - \varkappa_0)$	$V^2 = V_0 + 2a\Delta\varkappa$
$\Delta = \frac{v+v0}{2}$	$\tilde{V} = \frac{1}{2}(v_0 + v)$
	$\Delta\varkappa = \frac{1}{2}(v_0 + v)\Delta t$

Note that these equations are not valid unless (a) is a constant. In many cases $\varkappa_0 = 0$

Illustrations:

1. A car accelerates from rest to 30m/s. What is its average acceleration?

Solution: -

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v+v0}{t-t0} = \frac{30-0}{10} = 3 \text{ m/s.}$$

$\Longrightarrow$ an increase in velocity of 3m/s in each second of the 10 seconds time interval.

(2) An object moves according to the formula

$\varkappa = b + ct^{3}.$ What is the instantaneous acceleration at time t?

<u>**Solution:**</u>

1^{st} find v

$$V = \frac{dx}{dt} = \frac{d}{dt}(b + ct^3) = 3ct^2$$

Then, $a_{hnat} = \lim \longrightarrow 0 \frac{dv}{dt} = \frac{d}{dt}(3tc^2) = 6ct$

(3). A car initially at rest at a traffic light accelerates at $2m/s^2$ when the light turns green. After 4 secs, what are its velocity and position?

Solution:

$a = 2m/s^2$, $\Delta t = 4secs$ and v_0

$\therefore$ (i) $v = v_0 + a\Delta t = 0 + 2(4) = 8mls$

(ii) $\Delta x = v_0\Delta t + \frac{1}{2} a (\Delta t)^2 = 0 + \frac{1}{2}(2)(4)^2 = 16m$

(4). A car accelerates from rest with a constant acceleration of $2m/s^2$ onto a highway where traffic is moving at a steady rate of 24m/s.

(a). How long will it take for the car to reach a velocity of 24m/s?

(b). How far will it travel in that time?

(c). The driver does not want the vehicle behind to come closer than 20m nor force it to slow down. How large a break in traffic must the driver wait for

Solution

(a) i. e time need to reach the velocity v= 24m/s

$$V = V_0 + a\Delta t$$

$$= \Delta t = \frac{V + V_0}{a} = \frac{24}{2} = 12sec$$

(b) $\Delta x = V_0 \Delta t + \frac{1}{2} a (\Delta t)^2 = 0 + \frac{1}{2}(2)(12)^2 = 144m$

(c) The vehicle behind is moving at a constant velocity $V_0 = 24m/s$, so $a = 0$

$\therefore$ In 12sec., it moves a distance $\Delta x = V_0 \Delta t + \frac{1}{2} a (\Delta t)^2$

$$= 24 \times 12 + \frac{1}{2}\ 12 \times 0 = 288m$$

Since the entering car travels 144m in this time, the oncoming vehicles gains (288 - 144) m or 144m, If it is to come no closer than 20m, the break in traffic must be at least (144 + 20) m or 164m.

Assignment 1

A car reaches a velocity of 20m/s with an acceleration of $2m/s^2$. How far will it travel while it is accelerating if it (a) Initially at rest? (b). initially moving at 10m/s.

A train accelerates uniformly from rest to reach 54km/h in 200sec after which the speed remains constant for 300 sec. At the end of this time the train decelerates to rest in 150 sec. Find the total distance travelled.

A baseball pitcher throws a fastball with a speed of 44m/s. It is observed that in throwing the baseball, the pitcher accelerates the bell through a displacement of about 3.5m from behind, estimate the average acceleration of the ball during the throwing motion.

Falling Bodies and Gravity (By Galileo Galilei 1564- 1642)

At a given location on the Earth and in the absence of air resistance, all objects fall with the same uniform acceleration due to gravity , denoted by $g = 9.8m/s^2$, downward. "g" varies slightly due as a result of changes in latitude, elevation and density of local geological features.

When dealing with freely falling objects, we make use of the same equation as described in kinematic by replacing a with g and since the motion is vertical, we put y in place of x, y_0 in place x_0.

Note: It is arbitrary whether we choose y to be position in the upward direction or in the downward direction, but we must be consistent about it throughout a problem's solution.

The equations for falling bodies will be

$$V = v_0 + gt$$

$$Y = y_0 + v_0t + \frac{1}{2}gt^2$$

$$V^2 = V_0^2 + 2g(y - y_0)$$ and taken $a = +g$ (as downward).

Example: Suppose that a ball is dropped from a tower 70.0m high, how far it will have fallen after 2.0 sec.

$a = g = +9.8m/s^2$ since we have chosen downward as +ve

$\therefore V_0 = 0, y_0 = 0$

$y = 0 + 0 + \frac{1}{2}gt^2 = \frac{1}{2}(9.8) \times 2^2 = 19.6m$

Now suppose the ball in the above is thrown downward with a speed of 3.0mls instead of being dropped, what they would be its position and speed after 2.0 sec?

Solution:

$V_0 = 3.0m/s$ and $t = 2.0Sec$, $y_0 = 0$

$y = V_0t + \frac{1}{2}gt^2 = 3 \times 2 + \frac{1}{2} \times 9.8 \times 4$

$= 6 + 9.8 \times 2 = 25.6m$

Its speed after 2.0 Sec,

$V = V_0 + gt = 3.0 + 9.8 + 2 = 22.6m$

Example: A ball is thrown upward into the air with an initial velocity of 15.0m/s Calculate (a) How high it goes (b) How long the ball is in the air before it comes back to his hand.

Solution

Let y be +ve in upward direction and –ve in the downward direction.

Note the difference in convention) i.e. $a = -9.8mls^2$

So, to determine d highest height, $V = 0$, and $V_0 = 15.0m/s$

(1) $V^2 = v_0^2 + 2gy$

$Y = \frac{v^2 - v02}{2g} = \frac{0 - (15)^2}{2(-9.8)} = 11.5m$

(2) $y = y_0 + V_0t + \frac{1}{2}gt^2$, $y_0 = 0$

$y = 15t + \frac{1}{2}(-9.8)t^2$

$y - y_0 = V_0t + \frac{1}{2}gt^2$

Displacement not total distance travelled

$0 = 15.0t + \frac{1}{2}(-9.8)t^2$

$= 15t - 4.9t^2 = 0$

$= t\,(15 - 4.9t) = 0$

$t = 0 \quad$ or $\quad t = \frac{15}{4.9} = 3.06\text{Sec}$

t = 0 corresponds to initial point (A) and y = 0, while t = 3.06sec corresponds to C when the ball has returned to y = 0 i.e ball is in the air.

Assignment 2

1. A ball thrown upward with velocity of 15.0mls, (a) how much time it takes for the ball to reach the maximum height. (b) The velocity of the ball when it returns to the thrower's hand. (c) at what time t the ball passes a point 8.0m above the ground.

2. A ball is dropped from a window 84m above the ground, (a) when does the ball strike the ground? (b) What is the velocity of the ball when it strikes the ground?

(3) A ball is thrown upward at 19.6mls from a window 58.8m above the ground (a) How high does it go? (b) When does it reach its highest point? (c) When does it strike the ground?