$$\text{Elastic modulus} \equiv \frac{\text{Stress}}{\text{Strain}}$$

We consider three types of deformation and define thier elastic moduli: Young's modulus, shear modulus and bulk modulus.

<u>Young's Modulus</u>: Elasticity in length

This is a measure of the resistance of a solid to a change in its length when a force is applied to a face. Consider a long bar of cross- sectional area **A** and length **Lo**, (see Figure 2.2). When the bar is subject to equal and opposite forces $\mathbf{F_n}$ along its axis and bar to the end faces se fig (2.2b), its length changes to ΔL. The forces stretch the bar, thus tensile stress on the bar is

$$\text{Tensile stress} = \frac{Fn}{A}, \text{ also}$$

Tensile strain = $\underline{\Delta L}$, it is a dimensionless quantity.

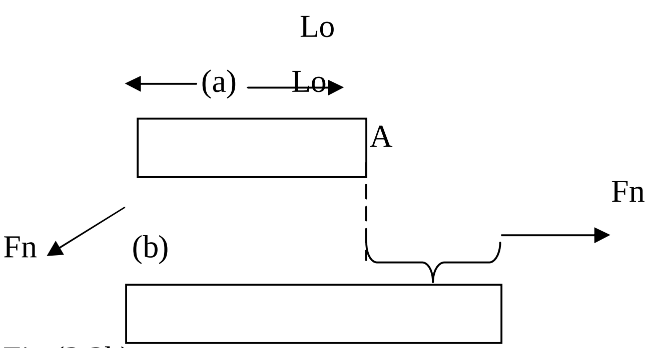


Fig (2.2b)

Young's Modulus **Y** is defined as the ratio tensile stress to tensile strian.

$$\text{Young's Modules} = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

$$= \frac{Fn}{A} \Big/ \frac{\Delta L}{Lo}$$

Unit is N/m^2.

Shear Modulus: Elasticity of shape

This is another form of deformation which when subjected to a force **F** tangential to one of the faces while the opposite face is fixed by a force such as the force of friction, **Fs**. Stress in the case is called a **Shear Stress**

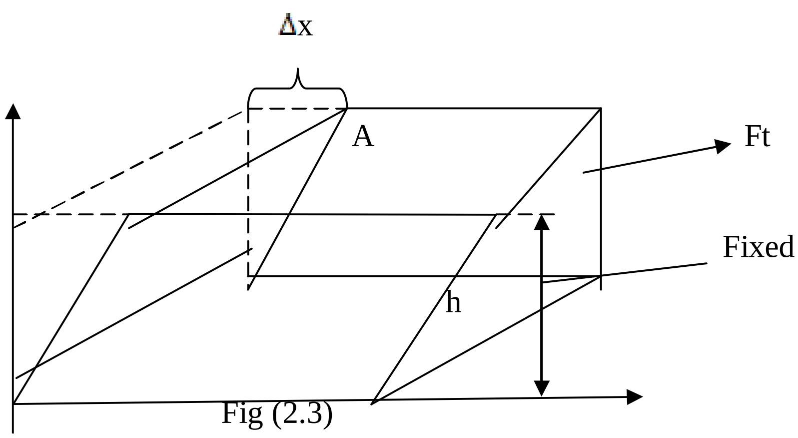


Fig (2.3)

Shear stress is the ratio of the tangential force to the area, **A**. of the face being sheared. Also, shear strain is defined as the ratio Δx / h, where Δx is the horizontal distance the sheared face moves and h is the height of the object

$$\textbf{Shear Modulus, S} = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{F/A}{\Delta x/h}$$

Unit = N/m

It is sometimes referred to as modulus of rigidity i.e deformation in shape without a change in volume. A rigid body (very high shear modulus) is a body whose shape cannot be easily altered.

<u>Bulk Modulus</u>: Volume Elasticity

This characterizes the resistance of a substance to change in volume.

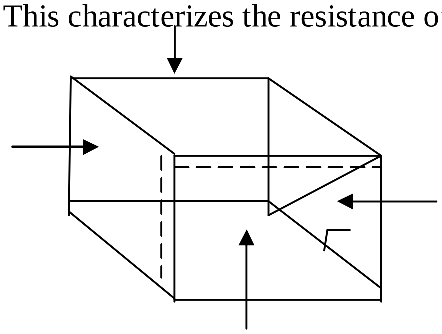


Figure 2.4

Consider a cube of some material, solid or fluid as in Figure 2.4; we see that all faces experience the same force, $\mathbf{F_n}$, normal to each face. This type of deformation changes the volume while the shape is not altered.

Hence volume stress, **ΔP**, is defined as the ratio of the magnitude of the normal force, **F**, to the area, **A.** So, also, volume strain is equal to the change in volume, ΔV, divided by the original volume, V.

$$\textbf{Bulk Modulus, B} = \frac{\text{Volume Stress}}{\text{Volume Strain}} = \frac{F/A}{\Delta V/V} = \frac{-\Delta P}{\Delta V/V} \qquad (+\text{ve } \Delta P)$$

-ve sign is to make B a +ve no., meaning increase in pressure *decreases* the volume (- ve ΔV) and vice – versa.

Note: That the reciprocal of the Bulk modules is called **compressidity** of material.

Example 1

A copper wire of length 1.5m and a radius of 0.5mm ,when subjected to a tension of 2000N, Cal. the change in length when Y = 1.4×10^{11} N/m^2.

Solution:

$$Y = \frac{F}{A} \Big/ \frac{\Delta L}{L_o} \quad , \text{Area} = \Pi r^2 = 7.84 \times 10^{-7}\ M^2$$

$$\Delta L = \frac{FL_O}{AY} = \frac{2 \times 10^3 \times 1.5}{(7.84 \times 10^{-7}\ m^2)(1.4 \times 10^{11})}$$

$$= 2.73 \text{ cm}$$

Example 2

A load of 102 kg is supported by a wire of length 2.0m and cross- sectional area 0.10cm^2. When the wire is stretched by a 0.22cm, find the tensile Stress, tensile Strain and Young's Modulus for the wire.

Solution:

$$\text{Tensile Stress} = \frac{F}{A} = \frac{mg}{A} = \frac{100\text{kg} \times 9.80\text{m/s}^2}{(0.10 \times 10^{-4}\text{m}^2)}$$

$$= 1.0 \times 10^8 \text{ N/m}^2$$

$$\text{Tensile Strain} = \frac{\Delta L}{L_O} = \frac{0.22 \times 10^{-2}\ m}{2.0\ m} = 0.11 \times 10^{-2}$$

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{1.0 \times 10^{8}\ N/m^2}{0.11 \times 10^{-2}} = 9.1 \times 10^{10}\ N/m^2$$

Example 3:

A solid lead sphere of volume $0.50m^3$ lowered to a depth of water of pressure $2.0 \times 10^7\ N/m^2$. If the bulk modulus is $7.7 \times 10^9\ N/m^2$, find the change in volume of the sphere?

Solution:

$$\text{Bulk Modulus, } B = -\frac{\Delta P}{\Delta V/V}$$

$$\Delta V = -\frac{V\Delta P}{B}$$

$$= -\frac{(0.50m^3)(2.0 \times 10^7\ N/m^2)}{7.7 \times 10^9\ N/m^2} = -1.3 \times 10^{-3}\ m^3$$

Note: This is large relative to atmospheric pressure, $1.01 \times 10^5\ N/m^2$ and the –ve sign indicates a decrease in volume.

Example 4: An Iron wire of diameter 0.8cm and length 3m is fixed at both ends so that it cannot expand and, then it is heated from 10^{0} to 100^{0}c. Calculate the force exerted on each end of the wire if $Y = 2 \times 10^{11}\ N/m^{-2}$ and $\propto = 18 \times 10^{-6}\ K^{-1}$ (Expansivity).

Solution:

Recall $$\propto = \frac{e}{\ell o\, \Delta t}$$

$$e = \propto \ell o\, \Delta t$$

$$F = \frac{YAe}{\ell o} = YA\propto\Delta t$$

$$= 2 \times 10^{11} \times \Pi\, (0.4 \times 10^{-3})^2 \times 18 \times 10^{-6} \times (100 - 10)$$

$$= 163\ N$$

<u>Example 5</u>: Two clamps are fastened near the ends of a rectangular steel rod 10cm long. The rectangular cross sectional area of the rod is $5cm^2$. A force of 80N is applied on each of these clamps parallel to this area but in opposite direction. Determine (i) Shear stress (ii) Shear strain and (iii) the relative displacement of the top surface with respect to bottom surface, (G = 5 x 1010 Nm^{-2})

<u>Solution:</u>

(i) $$\text{Shearing Stress} = \frac{F}{A} = \frac{80}{5 \times 10^{-4}} \; N/m^2$$

(ii) $$\text{Strain} = \frac{\text{Stress}}{G} = \frac{16 \times 10^{4}}{5 \times 10^{10}} = 3.3 \times 10^{-6}$$

(iii) $$\frac{\Delta x}{0.1} = \text{Strain}$$

$$\Delta x = 3.2 \times 10^{-7} \text{ m}$$

Exercises

1. What is the Young's Modulus of a cylindrical bone specimen of cross sectional area of 1.5 cm^2 if a load of 10kg produces a decrease of 0.0065% in its length?
2. A cylindrical copper wire and cylindrical steel wire, each of length 1.5m and diameter 20mm, are joined at one end to form a composite wire 3m long. The wire is loaded until its length becomes 3.003m. Find the strains in the copper - steel wire and applied force to the wire if $Y_c = 1.2 \times 10^{11}$ Nm^{-2} and $Y_o = 2.0 \times 10^{11}$ N/m^2
3. What increase in pressure would be needed to decrease thevVolume of $1m^3$ of water by 10^{-4} m^3 if its bulk modulus is 2 x 109 N/m^2?
4. A steel bar of length 4m and of rectangular section 1.5cm x 2.0cm supports a load of 100kg. By how much is the bar stretched?

Elementary Principles of Hydrostatics and Hydrodynamic:

<u>Fluid Dynamics</u> (Mechanics)

<u>Preamble</u>: Matter consists of three states: Solid, Liquid and Gases.

Liquid and gas are referred to as fluid. It has definite shape or volume.

<u>Pressure</u>: Study of fluid mechanics involves the density of a substance, defined as mass per unit volume.

If **F** is the magnitude of the normal force on the piston and **A** is the surface area of the proton, then the pressure, **P** of the fluid at the level to which the device has been submerged is defined as the rate of force to area.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

Suppose the normal force exerted by the fluid is **F** over a surface element of area δ**A** then the pressure at that point is:

$$P = \lim_{\delta A \to 0} = \frac{F}{\delta A} = \frac{\delta A}{dA} \qquad \text{---------------------} \ (3.0)$$

Unit = Pascal (pa) ⇉ N/m^2.

<u>Variation of pressure with Depth</u>

Pressure increases nearly with depth. Consider a liquid of density **ρ** at rest and open to the atmosphere as in Figure 3.0. A sample of liquid in a cylinder of cross section area **A** extending from the surface of the liquid to a depth h. Pressure exerted by the fluid on the bottom face is **P** and on the top face is $\mathbf{P_o}$, hence upward force is **PA** and downward force exerted is **PA.**

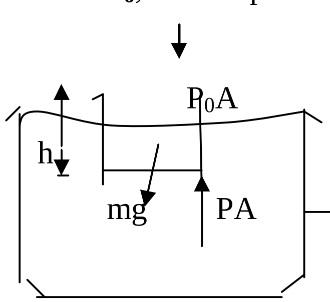


Figure 3.0

Absolute pressure

Mass of liquid in the cylinder is ⇉ M = PV = PAL. Also, the weight **w** of the liquid in the cylinder is w = mg = Pvg = PgAh.

For the cylinder to be in equilibrium:

Upward Force: $PA – P_o A = PghA = (P – p_o) A = phgA$

$$P = p_o + Pgh \qquad \text{----------- (3.1)}$$

Where, P_o = atm pressure = 1.00atm ≈ 1.01 x 10^5 Pa.

Pascal's Law: A change in pressure applied to an enclosed liquid is transmitted undiminished to every point of the liquid and to the walls of the container, i.e., pressure at every point in a liquid is the same

Fluid Dynamics (Fluids in Motion)

Study of properties of fluid as a function of time. Fluids in motion are characterized in two main types: Steady or Laminar and Non-Steady or Turbulent.

Study or Laminar: If each particles of the fluid follows a smooth path such that different particle never cross each other. In the case, the velocity of the fluid at any point remains constant in time.

Non steady or Turbulent: This is an irregular flow characterized by whirl pool like religion i.e. at certain critical speed; the fluid flow becomes non steady.

Viscosity: Degree of internal friction in the fluid, viscous force is associated with the resistance of two adjacent layers of the fluid to more relative to each other.

Stream Lines and Equation of Continuity

Streamline: Path taken by a fluid particle under steady flow. The velocity of the fluid particle in always tangent to the fluid particle is always tangent to the streamline as shown in Figure 4.0

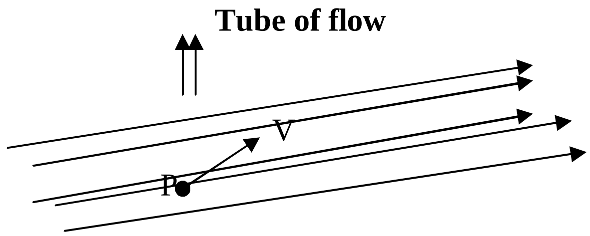
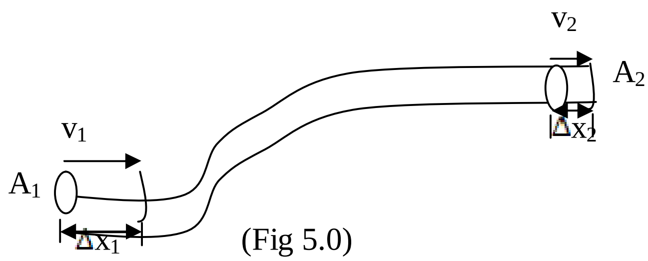


Figure 4.0:

Note: No two streamlines can cross each other

Consider the flow of an ideal fluid through a pipe of non-uniform size (See Figure 5.0), the particles in the fluid more along the streamlines in a Steady flow.



(Fig 5.0)

Suppose at interval of the Δt, the fluid at the bottom end of the pipe move a distance $\Delta x_1 = V_1 \Delta t$. If A_1 is the cross sectional area, then the mass is $\Delta xm_1 = PA_1\Delta x_1 = PA_1V_1\Delta$.

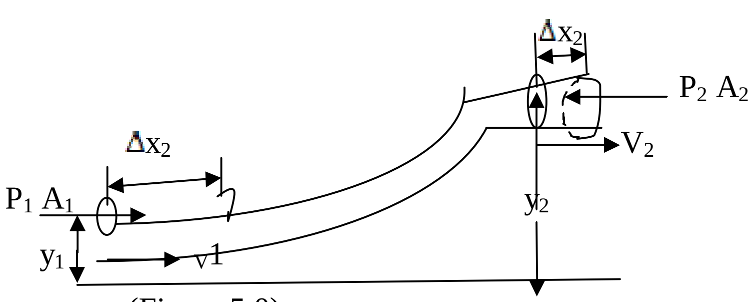
Similarly at the end of Area A_2, $\Delta m_2 = PA_2V_2\Delta t$.

By principle of conservation of mass, the time of steady flow

$\Delta m_1 = \Delta m_2$ or $PA1V1 = PA_2V_2$.

Hence, $A_1V_1 = A_2V_2 =$ Constant ------------------------- (3.2)

Equation (3.2) is called the Equation ofCcontinuity. This states that "The product of the area and the fluid speed at all points along the pipe is a constant for an incompressible fluid".



(Figure 5.0)

Consider the flow of an ideal fluid through a non uniform pipe is a time Δt as Shown in Figure 6.0. Work done by the force P_1A_1 on the lower end of the fluid is $W = F_1\Delta x_1 = P_1A_1\Delta x_1 = P_1\Delta V$, where

ΔV is the volume of section 1. Also work done by force P_2A_2 at the upper end in time Δt is $W2 = P_2A_2\Delta x_2 = - P_2\Delta V$. The work is –ve because the fluid force opposes the displacement.

Hence net work, $W = (P_1 - P_2)\,\Delta V$. ---------------------------------- (3.3)

If the Δm is the mass passing through the pipe in time Δt, then change in K.E is

$\Delta k = \frac{1}{2}(\Delta m)V_2^2 - \frac{1}{2}(\Delta m)V_1^2$ ---------------------------------- (3.4)

Change in gravitational potential energy is

$\Delta u = \Delta mg\, y_2 - \Delta mg\, y_1$

Applying the work – energy theorem in the form $W = \Delta k + \Delta u$ to the volume of fluid to give

$(P_1 - P_2)\,\Delta v = \frac{1}{2}(\Delta m)V_2^2 - \frac{1}{2}(\Delta m)V_1^2 + \Delta mg\, y_2 - \Delta mg\, y_1$ ------------- (3.5)

Dividing each term by Δv and also recall $P = \Delta m / \Delta v$, eqn. (3.5) becomes:

$P_1 - P_2 = \frac{1}{2}PV_2^2 - \frac{1}{2}PV_1^2 + Pg\, y_2 - Pg\, y_1$ --------------------------------- (3.6)

Rearranging, we have,

$P_1 + = \frac{1}{2}PV_1^2 + Pg\, y_1 = P_2 + \frac{1}{2}PV_2^2 + Pg\, y_2$ ---------------------- (3.7)

Hence, $P + = \frac{1}{2}PV_1^2 + Pg\, y = \text{constant}$ ---------------------- (3.8), This is **Bernoulli's equation** as applied to an ideal fluid. It states that "the sum of the pressure, (P), the kinetic energy per unit volume ($\frac{1}{2}Pv^2$), and gravitational potential energy per unit volume (Pgy) has the same value at all point along a streamline".

Note: When the fluid is at rest, $V_1 = V_2 = 0$, then eqn. (3.9)

Becomes $P_1 - P_2 = P_2(y_1 - y_2) = Pg\, h$.

<u>Application of Bernoulli's equation</u>: (Read it up)

<u>Examples 1</u>: Calculate the pressure at an ocean depth of 1000m. Assume the P of sea water is 1.024×10^3 kg/m^2 and take $P_o = 1.01 \times 10^5$ Pa

<u>Solution</u>:

$P = P_o + Pg\, h$

$= 1.01 \times 10^5 \text{ pa} + (1.024 \times 10^3 \text{ kg/m}^2) \times 9.8\text{m/s}^2 \times 1.0 \times 10^3\text{m}$

$\rightrightarrows \quad P = 1.01 \times 10^7$ Pa.

Example 2: A water hose 2.00cm in diameter is used to fill a 20.0 liter bucket. If it takes 1.00min to full the bucket, find the speed V at which the water leaves the hose? (1 L = 10^3 cm^3)

Solution:

The cross – sectional area of the hose is:

$$A = \pi r^2 = \pi \frac{d^2}{4} = \pi \frac{(2.00^2)}{4} cm^2 = \pi \ cm^2$$

Flow rate = 20.0 liters/min.

$$Av = 20.0 \frac{L}{m} = \frac{20 \times 10^3 cm^3}{60 \ sec.}$$

$$V = \frac{20.0 \times 103}{\pi \ cm^2 \times 60} = 106 \ cm/s$$

Exercises:

1. A spherical balloon filled with helium at 1 atm just lifts a 2kg load (which includes the mass of the balloon). What is its radius?
2. A 5.kg ball of density ρ = 6kg/m3 is completely sub merged in water. What is the tension in a string attached to the ball?