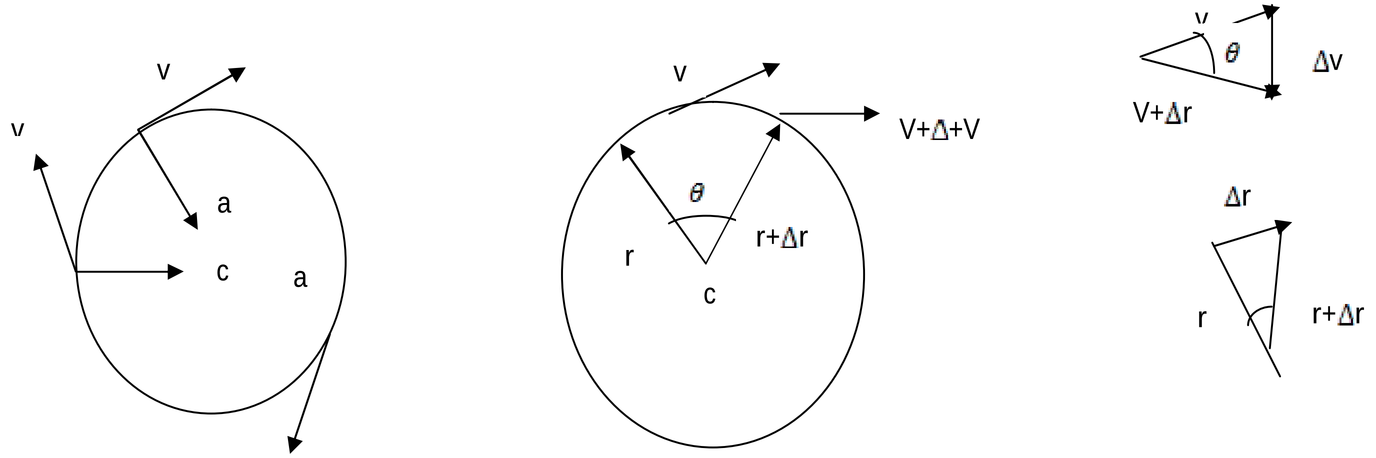
2.0 **Circular Motion**

An object moves in a straight line if the net force on it acts in the direction of motion or is zero. If the net force acts at an angle to the direction of motion or is zero. If the net force acts at an angle to the direction of motion at any moment, the object moves in curved paths. An object that moves in a circle at constant speed V is said to experience ***Uniform circular Motion.*** The magnitude of the velocity remains constant but the direction is continuously changing as the object moves around the circle.



2.1 Centripetal Acceleration: ***"Centre-Seeking"*** acceleration or radial acceleration (it's directed along the radius, towards the centre of the circle, denoted by a_r.

Consider the above diagrams, since they are similar (isosceles Δ) then their sides are proportional i.e.

$$\frac{1}{v}\left|\Delta v\right| = \frac{1}{r}\left|1r\right|$$, now divide by Δt,

$$\frac{1}{v}\left|\frac{\Delta v}{\Delta t}\right| = \frac{1}{r}\left|\frac{\Delta r}{\Delta t}\right|$$, taking the limit $\Delta t \longrightarrow 0$,

(Instantaneous acceleration and velocity), hence $\frac{1}{v} a_r = \frac{1}{r} v$

$$\frac{a_r}{V} = \frac{v}{r}, \qquad a_r = \frac{v^2}{r}$$, called the centripetal acceleration

Comments: Acceleration varies inversely with the radius, the smaller the circle the greater the acceleration. It also varies as V^2, i.e. it increases rapidly with the speed.

Example:

(1). A 150g ball at the end of a string is revolving uniformly in a horizontal circle of radius 0.6m. The ball makes exactly 2.0 revolutions in a second; what is its centripetal acceleration?

$$V = \frac{2\pi r}{t}$$

$$= \frac{2(3.14)(0.6)}{0.5} = 7.54\text{m/s}$$

$$\therefore\ a_r = \frac{V^2}{r} = \frac{(7.54)^2}{0.6} = 94.8\text{m/s}^2$$

Note: The period T of an object revolving in a circle is defined as the time required for one complete revolution.

$$V = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T}$$

(2). The moon is nearly circular orbit about the Earth has a radius of about 384,000km and a period of 27.3days. Determine the acceleration of the moon towards the earth.

Solution: To orbit the earth, the moon travels a distance $2\pi r$,

$$r = 3.84 \times 10^8\text{m}$$

$$\therefore\ v = \frac{2\pi r}{T} = \frac{6.28 \times 3.84 \times 10^8}{27.3 \times 24 \times 60 \times 60}$$

$$= 1.02 \times 10^3\text{mls.}$$

$$\text{Hence } a_r = \frac{v^2}{r} = \frac{(1.02 \times 10^3)^2}{3.84 \times 10^8} = 2.72 \times 10^{-3}\text{mls}^2$$

2.2: **Centripetal Force:** From Newton's 2^{nd}, law, F= Ma, an object that is accelerating must have a net force acting on it. Therefore, for a ball on the end of a string, moving in a circle must have a force applied to keep it moving in that circle.

Since, $a_r = \frac{v^2}{\ }$ $\therefore$ Centripetal force is $F = \frac{Mv^2}{\ }$

r r

Since, a_r is directed towards the centre at any moment, the net force too must be directed toward the centre of the circle.

(In vector form, $F = - M v^2$ ř, ř = ⟹ is a unit vector in that direction

r r

Example: A car travels on flat circular track of radius 200m at 30m/s and has a centripetal acceleration $a_r = 4.5 m/s^2$.

(a) If the mass of the car is 1000kg, what frictional force is required to provide the acceleration?

(b) If the co-efficient of static friction $\aleph_s$ is 0.8, what is the maximum speed at which the car can circle the track?

Solution

(a) Mass = 1000kg, $a = 4.5 m/s^2$

$F = Ma_r = 1000 \times 4.5 = 4500N$

(b) W = mg (i.e normal force N)

∴ Frictional force possible is $N_s N = N_s mg$

∴ $\frac{mv^2}{r} = N mg$

$= \sqrt{Nsrg}$

$= (0.8 x 200 x 9.8) = 39.6 m/s$

$\frac{F}{N} = N$

$F = u N$

$F = U_S N = N_s mg$

Digression

Comment: If the driver attempt to exceed 39.6m/s, the car will not be able to continue on the circular course and it will skid off.

2.3 Moment of Inertial

Moment: Turning effect of a force about an axis. And Torque is just equal and opposite forces. Despite the fact that $T = I \propto$ is similar in form to $F = Ma$, it is important to realize that both the torque **T** and moment of inertial **I** depend on the position of the axis of rotation. **"I"** also depends on the shape and mass of the rotating object.

To calculate the moment of inertial of a complex object, we must initially separate the object into N small pieces of mass m_1, m_2, m_3, ………m_N, with each piece having distance r_1, r_2, r_3, ………, r_N from the axis of rotation.

∴ Moment of inertial I for first piece is

$$I = m_1r^2 + m_2 r_2^2 + \ldots\ldots\ldots\ldots + M_N r_N^2$$

$\sum_{i=1}^{N} m1r2$,

Note that I is large when the force are far from the axis of rotation.

When the masses are arbitrarily small, the sum becomes an integral, given by

$I = \int r2dm$, so, for several shape and sizes, we have different moments of inertial. For instance, a uniform disk or cylinder of radius R rotating about the axis,

$I = \frac{1}{2} mR^2$ and for a rod of length **l** rotating about the centre,

$$I = \frac{1}{12} ml^2$$

Example: Two equal point masses M_0 are at the ends of a mass less than bar of length **l**. Find the moment of Inertial for an axis perpendicular to the bar through (a) the centre (b) an end

Solution (a): For an axis through an end, the mass at that end has r = 0 while the other mass is at a distance **l**, so

$$I = M_0 \left(\frac{l}{2}\right)^2 + M_0 \left(\frac{l}{2}\right)^2 = \frac{M_0 l^2}{2}$$

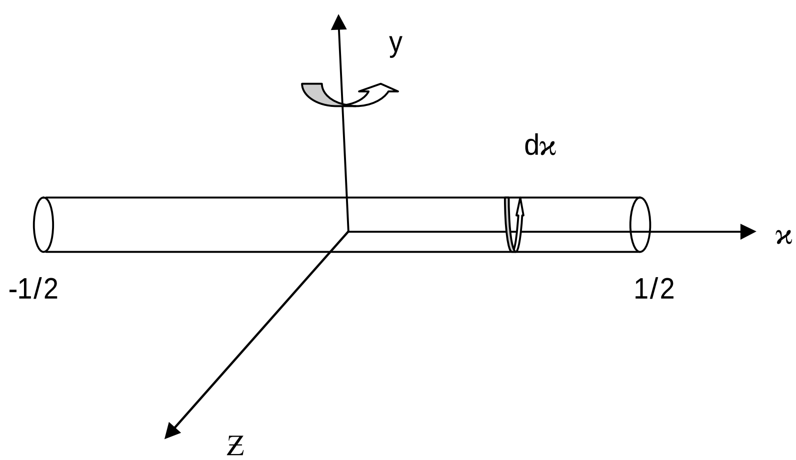
(b) For an axis through an end, the mass at that end has r = 0 while the other mass is at a distance l, so, $I = 0 + M_0 l^2 = M_0 l^2$

Comment: This shows that moment of Inertial depends on the position of the rotation axis.

2.4 <u>Centrifugal force:</u> This is equal and opposite to the centripetal force and therefore acts radially outwards. It is seen to be due to the tension in a cord, required to provide the motion in a circle.

Assignment 3

1. A 1000kg car rounds a curve on a flat road of radius 50m at a speed of 50km/h (14m/s). Will the car make the turn, or will it skid, if (a) the pavement is dry and the coefficient of static friction is N_s = 0.25?
2. A racing car starts from rest in the pit area and accelerates at a uniform rate to a speed of 35m/s in 11Sec; moving on a circular track of radius 500m. Assuming constant tangential acceleration, find (a) the tangential acceleration and (b) the centripetal acceleration when the speed is 30m/s.
3. Find the moment of Inertial of a thin rod of length **l** and mass m about an axis through its centre.



Simple Harmonic Motion

When an object moves back and forth repeatedly over the same path, it is said to be oscillating or vibrating. Examples are a Sheldon or swing, pendulum clock, violin string etc. S.H.M is characterized by several quantities like (1) Amplitude (maximum displacement of the oscillating object from equilibrium). Cycle (complete oscillation back and forth), Period T (time required for one complete oscillation). Frequency F (the number of cycles in a unit time).

In general, the period T and frequency F are related by F = $\frac{1}{T}$ in H_z

Now consider an object at the end of a coil spring, when displaced from its equilibrium position and released, the resulting oscillating motion is referred to as simple harmonic motion. The position, velocity and acceleration are related in a specific way which we now determine.

When a coil spring is stretched by application of force, the logarithm x and the applied force F are proportional.

$F = k\varkappa$, k is called the spring constant.

The spring exerts a restoring force that is opposite in direction

$F_r = -k\varkappa$

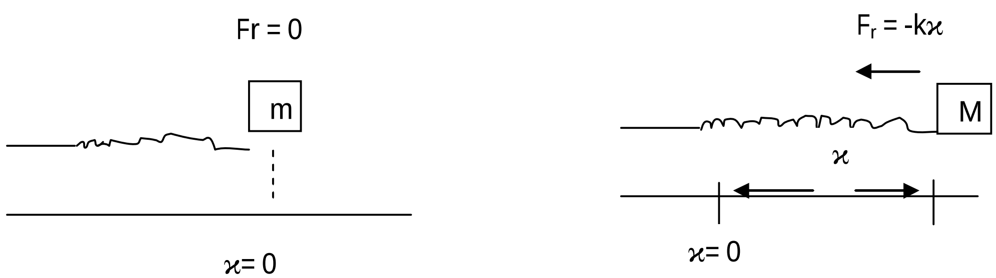
-ve sign indicates that the restoring force is always opposite to displacement

Take for instance, a mass resting on a frictionless table attached to a spring. Suppose the mass is pulled from its equilibrium point and it is released, then it moves under the influence of restoring force.

Definition of

$\therefore \quad F = ma$

$-kx = ma \quad , \quad a = -\frac{kx}{m}$ (simple harmonic motion)



Now recall that, $v = \frac{dx}{dt}$ and $a = \frac{dv}{dt} = \frac{d2x}{dt2}$

Hence $a = -\frac{kx}{m}$ $\Rightarrow$ 2^{nd} derivative of $\varkappa$ is proportional to $-\varkappa$

i.e $\frac{d2x}{dt2} = -\frac{k}{m}\varkappa$ by comparism

$\therefore \ \varkappa = -\ \varkappa$ Two functions that have this properly are sines and cosines

For instance,

$\varkappa$ = A co scot, where A and W are constants to be determine shortly.

$\frac{d}{dt}$ (cos ωt) = - ωsm ωt, V = $\frac{dx}{dt}$ = - A $\omega \sin \omega t$

Similarly, $\frac{d}{dt}$ (smωt) = $\omega \cos \omega t$

∴ a = $\frac{dv}{dt}$ = A ω^2 cos ωt

Recall S.H.M., equation, a = $\frac{-k}{m}$x, so by compassion

$\frac{-k}{m}$x = - A ω^2cos ωt = - ω^2 $\frac{A \cos \omega t}{x}$

∴ $\frac{-kx}{m} = - \omega^2$x

$\omega^2 = \frac{k}{m}$ or $\omega = \sqrt{\frac{k}{m}}$

A and ω A are the amplitude, maximum displacement in either direction from the equilibrium position.

Since $\omega = 2\lambda f$ and $f = \frac{1}{T}$

∴ $\omega = \frac{2\lambda}{T} = 2\lambda f$

Or $f = \frac{1}{T} = \frac{\omega}{2\lambda} = \frac{1}{2\lambda}\sqrt{\frac{k}{m}}$

Question: An object has a mass of 0.1kg and is on a flatless table. If a 5N force is applied, the spring in stretched 0.2m,

(a) What is the spring constant? (b) Find the characteristic frequency and period of oscillation that the mass is set in motion.

Solution:

a) F = kx = k = $\frac{f}{x} = \frac{5}{0.2} = 2.5\ N/m$

b) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{25}{0.1}} = 2.52.Hz$, T = $\frac{1}{f} = \frac{1}{2.52} = 0.397s$

= $0.397sec$

3.1 Energy in Simple Hamonia Motion

In S.H.M., like pendum, there is a continual interchange of potential and kinetic energy, i.e. when the pendulum is at its highest point, the velocity is zero and the energy is entirely potential.

Simply, when a mass oscillates on a spring, the total energy is constant and there is also a continual interchange of potential and kinetic energy. It is convenient to define potential energy to be zero at the equilibrium point. As the mass passes through **X** = 0, its energy is entirely kinetic.

The potential energy at a displacement **X** is equal to the work that must be done against the restoring force to stretch the spring to that extent.

Hence a displaced object, work done by a force **F** is $\int frds$ and the required force to stretch a spring is F = kx. Hence, work done in stretching the spring from 0 to R is

$$\omega = \int_0^x f dx = \int_0^x k_x d_x = \frac{1}{2} kx^2 =$$

∴ Potential Energy $u = \frac{1}{2}kx^2$

Total energy = $P.E + K.E = \frac{1}{2} mv^2 + \frac{1}{2}kx^2$

Example: A mass of 2kg on a spring is extended 0.3m from the equilibrium position and released from rest. The spring constant is 65N/m

(a) What is the initial potential energy of the spring?

(b) What is the maximum speed of the mass after it is released?

(c) Find the speed when the displacement is 0.2m

Solution (a) initially the displacement is 0.3m, so $u_0 = \frac{1}{2}$ kx^2

$$u_0 = \frac{1}{2} \text{kx}^2 = \frac{1}{2} \text{ x } 65 \text{ x } (0.3)^2 = 2.92J$$

(b) The energy is totally kinetic when the spring and the mass passes through the unstretched position x = 0. So the K.Σ $\frac{1}{2} mv^2$

$$\frac{1}{2} mv^2 = u_0$$

$$v = \sqrt{\frac{2u_0}{m}} = \sqrt{\frac{2\ (2.92)}{2}} = 1.71\ m/s$$

(c) When x = 0.2m, potential and kinetic energies are non–zero since total energy is conserved:

$$\therefore \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = u_0$$

$$v = \sqrt{\frac{2}{m}}\,(u_0 - \tfrac{1}{2}\,\mathrm{kx}^2)$$

$$= \sqrt{\frac{2}{2}}\,(2.92 - \tfrac{1}{2}\,(65)(0.2)^2 = 1.27\,m/s$$

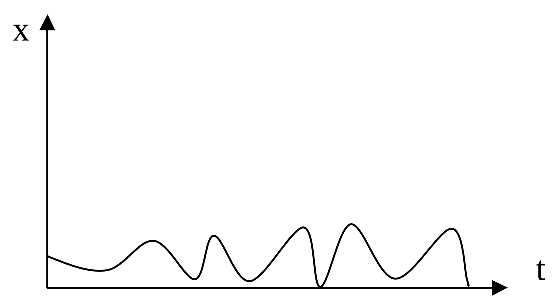
3.2. Damped Oscillation

Most real situation cannot be described precisely by the equations of S.H.M. because of the presence of dissipative forces such as friction or air resistance. For instance, a pendulum clock will gradually come to rest unless energy is supplied to replace the losses.

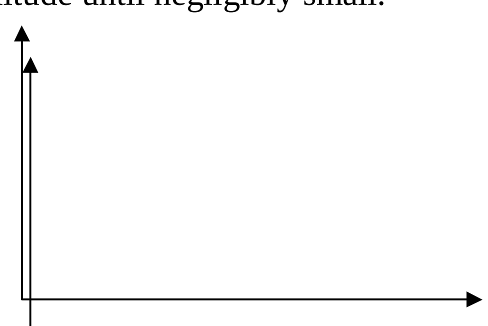
Damping is caused by dissipative forces, typically dependent on the velocity. Dissipative force $\mathbf{F_d}$ is linearly proportional to V i.e. F_d = rv, r = damping constant, while the minus sign indicates that the damping force opposes the motion.

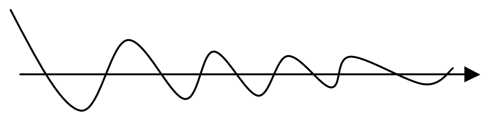
Now consider the effect of damping force in the equation of motion for a weight on a spring:

When r = 0, the oscillation continues with same amplitude indefinitely.

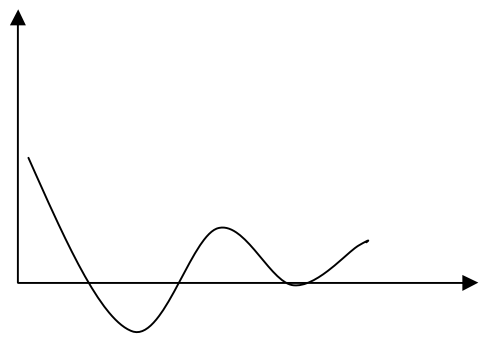


When a small amount of damping is present, oscillation is steedily decreases in amplitude until negligibly small.





If r is larger, then the oscillation is faster



But when very large, oscillation cannot occur and the body/weight returns to its equilibrium position without oscillation.

3.3 Forced Oscillation and Resonance

When a vibrating system is set in motion, it vibrates at its natural frequency. However, a system is often not left to merely oscillate on its own but may have an external force applied to it, which itself oscillates at a particular frequency.

For instance a mass on a spring when pulled, vibrates back and forth at a frequency f, the mass then vibrates at the frequency f of the external force, even if this frequency is different from the natural frequency of the spring, which we denote as $f_{0,}$ where f_0 is:

$f_0 = \frac{1}{2x}\sqrt{\frac{k}{m}}$, this is an example of forced oscillation.