SECTION THREE

Static Electricity

A basic knowledge of the atomic structure of matter is necessary to understand the phenomenon of static electricity.

An atom of any element consists of a central nucleus with one or more electrons revolving round it in fixed orbits. The nucleus is made up of two types of particles called neutrons and protons. An electron has a –ve charge while the proton has a +ve charge equal in magnitude to the –ve charge of the electron. A neutron however, has no charge.

An atom in the normal state contains an equal number of electrons and protons. In such an atom, the total +ve charge of the protons is neutralized by the total -ve charge of the electrons, and therefore, the atom is said to be electrically neutral.

A body is said to be +vely charged when it loses some (or at least one) of its electrons; -vely charged when it possesses more than its normal share of electrons; and neutral when it is in its normal state.

When one neutral body is rubbed against another, there is a transfer of electrons.

Elementary experiments on rubbed bodies indicate that

- a) There are two, and only two kinds of electric charge (+ve and -ve)
- b) Like charges repel, unlike charges attract

Thus when a glass rod is rubbed with silk, the rod becomes +vely charged, and when an ebonite rod is rubbed with fur, the rod becomes –vely charged.

If any of these are brought close to an oppositely charged body, attraction takes place.

Conductors and Insulators

All substances offer a certain degree of resistance to the flow of electric charge; they can be classified as either conductors or insulators according to the degree of resistance they offer to the passage of electrons.

- A charge placed on an insulator (or dielectric material) is confined to the region in which it
 was placed. An insulator has no charge carriers that are free to migrate within the
 boundaries of the body.
- A charge placed on a conductor may be allowed to spread over the whole surface of the body, because the body has charge-carriers which are free to migrate.
 - In metals, which are good conductors, the charges are carried by free or conduction electrons (-ve).
 - In electrolytes, which are intermediate conductors, the charge carriers are ions of both signs.
 - In semiconductors, the charge-carriers may appear to be +ve, -ve, or both.

Charge Quantization

The charge of an electron is -e, where $e = 1.6 \times 10^{-19}$ C.

No particle has been observed to carry a fraction of this charge. Any charge Q is Q = ne, where n is a +ve or –ve integer, and e is the fundamental quantum of electric charge.

Electric Force and Coulomb's Law

The electric force between two static charges is sometimes referred to as the Coulomb force. This force

- a) Obeys the superposition principle a third charge will not affect the amount of force exerted by two charges on each other.
- b) It is a conservative force.
- c) It is a central force.

Diagram showing i) isolated static +ve and –ve charges

ii) two +ve charges undergoing attraction

The electric force between two static charges Q and Q₀ (a test charge) is expressed by Coulomb's law given by

$$F = k \frac{QQ_0}{r^2}$$

 $F = k \frac{QQ_0}{r^2}$ Unit: N Dimension: [MLT⁻²] It is a vector quantity

where $k = \frac{1}{4\pi\epsilon_0}$ Given that $\epsilon_0 = 8.85 \times 10^{-12} \, \text{Fm}^{-1}$, then $k = 9 \times 10^9 \, \text{Nm}^2 \text{C}^{-2}$

Q is a static charge.

Unit: coulomb, C

 Q_0 is a test charge.

Unit: coulomb, C

r is the distance between the centres of the two charges.

Unit: metre, m

Electric Field Strength, E

E is called the electric field strength, the electric field intensity, the electric vector, or simply the electric field.

Suppose we place a test charge Q₀ at a point in an electric field where it experiences a force F, the the electric field strength, E at that point is defined by the equation

$$E = \frac{F}{O_0}$$

Unit: NC⁻¹ or Vm⁻¹ (from $E = \frac{V}{d}$) It is a vector quantity

- E represents a vector quantity whose direction is that of the force that would be experienced by a positive test charge.
- E is numerically equal to the force acting on unit charge placed at a point in the field established by Q.

- The magnitude of Q_0 must be small enough not to affect the distribution of the charges responsible for E.
- The dimensions of E are [MLT⁻³I⁻¹].

A resident charge sets up a field E, all around it within the space surrounding this charge. A test charge brought into this field set up by the charge experiences an electric force. The magnitude of which is, for point charges:

$$E = k \frac{Q}{r^2}$$

Electric Potential

Suppose an external agent does work W in bringing a test charge Q_0 from infinity to a particular point in an electric field, then the electric potential at that point is defined by the equation $V = \frac{W}{Q_0}$ Unit: JC⁻¹ or volt, V Dimension: [ML²T⁻³I⁻¹] It is a scalar quantity

 The potential at a point is numerically equal to the work done in bringing unit positive charge from infinity to that point provided the field is not disturbed by the presence of such a large charge.

Away from the charge that sets up a field, electric potential is established along the field lines. The magnitude of which is, for point charges:

$$V = k \frac{Q}{r}$$

Electric Potential Difference

Suppose an external agent does work W_{AB} in bringing a test charge Q_0 from a farther point B to a nearer point A in an electric field, then we can calculate the potential difference between the points A and B by the equation

$$V_{BA} = V_A - V_B = \frac{W_{AB}}{Q_0}$$

For point charges, this becomes

Diagram illustrating electric potential energy

$$V_{BA} = V_A - V_B = k \left(\frac{1}{a} - \frac{1}{b} \right) Q$$

Equipotential Surfaces

These are surfaces on which the potential is the same at all points. This means that no work has to be done in moving a test charge between any two points. Equipotential surfaces and electric field lines are perpendicular to each other at any crossing point. Equipotential surfaces due to a point charge are a family of concentric spheres.

Classwork 1

Current Electricity

An electric current consists of the movement of electric charge. A steady current exists when there is a systematic drift of charge carriers. The conventional direction of an electric current is that in which there is a net displacement of +ve charge over a given time interval.

Current is a primary or fundamental quantity. Its unit is the ampere, A.

Suppose a conductor carries a current I, then the rate of flow of charge Q past a given cross-section is defined by the equation

$$I = \frac{dQ}{dt}$$

The charge Q that passes a given cross-section in a given time is found from $Q = \int I dt$.

When the current does not vary with time we can use **Q** = It

Ohm's Law

Provided all physical conditions such as temperature are fixed, then over a wide range of applied potential differences, $V \alpha I$.

Circuit Equation

For any circuit, the e.m.f. = sum of potential differences in the circuit; that is

$$\mathcal{E} = I(R + r)$$

where E is the electromotive force, e.m.f. in volts, V;

I is the current flowing in the circuit, in ampere, A;

R is the sum of external resistances in ohms, Ω ;

and r is the sum of internal resistances of cells in the circuit, in ohms, Ω .

IR is the terminal potential difference, and Ir is the lost volt.

Hence $\mathcal{E} = \mathbf{IR} + \mathbf{Ir}$ that is, e.m.f. = terminal p.d. + lost volt

Power by a Circuit

The power deliverable by a circuit is given by $P = I^2R$

Other important parameters

Conductance, G = 1/R Unit: Ω^{-1} (Siemens, S)

Resistivity, $\rho = RA/I$ Unit: Ωm

Conductivity, $\sigma = 1/\rho$ Unit: $\Omega^{-1} \text{m}^{-1} \text{ (Sm}^{-1})$

Classwork 2

Series and Parallel Arrangements of Cells, Resistors, and Capacitors

Cells in Series

Appropriate Equation: $\varepsilon = Ir$

If n cells are arranged in series, then

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \ldots + \mathcal{E}_n$$

$$I = constant => I_1 = I_2 = I_3 = ... = I_n$$

$$r = r_1 + r_2 + r_3 + \ldots + r_n$$

Cells in Parallel

$$\varepsilon$$
 = constant => $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \dots = \varepsilon_n$

$$I = I_1 + I_2 + I_3 + \dots + I_n$$

$$1/r = 1/r_1 + 1/r_2 + 1/r_3 + \ldots + 1/r_n$$

Resistors in Series

Appropriate Equation: **V** = **IR**

If n resistors are arranged in series, then

$$V = V_1 + V_2 + V_3 + \ldots + V_n$$

$$I = constant => I_1 = I_2 = I_3 = ... = I_n$$

$$R = R_1 + R_2 + R_3 + ... + R_n$$

Resistors in Parallel

$$V = constant => V_1 = V_2 = V_3 = ... = V_n$$

$$I = I_1 + I_2 + I_3 + \dots + I_n$$

$$1/R = 1/R_1 + 1/R_2 + 1/R_3 + ... + 1/R_n$$

Capacitors in Series

Appropriate Equation: Q = VC

If n capacitors are arranged in series, then

$$Q = constant \Rightarrow Q_1 = Q_2 = Q_3 = \dots = Q_n$$

$$V = V_1 + V_2 + V_3 + \ldots + V_n$$

$$1/C = 1/C_1 + 1/C_2 + 1/C_3 + ... + 1/C_n$$

Capacitors in Parallel

$$Q = Q_1 + Q_2 + Q_3 + \ldots + Q_n$$

$$V = constant => V_1 = V_2 = V_3 = \ldots = V_n$$

$$C = C_1 + C_2 + C_3 + \ldots + C_n$$

Classwork 3

Alternating Current (a.c.) Theory

Representation

Current $I = I_m \sin \omega t$ or $I = I_0 \sin \omega t$

Voltage or e.m.f. $V = V_m \sin \omega t$ or $V = V_0 \sin \omega t$

 $\mathcal{E} = \mathcal{E}_m \sin \omega t$ or $\mathcal{E} = \mathcal{E}_0 \sin \omega t$

I, V, and E are called Instantaneous values of current, voltage, and e.m.f. respectively.

 I_m , V_m , and ε_m are called maximum (or peak) values of current, voltage, and e.m.f. respectively.

 I_m , V_m , and ε_m can be replaced by I_0 , V_0 , and ε_0 .

ω is called the pulsatance (Unit: rad. s⁻¹); and t is the time (Unit: s)

Effective or Root-Mean-Square (r.m.s.) Value

Current
$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.7071 I_m$$

Voltage or e.m.f.
$$V_{rms} = \frac{V_m}{\sqrt{2}}$$
 or $\varepsilon_{rms} = \frac{\varepsilon_m}{\sqrt{2}} = 0.7071 \, \varepsilon_m$

Reactance, X Unit: Ω

Appropriate Equation: V = IX

i) A Purely Resistive Circuit

-	Reactance	$X_R = R$
-	Energy stored	W = 0

- Average power dissipated $<P> = V_{rms}I_{rms}$

V and I are in phase

ii) A Purely Inductive Circuit

$$\begin{array}{lll} - & Reactance & X_L = \omega L \\ - & Energy stored & W = \frac{1}{2} L I_m^2 \\ - & Average power dissipated & = 0 \\ V \ leads I \ by \ \pi/2 & \end{array}$$

iii) A Purely Capacitive Circuit

-	Reactance	$X_C = 1/\omega C$
-	Energy stored	$W = \frac{1}{2} CV^2$
-	Average power dissipated	<p> = 0</p>
	V lags I by π/2	

Impedance, Z Unit: Ω

Appropriate Equation:
$$V = IZ$$
 or $Z = \frac{V_{rms}}{I_{rms}}$

RCL Series Circuit

Impedance, Z is given by
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Phase angle
$$\varphi$$
 is given by
$$\tan \, \emptyset \, = \, \frac{v_{L-\,V_C}}{v_R} \, = \, \frac{x_{L-\,X_C}}{R}$$

$$V_L = IX_L \qquad \qquad V_C = IX_C$$

Resonant frequency, f₀

Condition for resonance:

Z must be a minimum

$$X_L - X_C = 0$$
 or

$$f_{0=\frac{1}{2\pi\sqrt{LC}}}$$

Power factor, cos φ

$$\cos \Phi = \frac{R}{Z}$$

Average Power, P

$$P = I_{rms}V_{rms}\cos \phi$$

V leads I by the phase angle ϕ .

RC and RL Series circuits can thus be configured from the RCL structure.

Classwork 4