**SPECIAL THEORY OF RELATIVITY**

**INTRODUCTION: HISTORICAL PERSPECTIVE**

- In 1905 Albert Einstein (1879-1955), at the age of 26, - published four scientific papers that revolutionalized physics.
- One of the papers dealt with the photoelectric effect - responsible for the birth of quantum physics.
- Two of the papers dealt with special theory of relativity. Relativity represents the greatest intellectual achievement of twentieth century physics. Just as quantum theory showed that the classical concepts are to be revised in case of microscopic world, the relativity theory established that the classical notions are not applicable to bodies moving with velocities approximately nearer to that of light.
- Classical mechanics regarded space and time to be absolute and separate entities. It assumed the flow of time to be uniform in all situations. As such the moments of time and time intervals are supposed to be identical in all frames of reference. Similarly, lengths are assumed to be identical in all frames. An analysis of these concepts at high velocities revealed that they are not correct. The relativity theory leads to many unusual conclusions. It shows that the length of moving bodies contract in the direction of motion and the clocks in motion slow down. It is difficult to comprehend these conclusions because of our habit founded on routine experiences. At high velocities, space and time are no more separate but merge into space-time continuum. The results of experiments on high energy particles provide the proof for the predictions of relativity.

**SPACE, TIME AND MOTION**

- The concepts of space, length, time and mechanical motion appear to us as self-evident and obvious.
- However, Classical mechanics presumes the space to be homogeneous in all its parts and also isotropic. It means that the properties of space are identical at all points and in all directions at each point. Classical mechanics further supposes the existence of an absolute space which is absolutely motionless and irrelevant to the existence of any bodies. According to Newton "absolute space, in its own nature, without regard to anything external, remains always similar and immovable".
- In classical mechanics, time is understood as a measure of absolute duration which exists irrespective of physical bodies. Newton regarded that the true course of time is not liable to

change. In his opinion the course of time belongs to absolute category.

In Newton's theory of gravitation, the force $F = G m_1m_2/r^2$ is a force that acts instantaneously. If a force can act instantaneously, it means that a signal, or energy, could be transmitted instantaneously from $m_1$ to $m_2$. This violates one of the basic tenets of relativity: that no energy, not even a signal, can travel at a speed faster than the speed of light.

The position of a body at any instant of time is determined by measuring its distance from a set of coordinate axes at that particular instant of time. Hence, the coordinate axes x, y, z, and t are needed to specify fully the position of the body.

**Frames of Reference**

- Space cannot be thought of without physical bodies.
- To locate a body in space, the Cartesian coordinate system is used normally. The body may be a fixed body used for reference or it may be a body which is in motion.
- The choice of a frame of reference is determined by our own convenience.
- Coordinate system attached to the reference fixed body is called a "frame of reference" or a "coordinate **system".** It is impossible to attach a set of coordinate axes to empty space.
- For describing the motion of bodies on the Earth, we choose a frame of reference rigidly connected to the Earth, which is regarded as a fixed body. However, in reality the Earth is in circular motion. In investigation of the Earth's motion, we attach the coordinate system to the Sun. In studying the Sun's motion, we choose a reference frame connected to the stars. Sometimes, we choose the floor or walls of a room as the reference frame which may be at rest with respect to the Earth or which may be in motion if it is on a train or in a spacecraft.
- The choice of a frame of reference is arbitrary. Therefore, a passenger in a moving train may claim to be at rest and declare that the electric poles and trees are moving backward while a person standing on the ground claims that the train is moving forward. Both are equally right. On the basis of our daily experience, we intuitively choose a frame of reference attached to the Earth and describe the motion of various bodies. Thus, in the reference system attached to the Earth, it is the traincar that moves forward.

***The set of coordinate axes with reference to which the position of a particle is measured, and time scale with reference to which the time of these measurements is expressed are jointly known as a frame of reference.***

## INERTIAL FRAMES OF REFERENCE

- Basically, there are infinite number of reference frames available and any one of them can be used. But the laws of mechanics may take a different form in different reference frames. For instance, let us consider acceleration of a body relative to an arbitrary frame of reference. The acceleration of the body may be due to its interaction with other bodies or it may be due to the properties of the reference frame itself. If the acceleration of the body arises solely due to its interaction with other bodies, then the frame of reference is said to be an inertial reference frame. In an inertial frame of reference, a free body moves rectilinearly and uniformly, without exhibiting acceleration.
- Newton's first law states that every body continues in its state of rest or of uniform rectilinear motion, unless it is compelled to change that state by forces impressed on it. This is known as the law of inertia. This law, in effect, defines an inertial frame of reference.
- An **inertial frame of reference** is one in which a body, not subjected to a force, moves with constant velocity. A reference frame which moves with constant velocity relative to the distant stars is the best approximation of an inertial frame. In reality, the Earth is not an inertial frame because of its orbital motion about the Sun and rotational motion about its own axis. However, the Earth may be assumed to be an inertial frame in many situations.
- Any other reference frame moving rectilinearly and uniformly relative to an inertial frame is also inertial. Thus, there is a vast number of inertial reference frames moving relative to one another uniformly and rectilinearly.

## NON-INERTIAL REFERENCE FRAME

- A frame of reference which is in an accelerated motion with respect to an inertial frame of reference is known as a non-inertial frame of **reference.** The law of inertia is valid in an inertial frame, whereas it is not valid in an accelerated reference frame. A ball placed on the floor of a train will move to the rear if the train accelerates forward even though no forces act on it; likewise, a coin placed on a rotating turntable will slide to the periphery though no visible force pushes it away from the centre.
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**Examples of the idea of frames of reference**

i) A car C moving with uniform velocity in a straight line along a road. An observer sitting in the car can determine the position of an object A inside the car by choosing a frame of reference fixed in the car. If the object A is moving relative to the car, then the observer can also determine its velocity relative to the car. However, this velocity will be different from the velocity of the same object as measured by an observer standing by the road. This second observer will have to use a frame of reference fixed on the road. If A is moving in the same direction as C in C with a velocity of 1 $ms^{-1}$, the observer would measure it as 1 $ms^{-1}$. However, an observer on the road would measure it as 11 $ms^{-1}$ if C was moving with a velocity of $10ms^{-1}$.

ii) Consider another simple experiment: If the observer sitting in the car C throws a ball vertically downward on the floor of the car, assumed to be perfectly smooth, then to him the ball will appear to rebound vertically upward (Actually, the ball does not fall back to the same point at which it first hit the floor of the car. It describes an ellipse as it is in relative motion to C, and vice versa during its period of flight.). The movement of the ball is determined by Newton's laws of motion. If however, an observer standing by the roadside watches the ball, to him it will appear to follow a parabolic path going downward, and then a similar parabolic path upward after rebound. If the velocity of the car is known, then the observer by the roadside can determine the path of the ball with the help of Newton's laws of motion. If the reverse experiment is performed in which the observer by the roadside throws a ball vertically downward, to him the ball will appear to rebound vertically upward, while to the observer in the moving car C, the ball will appear to follow parabolic paths, both downward and upward.

It is usually assumed that all frames of reference attached to the earth are inertial frames. This is not strictly true. Newton believed in the existence of an absolute space; again it has been prove that absolute space does not exist.

In this study, our attention will be focused on the situation in which two frames are moving with constant velocity, v with respect to each other. Such frames are known as Inertial frames of reference. By limiting our considerations to the inertial frames of reference, we will only be studying a Special Theory of Relativity.

**Equivalent Frames**

Two frames of reference which are in uniform rectilinear motion with respect to one another are equivalent.

Newton's law of motion remains the same in frames of reference which are either at rest or are in uniform rectilinear motion relative to one another; that is they are *covariant*. This means that an observer at rest in a particular frame of reference will not be able to decide whether his frame is at rest or is in uniform rectilinear motion by performing some mechanical experiments within the frame

**GALILEO'S PRINCIPLE OF RELATIVITY**

- A reference frame for which Newton's laws of dynamics hold is called **inertial.** If a body is in uniform motion (v = constant) an observer in an inertial frame which is at rest with respect to the body will find that the acceleration and the resultant force on the body are zero *(i, e.,* $a = 0$ and $F = 0$*).* An observer in another frame of reference moving with uniform velocity in a straight line with respect to the first frame will also find that $\boldsymbol{a} = 0$ and $F = 0$ for the body. Consequently, the second reference frame is inertial to the same degree as the first.
- It follows that any reference frame moving with uniform velocity with respect to an inertial frame is also an inertial frame.
- Thus, reference frames fixed in a railway car or a ship, which travels with a uniform velocity in a straight line, are inertial frames. Experience shows that in a railway car or on a ship travelling with uniform velocity, it is equally easy to move in any direction as it is on the Earth. A body released at a height falls vertically downward. Thus, the results of an experiment performed in a uniformly moving vehicle will be the same as those from the same experiment performed in the stationary laboratory. Therefore, it is not possible for us to tell by any experiment whether we are at rest or moving with uniform velocity.
- It follows that the laws of mechanics have the same form in all inertial frames and none of the reference frames have any advantage over the others. It implies that there is no "exclusive" or "pre-ferred" frame of reference and every inertial frame is as good as the other. Absolute rest or absolute motion of bodies has no sense. We can only speak of their relative motion in some inertial frame of reference.
- This basic law of nature was recognized by Galileo and is summed up in the form of principle of

relativity. Galileo's principle of relativity states that *the laws of mechanics are the same in all inertial frames of reference.*

## GALILEAN TRANSFORMATIONS

- The transformation from one inertial frame of reference to another is called a **Galilean transformation.** Knowing the laws of motion of a body in a reference system S, one can derive the laws of motion of the same body in another inertial system S'.

### Galilean Transformation Equations

With the help of the underlisted transformation relations, it is possible to transform the coordinates and time from the (x, y, z, t) frame to the (x', y', z', t'), and vice versa.

| S' | | S |
|---|---|---|
| $x' = x - vt$ | => | $x = x' + vt'$ |
| $y' = y$ | | $y = y'$ |
| $z' = z$ | | $z = z'$ |
| $t' = t$ | | $t = t'$ |

where v is the relative speed of S' frame whose motion is parallel to the x – axis of the frame S, and in the direction of increasing x values.

*Example:* A train moving with a speed of 72 $kmh^{-1}$ along a straight rail track, passes a station at midnight (00.00 hr.). Twenty-five seconds later, a lightning bolt strikes at a point on the track 1.5 km away from the station in the direction of motion of the train. What are the coordinates of the point at which the lightning struck as measured by an observer at the station and by another observer in the train?

### Electromagnetic Waves and the Ether

**The Michelson-Morley Experiment**

## FAILURE OF GALILEAN TRANSFORMATIONS

- The laws of mechanics are invariant under a Galilean transformation. However, the laws of electromagnetism are found to change their form under a Galilean transformation. This fact can be demonstrated with the following simple example.
- If we consider two equal and like charges $q$ which are, stationary in one reference frame $S$. They repel each other according to Coulomb's law. Now, suppose these charges are examined by an observer in a reference frame S' moving with velocity u along a line perpendicular to the line joining the two charges. The observer will find the two charges moving with velocity -u. The two moving charges constitute two currents in the same direction and two parallel currents attract each other. Therefore, the observer in the moving frame finds that the charges exert a mutual repulsive force as well as an attractive force.
- It means that either the Galilean transformaion is wrong or Maxwell's equations are wrong. However, it was found that Maxwell's equations are in total agreement with all known experiments.
- Albert Einstein was convinced that Maxwell's equations are true and must yield the same result in all inertial frames of reference.

## EINSTEIN'S PRINCIPLE OF RELATIVITY

- Based on thorough analysis of all experimental and theoretical data accumulated by the beginning of the twentieth century, Einstein concluded that the classical concept of space and time are to be revised. In 1905, Einstein published a paper entitled "On the Electrodynamics of Moving Bodies" in the seventeenth volume of *Annalen der Physik*, a German scientific journal. In this paper, he made the revolutionary proposal that the Newtonian concepts of space and time are to be revised and propounded the special theory of relativity. He suggested that the experimental failure to detect uniform motion relative to an absolute reference system simply means that there is no such system at all and that all the inertial frames are completely equivalent.

The special theory of relativity is based on the following two postulates:

*1. The principle of relativity:* All the laws of physics are the same in all inertial frames of reference.

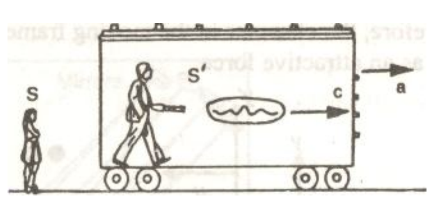
*2. The principle of independence of the velocity of light:* The speed of light in a vacuum is indepedent of the motion of the light source or receiver.

The term "special" implies that this theory considers phenomena only in inertial reference frames.

The first postulate is, in effect, a generalization of Galilean principle of relativity to cover all physical processes. All physical phenomena proceed identically in all reference frames. All physical laws are obsolutely identical in all inertial systems. Basically, no experiment can distinguish one of the frames as preferable. Thus, Einstein's principle of relativity establishes the complete equality of all inertial frames and rejects the Newton's ideas of absolute space and absolute motion.

The second postulate states that the velocity of light in a vacuum has the same value for all observers and is independent of their motion or of the motion of the light source. In contrast to all other velocities which change on transition from one reference frame to another, the velocity of light in a vacuum is invariant. This invariance of velocity of light requires that we modify some of our intuitive, everyday notions of space and time.

Let us take once again the example of an observer in a railcar moving with a velocity u. Suppose a flash of light is sent by him. The flash travels with a velocity c relative to the observer in the railcar. What will be the speed of the flash relative to the stationary observer on the ground? We expect the velocities to add as in classical case and the speed of the flash should be(c + u). Acording to the Einstein's second postulate, both observers would measure the speed of light to be c. This to the observer in a moving railcar the pulse should be c + u relative to a stationary observer.




Classically, the concluded speed appears to violate our common sense.

The second postulate explains the null result of observation made by the Michelson-Morley experiment clearly. It was presumed in the experiment that when light travels against the ether wind, its speed would be c - u and after reflection at mirror *M*,, it returns to *M* with a speed c + u. Because of the differences in speed, the beams *A* and *B* would acquire a phase shift and hence a fringe shift was predicted. According to Einstein's second postulate the value of c is the same in all directions irrespective of the direction of the motion of the Earth. Consequently, the path difference between beams *A* and *B* would be zero and as such shift in the fringe pattern does not take place.

**The Einstein Postulates**

In 1905, Albert Einstein proposed a Special Theory of Relativity which provided adequate explanation for the null result (no shift of interference fringes) of the Michelson-Morley experiment. Einstein based his theory on two postulates which made the null result obvious. The postulates are:

i) The laws of physics are the same (invariant) in all inertial frames of reference.

ii) The speed of light, c in empty space is the same for every inertial frame, and is independent of the source of light relative to the observer.

An important implication of the first postulate is that there is no particular preferred reference frame, at rest or otherwise, to which the velocity of objects could be measured.

The second postulate, in particular, when applied to the derivation of the equations in Michelson-Morley experiment gives $\Delta T = 0$. It follows therefore that n =0, which is in line with the observation.

**THE LORENTZ TRANSFORMATIONS**

- In Newtonian mechanics the Galilean transformation equation, relate the space and time coordinates in one inertial frame to those in the other frame. the equations are not valid for cases where v approaches the value of c. The transformation equations that apply for all speeds up to c and incorporate the invariance of speed of light were developed in 1890 by the German Physicist H.A. Lorentz (1853-1928). They are known as the **Lorentz transformations.** Their real physical significance was later established by Einstein.

(a) **Coordinate transformations**

Let us consider two inertial reference frames *S* and *S'* in which the standards for measuring distances and time are the same. Let the reference system *S* be stationary while the system *S'*, moves with constant velocity v relative to system S along the directions of the axes x and x'. The axes x and x' are in one line and the axes y and y' and z and z' are parallel. Let us assume that at the initial instant, *t* .= *t'* = 0 and the origin 0 and 0' of the reference systems coincide. We further assume that at the initial instant t = t' = 0, a light source placed at the origin emits a light pulse (flash). The light pulse (flash) travels in the form of a spherical wave at the speed c in both the reference frames. Then during the time interval t' the light pulse travels a distance of *ct* in the reference frame *S*. Similarly, in the reference frame S' , the light pulse travels a distance of ct' during the corresponding time interval t' . In both the frames the points reached by the light pulse at t and t' respectively lie on spherical surfaces of radii *ct* and *ct'*.

**The Lorentz Transformation Equations**

With the Einstein's postulates, it was necessary to modify the Galilean transformation equation describing events in one frame as measured by an observer in another inertial frame. In particular, it required that the intuitive (common sense) concept of time be modified.

Lorentz developed a set of transformation equations which satisfied the two postulates of Einstein, and gave a formal description of the null result of Michelson-Morley.

| S' | | S |
|---|---|---|
| $x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$ | => | $x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}}$ |
| y' = y | | y = y' |
| z' = z | | z = z' |
| $t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$ | | $t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$ |

It should be noted that the Lorentz transformation equations reduce to the ordinary Galilean equations when the relative velocity, v of S and S' is small compared with the velocity of light, c that is, v<< c.

These equations were derived by Lorentz in connection with electromagnetic phenomena. Einstein pointed out that they have a universal character because they involved only spatial coordinates and time.

*Example:* The coordinates of an event, as given by an observer attached to a frame S are x = 100m, y = 10m, z = 1m, at t = 5 × $10^{-6}$s. What are the coordinates x', y', z', and t' of this event as determined by a second observer on frame S' moving relative to S at a velocity of – 0.6c along their common x – x' axis?

The following important conclusion may be drawn from the Lorentz transformation:

- The Lorentz coordinate transformation differs drastically from the Galilean transformation. However, in the limiting case u << c, both the transformation laws coincide. It means that the theory of relativity does not reject the Galilean transformation but includes it as a special case.
- It may be seen that at u > c the radicands of the Lorentz transformation become negative and the formula lose physical meaning. It means that bodies cannot move with a velocity exceeding that of light in a vacuum. Thus no mechanical or electromagnetic agent can transport energy from one point to another with a speed exceeding c.
- If u = c, the radicands become equal to zero and in this case also, the formulae do not have physical meaning. It means that there is no reference frame in which a photon is at rest.
- The transformation law for the velocities differs radically from the ordinary addition rule known in classical mechanics. In the relativistic case (u - c), the sum of the components of the velocity along the x-axis is multiplied by the quantity $(1 + u_x u / c^2)^{-1}$ which is dependent on $uX$ , u and c. The components of the velocity along they- and taxes also change, because the time intervals change and $dt * dt'$ .
- The Newtonian addition rule for the velocity does not apply to the speed of light. If we have $ux = c$ for the speed of light in system $S'$, then in system $S$ we also have $u_x = c$
- If a light pulse propagates along the $y'$ -axis in S' frame $(u_y = c)$ , then the components of velocity of light in system S are

$$\mathbf{u_X = u} \quad \text{and} \quad u_Z = 0$$

Consequently, a ray of light which is normal to the x'-axis in system S' has a different direction in system S.

Of course, the magnitude of the speed of light remains c.

$$\sim ux + uy =[u2 + c^2 (1-u^2/c^2)]^{1/2} = 0$$

This change of the direction of the rays of light accounts for the aberration of starlight which is an observed seasonal change in the position of stars. It appears due to the Earth's orbital motion.

## CONSEQUENCES OF SPECIAL RELATIVITY

- The predictions of the special theory of relativity are very strange and startling. The strange effects, which we call relativistic effects, appear to conflict with our commonsense. Actually, relativistic effects require speeds that are close to the velocity of light. The relative velocities in our everyday experience are far smaller than the velocity of light. Therefore, we do not commonly observe the strange effects. The special relativity theory predicts that an observer will measure different times and length in different inertial frames. An observer finds that a clock in the moving system appears to run more slowly. When comparing two events, he finds the events which are simultaneous in his frame occur at different times in a moving system. The observer further finds that moving stick contract along the line of motion. For instance, a metrestick is measured to be shorter in the direction of motion in a moving frame. It means that we must give up the intuitively obvious notion of absolute time and absolute length on which Newtonian mechanics is based. The new mechanics based on relativity is called **relativistic mechanics.** In relativistic mechanics, there is no such thing as absolute time and absolute length. Events that occur simultaneously at different locations in one frame are not simultaneous in another frame.
- It should not be construed from the above discussion that Newtonian mechanics is incorrect. Newtonian mechanics is very much correct and valid for macroscopic bodies moving with speed much less then c. However, in case of the motion of high-speed particles, the relativistic mechanics is to be employed. In fact, the laws of relativistic mechanics become the same as Newton's laws for cases of $u \ll c$. It means that Newtonian mechanics is the limiting case of relativistic mechanics.

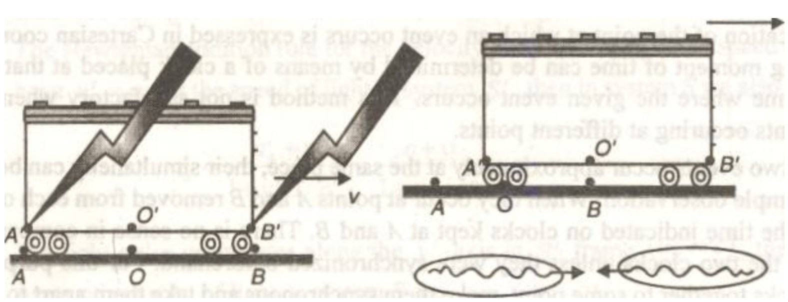
## SIMULTANEITY OF EVENTS

- Before proceeding to analyse the consequence of relativity, let us first understand the concept of simultaneity of two events occuring at different places.
- The location of the point at which an event occurs is expressed in Cartesian coordinates. The corresponding moment of time can be determined by means of a clock placed at that point of the reference frame where the given event occurs. This method is not satisfactory when we have to compare events occuring at different points.
- When two events occur approximately at the same place, their simultaneity can be detected by means of a simple observation. When they occur at points $A$ and $B$ removed from each other, one has to compare the time indicated on clocks kept at $A$ and $B$. There is no sense in comparing the time

indicated on the two clocks unless they were synchronized beforehand. For this purpose, one can bring the clocks together to some point, make them synchronous and take them apart to points A and B.

- A more elegant method to synchronize the clocks positioned at different points of a reference frame is by using light or radio signals. It can be done as follows: an observer located at the origin O of a given reference frame sends a signal at the moment $t_o$ as per his clock. At the moment when this signal reaches the clock located at $A$ at a known distance $r$ from the point 0, the clock is so set that it registers $t = t_o + r/c$ which takes into account the signal delay. The same procedure is followed at different points of the reference frame. The repetition of signal after specific time intervals permits al I observers to synchronize the rate of their clocks with that at the origin O. Having done this we can claim that all the clocks of the reference frame show the same time at each moment.
- In classical mechanics, temporal relationship between events are assumed to be the independent of the reference frame. It implies that two events that occur simultaneously in one reference frame, will also be simultaneous in all other frames moving with constant velocity relative to the first frame.
- According to Einstein, time and time interval measurements depend on the reference frame in which they are made.
  - *Two events that are simultaneous in one inertial frame of reference are in general not simultaneous in another reference frame moving with respect to the first.*
  - Therefore, simultaneity is not an absolute concept, but is a relative concept.

Einstein devised the following thought experiment to explain this concept of relativity of events. Let us consider a railcar moving with uniform velocity. Let two lightning bolts strike at its two ends as shown in Fig. 12. It leaves marks on the railcar as well as on the ground. Let $A$ and $B$ be the marks left on the ground $A'$ and $B'$ be the marks on the railcar. An observer is on the ground at 0 midway between $A$ and $B$. An observer at 0 midway between $A'$ and $B'$ *is* moving with the railcar. If the two light signals reach the observer at 0 at the same time he concludes that the events at A and $B$ occurred simultaneously.




a) b)

A moving railcar is hit by two lightning bolts. (a) The events appear to be simultaneous to the observer 0 on ground.

(b) The events appear to have occured at different times to the observer O' in the railcar.

**Lorentz Contraction and Time Dilation**

**Length Contraction**

Suppose an object has an x – component length L0 when at rest relative to an observer (L0 is called the proper length). If the object is now given an x – directed speed v, it will appear to the stationary observer to have been shortened in the x – direction (but not in the y – or z – directions). Its observed shortened x – length will now be

$$L = \left[\sqrt{1-\left(\frac{v}{c}\right)^2}\right] L_0 \qquad \text{or} \qquad L = \gamma L_0$$

where $\gamma = \sqrt{1-\left(\frac{v}{c}\right)^2}$ or $\gamma = \sqrt{1-\beta^2}$ where $\beta = \frac{v}{c}$

$\gamma = \sqrt{1-\left(\frac{v}{c}\right)^2}$ is called the ***relativistic factor***

*Example:* A metre rule passes by with a velocity of 0.6c. How long would it appear?

**Time Dilation**

Two identical clocks sitting side by side tick out time in unison. However, if one clock is now accelerated to a high speed v, so that it moves past the stationary clock and stationary observer, then the moving clock appears to the stationary observer to tick out time too slowly. While the stationary clock ticks out a time $t_s$, the stationary observer will measure the moving clock to have ticked out a time $t < t_s$ .

where $t = \left[\sqrt{1-\left(\frac{v}{c}\right)^2}\right] t_s$ or $t = \gamma t_s$

The time taken for an event to occur, as recorded by a stationary observer at the site of the event, is called the ***proper time***. All observers moving past the site record a longer time for the event to occur. Hence, the proper time for the duration of an event is the smallest measured time for that event.

**THE TWIN PARADOX**

- The time dilation effect leads to the famous "Twin Paradox" of special relativity. Let us consider a hypothetical experiment involving twin sisters Bola and Tola. After celebrating their $20^{th}$ birthday, the adventurous twin Tola sets out on a space voyage leaving behind her sister Bola on Earth. Her spaceship travels at a speed close to the speed of light ($v = 0.9998$ c). After an year she returns back to the Earth to celebrate her 21st birthday. On return she is shocked to find her sister Bola to be a 70 years old feeble lady while she herself is only 21 year old. It means while Tola has aged only one year, her twin sister has aged about 50 years. This is a paradox which challenges our common sense.
- As reckoned in the reference frame of Earth, the clocks and all the physical and biological processes on the spaceship slow down due to time dilation. Hence, Tola ages at a slower rate and at the end of her trip, when she joins her twin on Earth, she will be younger than Bola. But this explanation appears to contradict the postulate of special theory of relativity which asserts that all inertial frames are equivalent. The aging effect seems to provide a way of distinguishing among the frames. Further, from the point of view of the spaceship, the Earth is in motion and hence the clocks and biological processes on the Earth are slowed down due to time dilation. This leads us to the conflicting conclusion that Bola ages at a slower rate than Tola and when the twins are united, Bola will be younger than Tola.
- All these contradictions disappear when we take into account of the fact that Tola the space traveller, was not in an inertial frame most of the time. She experienced a series of accelerations when leaving the Earth and decelerations when coming back to the Earth. Thus, she was in an accelerated frame for a greater part of her trip. Hence, the predictions based on special theory of relativity are not valid in her reference frame. On the other hand, Bola is in an inertial frame all the time and, therefore, her predictions are reliable. Therefore, Tola will indeed by younger than Bola on returning to Earth.

*Example:* An observer attached to a frame S' moving with a speed of 0.6c relative to a space station (designated frame S) travels a distance of 9 light years to another space station. On reaching this station, he immediately returns to the first station S. Compare the age of the travelling observer, upon his return, to that of his twin brother who remained on S.

## THE RELATIVISTIC MASS

- In Newtonian mechanics, the momentum of a body is defined as p = mu and the mass of the body is supposed to be independent of its velocity. Further, it is supposed that the total momentum of an isolated system is conserved. Because of the immense significance of the conservation laws, the law of conservation of momentum is regarded as fundamental in the theory of relativity.
- In order that the total momentum of an isolated system remains constant, it is found that the mass of a body must depend on the velocity of the body. The dependence of mass m on the velocity of the body is governed by the following relation:
- It is seen that the relativistic mass is greater than the rest mass. It depends on the velocity of the body. It means that the mass of the same particle is different in different inertial reference frames. The rest mass $m_p$ , on the other hand, is the same in all reference frames.
- At velocities very small in comparison with c, the mass may be regarded as independent of the speed of the body. With increasing speed, the mass of the body steadily increases and requires a steadily increasing force to impart a constant speed to the body. At u = c, the mass becomes infinite and hence it is impossible to make a body move at the speed of light.
- Relativistic mass increases are significant only at speed approaching that of light. At a speed u = 0.1c, the mass increases only by 0.5% but at speeds of 0.9 c, the increase is more than 100%. The relativistic effects are of no significance in space flight. Atomic particles such as electrons, protons etc., have such high speeds which cause relativistic effects.
- There are experimental observations of the increase in mass of high speeds. It is found that a greater magnetic field is required to deflect a high-speed charged particle than that would be required if its mass is constant. Thus $m = \frac{m_0}{\gamma}$

## THE RELATIVISTIC MOMENTUM

- The momentum of a relativistic particle is given by $p = mv = \frac{m_0}{\gamma} v$
- This is known as the **relativistic momentum** of the particle. The momentum thus defined obeys the law of conservation regardless of the inertial reference frame chosen.
- For velocities v << c, yields the classical momentum p = $m_o$v. The velocity dependence of the classical and relativistic momentum of a particle are depicted. The difference between the moments grows substantially as the velocity of a particle approaches that of light.

The relativistic energy is thus given by $E = K + m_0 c^2$

**MASS - ENERGY EQUIVALENCE**

We rewrite previous equation as $mc^2 = E_k + m_o c^2$

If we interpret $mc^2$ as the total energy $E$ of the body, then the equation indicates that the total energy of a freely moving body consists of its rest energy plus its energy due to motion $E_k$ . When the body is at rest, it possesses the energy $m_o\, c^2$. Therefore, $m_o\, c^2$ is called the **rest energy** $E_0$. Thus $E = E_0 + E_k$

When $E_k = 0$, the body is motionless. However, it possesses the energy $E_0 = m_o\, c^2$

These equations are known as **mass-energy equivalence relations.** They imply that energy manifests as mass. This is one of the most remarkable results of Einstein's theory. Till the special theory of relativity was propounded, energy and mass were believed to be independent. The theory of relativity showed that mass is a form of energy.

The validity of the equation is evident in nuclear processes such as fusion and fission.

**EXPERIMENTAL BASIS AND EVIDENCES OF QUANTUM THEORY**

Phenomena such as motions of mechanical objects involving distances larger than $10^{-6}$m can be explained satisfactory by laws of classical physics which is based on the following basic laws.

i) Newton's laws of motion
ii) The inverse square law of gravitational attraction between two bodies
iii) Coulomb's inverse square law of attraction or repulsion between two electrically charged bodies
iv) The law of force on a moving charge in a magnetic field i.e. Lorentz force.

However, certain phenomena could not be explained by classical physics, and due to these failures, Quantum mechanics emerged as a new discipline because of the need to describe these phenomena that could not be explained using Newtonian mechanics or classical electromagnetic theory. These phenomena include the spectral distribution of energy in black body radiation, photoelectric effect, phenomena involving distances of the order of $10^{-10}$m etc.

The failure of classical physics to explain the distribution of energy in the spectrum of a black body led Max Planck to propose the quantum hypothesis in 1900, which marked the beginning of quantum theory.

**Concept of a Black Body**

A black body is one that absorbs all the radiation (of all wavelengths) that falls on it, and reflects or transmits none. A simple black body can be made by punching a small hole in a box. it is also about the best radiator, as a good absorber its also a good radiator energy. The radiation emitted is called black-body radiation, the full radiation, cavity radiation, or temperature radiation. It has the characteristics feature that the intensity of each frequency has some well-defined value which is determined by the temperature. If the cavity is at (say) 2000K, the small hole emits visible radiation whose colour is characteristic of that temperature. The quality and intensity of the radiation escaping from a black body hole does not depend on the nature of the particular surface from which it escapes but only on its *temperature*. When the body is made hotter, its radiation becomes not only more intense but also more nearly white. When we speak of the quality of radiation, we mean the relative intensities of the different wavelengths in it, the proportion of red to blue for example.

**Black Body Radiation**

Planck (1900) in an attempt to explain the distribution of energy in the black-body spectrum suggested that when radiation was emitted or absorbed, the emitting or absorbing oscillator always showed a discrete sudden change of energy $\Delta E$. $\Delta E$ is related to the radiation frequency by $\Delta E = h\nu$. h is called the Planck constant ($6.6 \times 10^{-34}$Js). For visible light, the quantum of energy, or photon carries energy $\Delta E = h\nu = (6.6\times10^{-34}\text{Js}) \times (10^{15}\text{s}^{-1}) = 10^{-19}\text{J}$.

The exact value of $\Delta E$ depends on the frequency (colour) of the light.

Einstein (1905) extended Planck's original idea by suggesting that emissions were not only generated discontinuously, but that they could exhibit particle behavior while being absorbed.

**Properties of blackbody radiation:**

- Radiation emitted by a blackbody is isotropic, homogeneous and unpolarized;
- Blackbody radiation at a given wavelength depends only on the temperature T;
- Any two blackbodies at the same temperature emit precisely the same radiation;

- A blackbody emits more radiation than any other type of an object at the same temperature;

**Nature of Radiation:** Thermal Radiation is the energy that travels from one place to another by means of electromagnetic wave motion. When absorbed by matter, it may increase the vibrational or translational kinetic energy of atoms or molecules, this increase of internal energy will usually become apparent as a temperature increases. The vibrational frequencies of atoms at room temperature are about $10^{14}$Hz. A wave of frequency of about $10^{14}$Hz would cause resonance to occur, and in general, would thus be efficient at transferring the electromagnetic wave energy to the electrically charged particles of which matter is composed. Waves whose frequency is close to $10^{14}$Hz are called infra-red waves. They are both radiated and absorbed by bodies at normal temperatures. Infra- red radiation is emitted when thermal agitation causes changes in the vibrational and rotational energy states of molecules.

A diathermanous body, such as calcium fluoride prism is one that absorbs little of the radiation passing through it (compare with a body translucent to visible light).

An adiathermanous body, such as a mass of water is one that absorbs strongly the radiation passing through it (compare with a body that is opaque to visible light). For example, a glass is diathermanous when $0.4x10^{-6}$m < λ < $2.5 \times 10^{-6}$m but diathermanous for longer wavelengths such as $10^{-5}$m. This fact can be used to explain the action of greenhouse.

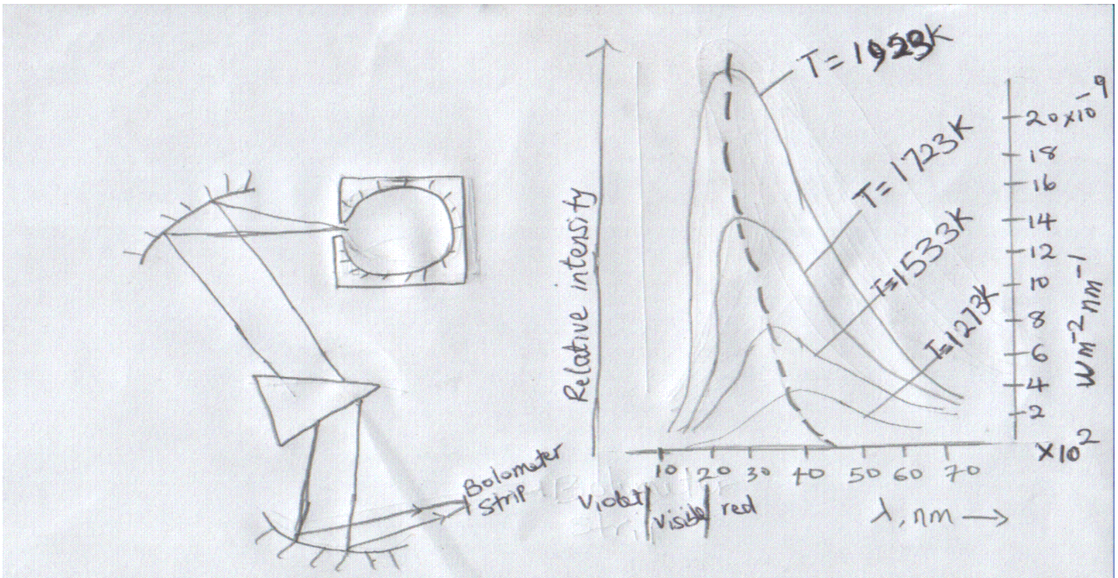
**Detection of Radiant Energy:**

Photodetectors depend upon photoelectric effect and photoconductivity. They respond, if a photon of incident radiation has a certain minimum energy (a maximum λ). Thermal detectors are thermometers, that is, they respond to the temperature changes that accompanies the absorption of all frequencies. The human skin is one example, others include:

i) The sensitive differential air thermometer,
ii) Radiomicrometer, in which a thermocouple is incorporated into a moving-coil galvo suspended by quartz fibre.
iii) The thermopile which consists of about 25 or more thermocouples joined in series, radiation falls on the blackened hot junction, while the cold junction remains shielded.

**DISTRIBUTION OF ENERGY IN THE SPECTRUM OF A BLACK BODY**

To study the quality and distribution of radiant energy from a black body over different wavelengths at constant temperature, Lummer and Pringsheim in 1899 heated a black body represented by a sphere as shown (in the figure 1, below) to $2000^0$C and measured its temperature with a thermopile. The beam coming out of the hole was passed through a diffraction grating, which sent the different wavelengths/frequencies in different directions, all towards a screen. To measure the intensities of the various wavelengths, they used an infrared spectrometer and a bolometer.



Fig(1): Lummer and Pringsheim apparatus for studying black body radiation (diagramatically) and the distribution of intensity in blackbody radiation.

Each curve gives the relative intensities of the different wavelengths, for a given temperature of the body. The actual intensity of the radiation are shown on the right of the graph, fig 1. The curves showed that as the temperature rises, shorter wavelengths increase more rapidly. Thus the radiation becomes as we have already observed, less red, that is to say, more nearly white. The curve for sunlight has its peak at about 5 x $10^{-7}$m in the visible green, from the position of this peak we conclude that the surface temperature of the sun is about 6000K, see fig 2, stars which are hotter than the sun, such as Sirius and Vega look blue, not white, because the peaks of their radiation curves lie further towards the visible blue than does the peak of sunlight.

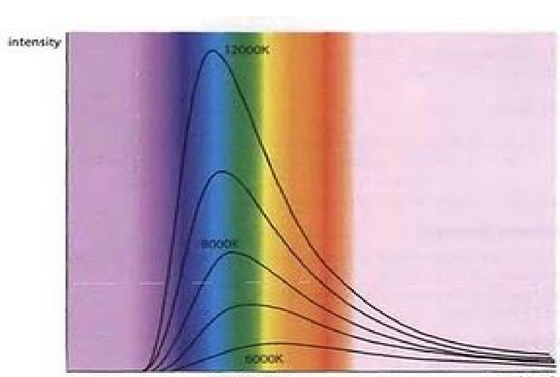


Fig 2: shows the surface temperature of the sun to be about 6000K

It is meaningless to speak of the intensity of a single wavelength. The slit of the spectrometer always gathers a band of wavelengths, the narrower the slit, the narrower the band, we therefore always speak of a given band. Hence,

Energy radiated ($m^{-2}s^{-1}$) in band $\lambda$ to $(\lambda + \Delta_\lambda) = E_\lambda \Delta_\lambda$

$E_\lambda$ is called the emissive power of a black body for the wavelength $\lambda$ at the given temperature. Therefore,

$$E_\lambda = \frac{\text{Energy radiated } (m^{-2}s^{-1}) \text{ in band } \lambda \text{ to } (\lambda + \Delta\lambda)}{\textit{bandwidth} \quad \Delta\lambda}$$

$$= \frac{\text{Power radiated } (m^{-2}) \text{ in band } \lambda \text{ to } (\lambda + \Delta\lambda)}{\textit{bandwidth} \quad \Delta\lambda}$$

$E_\lambda$ is expressed in watts per $m^2$ ($Wm^{-2}$) per Angstrom unit ($\overset{o}{A}$) SI unit may be $Wm^{-2}$ per nanometer ($10^{-9}$m).

The quantity $E_x\Delta\lambda$ is the area beneath the radiation curve between the wavelength $\lambda$ and $\lambda + \Delta\lambda$, (fig 3). Thus the energy radiated per $m^2$ per second between those wavelengths is proportional to that area. Similarly, the total radiation emitted per $m^{-2}s^{-1}$ over all wavelengths is proportional to the area under the whole area.

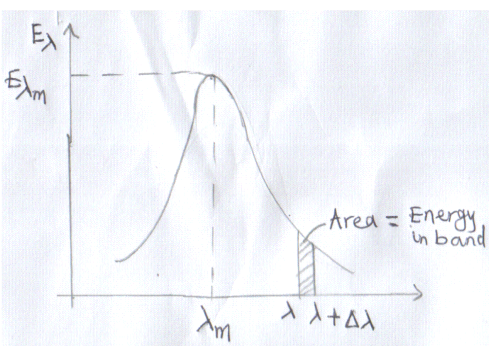



Fig 3: energy radiated per $m^2$ per second between the shown wavelengths

**Laws of Black Body Radiation**

The curves showing the distribution of intensity in Black body radiation can be explained only by Planck's quantum theory of radiation. Both theory and experiment lead to three generalizations, which together describe well the properties of black body radiation.

1. **Wien's Displacement Law**

i. Wien's displacement law states that there is an inverse relationship between the wavelength of the peak of the emission of a black body and its temperature when expressed as a function of wavelength, i.e $\lambda_M = \frac{b}{T}$

where $\lambda_m$ is the peak wavelength, T is the absolute temperature of the black body, and b is a constant of proportionality called Wien's displacement constant, equal to $2.8978 \times 10^{-3}$ m·K, that is, at a constant temperature T, when wavelength $\lambda$ is increased , the energy emitted E, first increases, reaches a maximum and then decreases i.e at a particular temperature, the spectral radiancy $E_\lambda$ is a maximum at a particular wavelength $\lambda_m$.

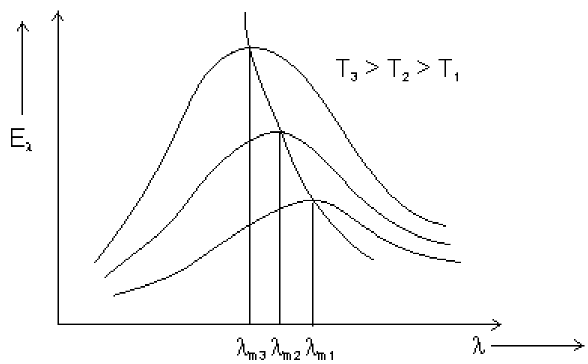



Fig 4: Wien's displacement Law

As the temperature increases , the maximum radiancy of energy occurs at shorter wavelength.

ii. If $E_{\lambda m}$ is the height of the peak of the curve for the temperature T, then

$$E_{\lambda m} \quad \alpha \quad T^5$$

iii. The curve showing the variation of $E_\lambda$ with $\Delta\lambda$ at constant temperature T obeys the Planck's formula,

$$E_\lambda = \frac{8\pi hc}{\lambda^5\left(e^{hc/\lambda KT} - 1\right)}$$

$$E_\lambda = \frac{c_1}{\lambda^5\left(e^{c_2/KT} - 1\right)}$$

Where $C_1 = 8\lambda hc$ and $C_2 = hc/\lambda$

**2. Stefan's Boltzmann's Law**

The total radiant energy emitted E per unit time by a black body of surface A is proportional to the fourth power of its absolute temperature.

$E \alpha T^4$

That is, the total area under each graph of the plots of $E_\lambda - \lambda$ graph at temp T, fig 3, should be proportional to the corresponding value of $T^4$.

or $E = \sigma AT^4$ , $\sigma$ = Stefan's constant ($5.7X10^{-8}Wm^{-2}K^{-4}$).

For a body which is not a black body, then $E = \varepsilon\sigma AT^4$ where $\varepsilon$ = emmisitivity of the Black Body, ($\varepsilon$, emmisivity =1 for a blackbody)

Note: Emmisitivity and absorptive power have the same value.

**Prevost's Theory of Heat Exchange**

In 1792, Prevost applied the idea of dynamic equilibrium to radiation. The asserted that a body radiates heat at a rate which depends only on the nature of its surface and its temperature, and that it absorbs heat at a rate depending on the nature of its surface and the temperature of its surroundings.

When the temperature of a body is constant, the body is loosing heat by radiation and gaining it by absorption at equal rates.

In simple terms it means that if a hot body A is placed in an evacuated enclosure B, at a lower temperature. Then A, cools until it reaches the temperature of B. if a body C, cooler then B, is placed in B, then C warms up to the temperature of B. we conclude that radiation from B falls on C, and therefore also on A, even though A is at a higher temperature. Thus A and C each come to equilibrium at the temperature. Thus A and C each come to equilibrium at the temperature of B when each is absorbing and emitting radiation at equal rates. (Zeroth Law of thermodynamics).

Now, suppose that, after it has reached equilibrium with B, C is transferred from B to a cooler evacuated enclosure D. it loses heat and cools to the temperature of D, therefore, it is radiating heat. However, if C is transferred from B to a warmer enclosure F, then C gains heat and warms up to the temperature of F. It is however unreasonable to suppose that C stops radiating when it is transferred to F. It is more reasonable to suppose that it goes on radiating, but while it is cooler than F, it absorbs more than it radiates.

***Net loss of thermal Energy***

If a body of surface area A is kept at absolute temp T in a surrounding of temperature $T_0(T_0 > T)$, then the energy emitted by the body per unit time is:

$E = \varepsilon\sigma AT^4$

and the energy absorbed per unit time by the body is:

$E_o = \varepsilon\sigma AT^4_o$

Net loss of thermal energy per unit time.

$\Delta E = E - E_0 = \varepsilon\sigma A(T^4 - T^4_o)$, but for a **Blackbody**, $\Delta E = E - E_0 = \sigma A(T^4 - T^4_o)$.

**Newton's Law of Cooling:**

For a small temperature difference between a body and its surroundings, the rate of cooling of the body is dirctly propertional to the temperature difference. If a body of temperature T and surface area A is kept in a surrounding temperature $T_0(T_0 < T)$. Then net loss of thermal energy per unit time.

$\frac{dQ}{dt} = \varepsilon\sigma A(T - T_o)$, T-temperature of the body, $T_o$ – temperature of the surrronding.

Since for a cooling body, the rate of heat loss is proportional to the difference in temperature between the body and its surroundings. If the temperature difference is small,

$T = T_o + \Delta T$

$$\Rightarrow \varepsilon\sigma A[(T_o + \Delta T)^4] = \varepsilon\sigma A[T_o^4(1+\frac{\Delta T}{T_o})^4 - T_o^4]$$

$$\Rightarrow \varepsilon\sigma AT_o^4[1+4\frac{\Delta T}{T_o} + higher \quad powers \quad of \frac{\Delta T}{T} - 1]$$

$$= 4\,\varepsilon\sigma AT_o^3\Delta T$$

Now, rate of loss of heat at temperature T: $\frac{dQ}{dt} = -mc\frac{dT}{dt}$

$$mc\frac{dT}{dt} = \; -4\varepsilon\sigma AT_o^3[T - T_o]$$

$$\frac{dT}{dt} = \frac{-4\varepsilon\sigma AT_o^3[T - T_o]}{mC}$$

$$\frac{dT}{dt} = -\,K[T - T_o]$$

$K = \frac{4\varepsilon\sigma AT_o^3}{mC}$, Note that for a blackbody, $\varepsilon$ =1

$\frac{dT}{dt}\,\alpha\;[T - T_o]$.

### 3. Kirchoff's Law

Most bodies are coloured, they transmit or reflect some wavelengths better than others. They must absorb these wavelengths weakly, and hence they must also radiate them weakly.

Since most radiation sources are not blackbodies. Some of the energy incident upon them may be reflected or transmitted. The ratio of the radiant emittance *W'* of such a source and the radiant emittance *W* of a blackbody at the same temperature is called the emissivity ε of the source.

That is, ε = *W'/ W, see figure 5.*

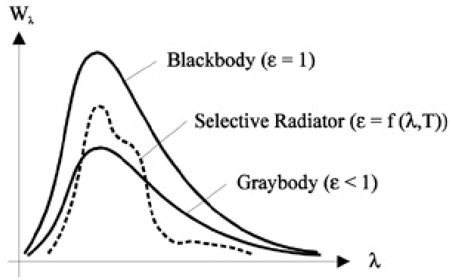



Fig 5: Diagram showing ε of some source

This can also be defined thus, the energy falling per $m^2$ per seconds on the body in the waveband $\lambda$ to $\lambda + \Delta\lambda$ is $E_\lambda\Delta\lambda$, where $E_\lambda$ is the emissive power of a black body in the neighbourhood of $\lambda$ at the temperature of the enclosure. If the body absorbs a fraction $a_\lambda$ of this, we call $a_\lambda$ the spectral absorption factor of the body, for the wavelength $\lambda$. In equilibrium, the body emits as much radiation in the neighbourhood of $\lambda$ as it absorbs; thus

Energy radiated = $a_\lambda E_\lambda \Delta_\lambda$ watts per $m^2$

We define the spectral emissivity $e_\lambda$ of a body by the equation

$$e_\lambda = \frac{\text{energy radiated by the body in the range } \lambda \text{ to } \lambda + \Delta\lambda}{\text{energy radiated in the same range by black body at the same temperature}}$$

$$= \frac{\text{energy radiated by the body in the range } \lambda \text{ to } \lambda + \Delta\lambda}{E_\lambda\Delta\lambda}$$

$$= \frac{a_\lambda\, E_\lambda\Delta\lambda}{E_\lambda\Delta\lambda}$$

$e_\lambda \quad = \quad a_\lambda \quad = \quad$ Kirchhoff's Law

Kirchhoff's Law: The spectral emissivity of a body for a given wavelength is equal to its spectral absorption factor for the same wavelength.

## 4. Sun As An Enegy Source, Solar Constant:

- Solar flux reaching the earth is a function of time determined by
  1) the orbital characteristics of the earth and the sun (i.e., eccentricity; obliquity, and periodic precession)
  2) the sun properties (e.g., solar surface activity).

**NOTE:**

a) Sun is a gaseous sphere consisting of hydrogen, helium, iron, silicon, etc.

Solar energy: nuclear fusion (conversion of four hydrogen atoms to one helium atom)

b) Temperature of sun's photosphere is about 5800 K.

b) Sunspots are cooler regions of the sun (with T = 4000K). Period between sunspot maxima is about 11 years (called **11-year-cycle**).

**Solar constant, $S_0$,** is defined as total flux of solar energy, reaching the top of the atmosphere, per unit surface normal to the solar beam at the mean distance between the sun and the earth.

The sun emits about $F_{sun} = 6.2\times10^7$ W/m$^2$. On the basis of energy conservation law, we have

$$F_{sun}\, 4\, \pi\, r_{sun}{}^2 = S_0\, 4\, \pi\, d_0{}^2$$

where $r_{sun}$ is the radius of the sun ($6.96\times10^5$ km), and $d_0$ is the mean distance between the sun and the earth ($1.5\times10^8$ km). Hence,

$$S_o = F_{Sun}\,(r_{Sun}/d_o)^2$$

Mean measured value $S_o = 1366$ W m$^{-2}$ with the measured uncertainty $\pm$ 3Wm$^{-2}$

**Actual solar flux at the top of the atmosphere** at a given time is

$$F_o = S_0 \left( \frac{d_o}{d} \right)^2 \cos(\theta_o)$$

where $\mathbf{d_o}$ is the mean distance from the center of the sun to the earth and **d** is the actual distance on a given day (depends of the earth orbit).

***Questions***

1. The tungsten filament of an electric lamp has a length of 0.5m and a diameter of 6 x $10^{-5}$m. The power rating of the lamp is 60W. Assuming the radiation from the filament is equivalent to 80% that of a perfect black body radiator at the same temperature, estimate the steady temperature of the filament. (Stefan's constant = 5.7 x $10^{-8}$Wm$^{-2}$k$^{-4}$).

***Solution:***

When the temperature is steady,

Power radiated from filament = power received = 60W

$0.8 \times 5.7 \times 10^{-8} \times 2\pi \times 3 \times 10^{-5} \times T^4 = 60$

(Since 80% = 0.8, and surface area of a cylindrical wire is 2πrh

$\therefore \quad T = \left(\frac{60}{0.4x5.7x10^{-8}x2\pi x3x10^{-5}}\right)^{1/4}$

T = 1933K

2. A metal sphere with a black surface and radius 30mm is cooled to -73$^0$C (200K) and placed inside an enclosure at a temperature of 27$^0$C (300K). Calculate the initial rate of temperature rise of the sphere, assuming the sphere is a black body (Assume density of metal = 8000kgm$^{-3}$, specific heat capacity of metal = 400Jkg$^{-1}$K$^{-1}$, and Stefan's constant = 5.7 x $10^{-8}$Wm$^{-2}$K$^{-4}$).

**Solution:**

Energy per second radiated by sphere = $\sigma A (T^4 - T_0^4)$

Where A – the surface area ($4\pi r^2$) of the sphere of radius r,

T = 200K, and $T_o$ = 300K

Since the temperature of the surroundings is greater than that of the sphere, the energy per second, Q, gained from the surroundings is given by

$$Q = \sigma 4\pi r^2 (300^4 - 200^4).$$

The mass m of the sphere = volume x density = $\frac{4}{3}\pi r^3 \rho$, where $\rho$ is the density.

If C is the specific heat capacity of the metal, and $\theta$ is the initial rise per second of its temperature, then

$$Q = mc\theta = \frac{4}{3}\pi r^3 \rho C\theta = \sigma 4\pi r^2 (300^4 - 200^4)$$

Simplifying, $$\theta = \frac{\sigma (300^4 - 200^4) \times 3}{r \rho C}$$

$$= \frac{5.7 \times 10^{-8} \times (300^4 - 200^4) \times 3}{30 \times 10^{-3} \times 8000 \times 400}$$

$$= 0.012\ K^{-1}$$

## Limitations of Wien's and Rayleigh-Jeans Radiation Formulae

### *Limitations of Wien's radiation Formula*

The harmony between theory and experiment did not last long in the Wien's formular. To Planck's consternation, experiments performed in Berlin showed that the Wien's law did not correctly describe the spectrum at very low frequencies but explains the experimental results fairly well for low values of $\lambda T$.

That is, Wien derived the law of energy distribution in the blackbody spectrum proceeding from clasical concepts. However, as was soon made clear, the formula of Wien's radiation law was correct only in the case of short ( in relation to the intensity maximum ) waves, his expression was invalid at high temperatures and long wavelength. Nevertheless, the laws of Wien have played a considerable part in the development of quantum theory.

**Limitations of Rayleigh-Jeans Law**

Lord Rayleigh in 1900 applied the principle of equipartition of energy to the electromagnetic vibrations. J.H Jeans also contributed to the experiment, by his attempt to the deduction of a formula for energy per unit volume inside an enclosure with perfectly reflecting walls.

The law showed that the energy density, $U_V dV$ i.e. the amount of energy per unit volume of the enclosure in the frequency range from $\upsilon$ to $\upsilon$ +d$\upsilon$ is given by $U_\upsilon d\upsilon = \frac{8\pi\upsilon^2 kT}{C^3} d\upsilon$, where k- Boltzmann's constant and c – Speed of light in free space.

The Rayleigh-Jeans formula can be transformed in terms of the wavelength λ by using the relation, $\upsilon = \frac{C}{\lambda}$ and $d\upsilon = \frac{-C}{\lambda^2} d\lambda$.

The energy $U_V dV$ contained in a frequency interval between $\upsilon$ and $\upsilon$ +d$\upsilon$ is equal to that contained in a corresponding wavelength interval between $\upsilon$ and $\upsilon$ +d$\upsilon$ and an increase in frequency corresponds to a decrease in λ.

Therefore: $U_\lambda d_\lambda = - U_V dV$

$$= \quad \frac{8\pi}{C^3}\left(\frac{C}{\lambda}\right)^2 KT\left(\frac{-C}{\lambda^2}\right)d\lambda \qquad = \frac{8\pi kT}{\lambda^4} d\lambda$$

This equation is another form of the Rayleigh-Jeans law

Recall the following:

- Wien's displacement formula/Law: $\lambda_m T = \frac{hc}{4.9651K} = 2.898 \times 10^{-3}$mK,

which is obtained by Finding $\frac{dU_\lambda}{d\lambda} \Big|_{\lambda m} = o$ where $U_\lambda = \frac{8\pi hc}{\lambda^5} . \frac{1}{e^{hc/\lambda KT} - 1}$

It is used to determine the temperature of a black body by determining the wavelength $\lambda_m$ at which the intensity of the radiation is maximum.

- Stefan – Boltzman law: E = σT4 where σ $\frac{2\pi^5 k^4}{15h^3c^2} = 5.67 \times 10^{-8}$ $Wm^{-2}K^{-4}$, derived from E = $\frac{cU}{4}$ where U = $\frac{4}{c}\sigma T^4$ and U = $\int_0^\infty U_\lambda d\lambda = \int_0^\infty \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda KT} - 1} d\lambda$

The work of Wien, Rayleigh and Jeans left the physics world in a dilemma until Max Planck made a radical change in classical physics by introducing the concept of quantization.

Rayleigh-Jeans Law explains the experimental facts for very long wavelengths but not for shorter wavelengths. According to the law, as λ decreases the energy density $U_\lambda$ will continually increase, and as λ tends to zero, $U_\lambda$ approaches infinity. This is contrary to experimental results. The law leads to an absurd result U = ∞, which shows that for a given quantity of radiant energy, all the energy will finally be confined in vibrations of very small wavelengths .

$$\left| \right| \quad U = \int_0^\infty \frac{8\pi KT}{\lambda^4} d\lambda = \quad 8\pi KT \left[ -\frac{1}{3\lambda^3} \right]_0^\infty = \infty \left| \right|$$

But Experimental results shows that $U_\lambda d_\lambda \Rightarrow 0$ as $\lambda \Rightarrow 0$. This discrepancy between theoretical conclusion and experimental result is sometimes known as "ultraviolet catastrophe". It is mainly because of the assumption that energy can be absorbed or emitted by the atomic oscillators continuously in any amount.

**Plancks Theory of radiation**

The failure of weins and Rayleigh-Jeans classical approach to provide satisfactory explanation of the distribution of energy in the spectrum of a black body led Max Planck to propose the quantum hypothesis/theory of radiation.

He assumed that the atoms in the walls of a black body behave like simple harmonic oscillators, and each has a characteristics frequency of oscillation. The following assumptions about the atomic oscillator were also made:

1. A simple harmonic oscillator cannot have any arbitrary values of energy but only those values of the total energy E that are given by the relation E = nhv, where n = o,1,2,3,………,n is called the quantum number, v is the frequency of oscillation, and h = 6.626 x $10^{-34}$Js is Planck's constant. In this relation, hv is the basic unit of energy and is called a quantum of energy. Thus the relation shows that the total energy of an oscillator is quantized.

2. As long as the oscillator has energy equal to one of the allowed values given by the relation E = nhν, it cannot emit or absorb energy. Therefore, the oscillator is said to be in a stationary state or a quantum state of energy.

The emission or absorption of energy occurs only when the oscillator jumps from one energy state to another.

If the oscillation jumps down from a higher energy state of quantum number $n_2$ to a lower energy state of quantum number $n_1$, the energy emitted is given by

$E_2 - E_1 = (n_2 - n_1)\, h\nu$

If $n_2 - n_1$ = unity, then $E_2 - E_1 = h\nu$

Similarly, an oscillator absorbs a quantum hν of energy when it jumps up to its next higher energy state.

According to planck, the quantum theory is applicable only to the process of emission and absorption of radiant energy. In 1905, Einstein extended Planck's quantum theory by assuming that a monochromatic radiation of frequency ν is consists of a stream of photons each of energy hν and the photons travel through space with the speed of light.

**Planck's Radiation Law**

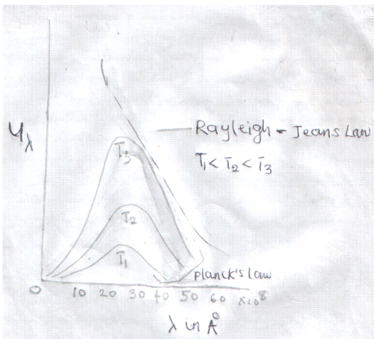
On the basis of the quantum theory, Planck obtained the formula for the average energy of an oscillator. $E = \frac{h\nu}{e^{h\nu/KT} - 1}$

Then assuming that the average value of the energies of the various modes of oscillation in black body radiation is given by the above, Planck obtained the equation .

$$U_\nu d_\nu = \frac{8\pi h\nu^3}{C^3} \cdot \frac{1}{e^{h\nu/KT} - 1} d\nu$$

Where $U_\nu\, d_\nu$ is the energy per unit volume in the frequency range ν and $\nu + d_\nu$ In terms of the wavelength of the radiation, the last equation becomes

$$U_\lambda d_\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hC/\lambda KT} - 1} d\lambda\ , \{\text{Since } \nu = \frac{c}{\lambda} \quad and \quad d\nu = \frac{-c}{\lambda^2} d\lambda\}$$




The last two equations are the two forms of Planck's radiation law. From Planck's law in the form $U_\lambda d_\lambda$, the Rayleigh- Jeans law, Wiens law and the Stefan – Boltzmann formula are obtained as mathematical consequences. The success of Planck's hypothesis was the beginning of quantum mechanics

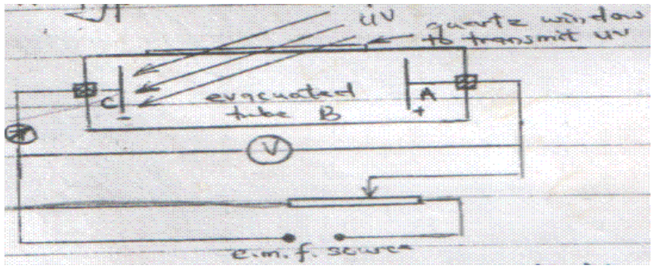
**Photoelectric Effect**

When an electromagnetic radiation of sufficiently high frequency such as ultraviolet and x-rays is incident on a metal surface, electrons are emitted from it. This phenomenon is known as the photoelectric effect, and the emitted electrons are called photoelectrons.

This effect can be produced by the whole range of electromagnetic spectrum most metals show this effect when exposed to ultraviolet rays or X-rays.

The photoelectric effect was discovered by Heinrich Hertz in 1887 with further experimental study undertaken by Hallwachs in 1888. In 1899, Lenard showed that the carriers of electricity emitted from a metal surface, under the action of ultraviolet light, were electrons. The photoelectric effect is one of the process which exhibit the particle characteristics (nature) of waves.

**Experimental study**




A typical set up is shown above to investigate the photoelectric effect.

A photosensitive metal plate C, the cathode, is mounted opposite to another metal plate A in a highly evacuated tube. The plates A and C form two electrodes to which a variable potential difference can be applied. Ultraviolet light from a source is transmitted through the quartz window and made to fall on the surface C.

When the potential difference between A and C is such that C is at a negative potential with respect to A, negative ions (photoelectrons) are emitted from C and accelerated towards A. The resulting photoelectric current, I flowing in the circuit is measured by a highly sensitive micro-ammeter, or galvo such as the d.c electrometer, and the accelerating potential difference is measured by the vacuum tube voltmeter, V.

***Experimental results of the photoelectric emission***

- For a given metal and frequency of incident radiation, the rate at which photoelectrons are ejected is directly proportional to the intensity of the incident light.
- For a given metal, there exists a certain minimum frequency $\nu_{o,}$ of incident radiation below which no photoelectrons can be emitted. This frequency is called the threshold frequency, the value of $\nu_o$ depends on the material and the nature of the emitting surface. The corresponding wavelength is called the threshold wavelength.
- For a given metal of particular work function, increase in intensity of incident beam increases the magnitude of the photoelectric current, though stopping voltage remains the same.
- For a given metal of particular work function, increase in frequency of incident beam increases the maximum kinetic energy with which the photoelectrons are emitted. Thus the stopping voltage increases. (In practice the number of electrons does change because the probability that each photon results in an emitted electron is a function of photon energy.)

- Above the threshold frequency, the maximum kinetic energy of the emitted photoelectron depends on the frequency of the incident light, but is independent of the intensity of the incident light so long as the latter is not too high.
- The time lag between the incidence of radiation and the emission of a photoelectron is very small, less than $10^{-9}$ second.
- The direction of distribution of emitted electrons peaks in the direction of polarization (the direction of the electric field) of the incident light, if it is linearly polarized.

**Mathematical description**

Einstein in 1905 applied Planck's quantum theory and made the following two assumptions.

1. A radiation of frequency ν consists of a stream of discrete quanta each of energy hν.
   Where h is Plank's constant,
   These quanta are called photons. The photons move through space with the speed of light.
2. When a photon of energy hν is incident on a metal surface, the entire energy of the photon is absorbed by a single electron without any time lag. The probability of its absorbing two or more photons at the same time is negligible.

The light of frequency ν illuminating the photosensitive material may be regarded as a stream of photon; each photon carrying an energy hν. That is, when a photon encounters an electron in the material, it gives up all its energy to the electron, and the electron acquires the energy. Each free electron in the metal is prevented from leaving its surface because of a potential barrier. The amount of energy required by an electron to surmount the potential barrier is equal to the work function ø, of that metal. Therefore, when an electron absorbs the photon energy hν, part of the energy is spent in overcoming the potential barriers and the remaining part of the energy goes into its kinetic energy. According to the law of conservation of energy

Energy of photon = Energy needed to liberate the electron + Maximum kinetic energy of the liberated electron, that is,

The maximum kinetic energy $K_{max}$ of an ejected electron is given by

$$K_{MAX} = hf - \phi$$

where h is the Planck constant and f is the frequency of the incident photon. The term $\phi$ = $hf_0$ is the work function (sometimes denoted by W), which gives the minimum energy required to remove a delocalised electron from the surface of the metal. The work function satisfies

$$\phi = hf_0$$

where $f_0$ is the threshold frequency for the metal. The maximum kinetic energy of an ejected electron is then $K_{MAX} = h(f - f_0)$.

Kinetic energy is positive, so we must have $f > f_0$ for the photoelectric effect to occur.

$\phi$ ***Stopping potential***

The relation between current and applied voltage illustrates the nature of the photoelectric effect. For discussion, a light source illuminates a plate P, and another plate electrode Q collects any emitted electrons. We vary the potential between P and Q and measure the current flowing in the external circuit between the two plates.

If the frequency and the intensity of the incident radiation are fixed, the photoelectric current increases gradually with an increase in positive potential until all the photoelectrons emitted are collected. The photoelectric current attains a saturation value and does not increase further for any increase in the positive potential. The saturation current depends on the intensity of illumination, but not on its wavelength.

If we apply a negative potential to plate Q with respect to plate P and gradually increase it, the photoelectric current decreases until it is zero, at a certain negative potential on plate Q. The minimum negative potential given to plate Q at which the photoelectric current becomes zero is called stopping potential or cut off potential.

i. For the given frequency of incident radiation, the stopping potential is independent of its intensity.

ii. For a given frequency of the incident radiation, the stopping potential $V_S$ is related to the maximum kinetic energy of the photoelectron that is just stopped from reaching plate Q. If m is the mass and $v_{max}$ is the maximum velocity of photoelectron emitted, then

$$K_{MAX} = \frac{1}{2} m v_{MAX}^2$$

If e is the charge on the electron and $V_S$ is the stopping potential, then the work done by the retarding potential in stopping the electron = $eV_S$, which gives

$$\frac{1}{2}mv_{MAX}^2 = eV_S$$

The above relation shows that the maximum velocity of the emitted photoelectron is independent of the intensity of the incident light. Hence,

$$K_{MAX} = eV_S$$

The stopping voltage varies linearly with frequency of light, but depends on the type of material. For any particular material, there is a threshold frequency that must be exceeded, independent of light intensity, to observe any electron emission.

**Failure of classical theory**

1. According to calassical wave theory, photoelectric effect should occur for any frequency of light, provided that its intensity can cause vibrations of electrons to the amplitude required, so as to free them from the material. Therefore, if at all a threshold for photoelectric effect existsexihibited, it must be for intensity.

As the frequency and wave intensity are not related in classical theory, the frequency is not expected to influence the emission of photoelectrons. Therefore, a threshold frequency should not be exhibited. However, these are contrary to the experimental observations. Hence classical theory cannot account for the existence of threshold frequency.

2. The non – dependence of the maximum kinetic energy of the photoelectrons on the intensity of the incident radiation.

   The classical wave theory expects that electrons would be emitted with higher velocities when the intensity of light is large. The velocity, and therefore, the kinetic energy of photoelectrons would be required to be proportional to the incident wave intensity. The oscillating electric vector of the incident radiation should cause an electron in the metal to oscillate. If the amplitude of oscillation is sufficient, the electron is emitted from the metal surface. If the amplitude of oscillation of the incident wave is increased, the electron should be emitted with greater energy. The K.E of photoelectrons and hence the stopping potential should be independent of the frequency. This is in contradiction with observation. A feeble light of a higher frequency produces a smaller number of photocurrent but they have much greater K.Es. The incident radiations of the same frequency greater than the threshold frequency but of different intensities give rise to emission of photoelectrons with exactly the same maximum K.E.

3. Classically, electrons at the surface of a metal would require some time to absorb enough energy from an incident radiation in order to escape from the surface. However, in reality, the electrons are emitted almost instantaneously, taking a time of the order of $10^{-9}$ seconds- negligible, i.e absence of time – lag between irradiation of a surface and the start of the emission.

4. The dependence of photocurrent on intensity can be explained on the basis of classical theory. The larger electric field of a more intense wave would interact with more electrons on the surface of the material and would dislodge more electrons therefore, the more intense the wave, the more are the number of photoelectrons.

   This is the only feature that could be explained by classical wave theory.

**Explanations:**

1. **Existence of threshold frequency**

   According to the photoelectric equation, the K.E. of a photoelectron is given by

$$K_{MAX} = hf - \phi$$

If the energy of the incoming photon is equal to or greater than $\phi$, the electron will be ejected from the material. On the other hand, if hf < $\phi$ the photon will not have enough energy and photoemission does not occur no matter how intense the incident light may be The minimum energy that a photon should have to cause photoelectric effect is

$$hf_0 = \phi$$

Hence an electron can only be emitted when the frequency of the incident light is greater than

$$f_o = \frac{\phi}{h}.$$

2. **The non – dependence of max. K.E. on intensity**.

The K.E of a photoelectron is determined b $K_{MAX} = hf - \phi$.

In the case of a monochromatic light of frequency f, the photon energy *hf* is a fixed quantity. Therefore, by increasing the intensity of the light, only the no. of photons increases and hence the no. of ejected electrons increases, but the K.E does not vary.

3. ***Absence of time lag***

The energy of a photon is not distributed over an area, as in the wave theory. It is in the form of a concentrated bundle. Therefore, when a photon is incident on an electron, all its energy is absorbed by the electron without any time lag hence the subsequent ejection at the same instance.

Photoelectric effect in essence, shows that an electromagnetic radiation also has particle properties.

**Question** 1: Light of a wavelength 2000 $\overset{0}{A}$ falls on an aluminum surface with work function 4.2 eV. Calculate i. The threshold wavelength and ii. The stopping potential.

**Solution:**

i. To calculate the threshold wavelength, $hf_0 = \phi$, C = fλ, $f_o = \frac{C}{\lambda}$, $\phi = \frac{hC}{\lambda_0}$, $\therefore \lambda_0 =$

$$\frac{hC}{\phi} = \frac{12400}{4.2} = 2952\overset{0}{A}$$

ii. $hf = \phi + eV_S$

$$eV_S = hf - \phi, \quad eV_S = \frac{hC}{\lambda} - \phi \quad \Rightarrow \frac{12400}{2000} - 4.2 = 6.2 - 4.2 = 2.0eV$$

$$V_S = \frac{2.0}{e}eV = 2.0V\ .$$

**Question 2:** The Photo-electric threshold wavelength of silver is 2762 $\overset{0}{A}$, Calculate:

a) The max K.E. of the ejected electrons , b) the max velocity of the electrons, and (c) the stopping potential in volts for the electrons when the silver surface is illuminated with ultraviolet light of wavelength 2000 $\overset{0}{A}$.

**Solution:**

$\lambda = 2000 \times 10^{-10}$m , $= 2 \times 10^{-7}$m, $\lambda_o = 2762 \times 10^{-10}$m $= 2.762 \times 10^{-7}$m.

From hf = $hf_0$ +$K.E_{max}$,

a) $K.E_{max} = h(f-f_0)$ , and since $\frac{C}{\lambda} = \upsilon$, C- speed of light, ν = f- frequency , wave length.

Then, $hc\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right)=6.63x10^{-34}x3x10^{8}\left(\frac{10^7}{2}-\frac{10^7}{2.762}\right)$

$\therefore$ K.E$_{max}$ = 2.745x10$^{-19}$J.

b) ½ mV$^2_{max}$ = K.E$_{max}$ $\Rightarrow$ V$_{max}$ = $\sqrt{\frac{2K.E_{Max}}{m}}$ $= \sqrt{\frac{2x2.745x10^{-19}}{9.11x10^{-31}}}$

$\Rightarrow$ V$_{max}$ =7.763x10$^5$ ms$^{-1}$ .

C) From $\frac{1}{2}mV^2{}_{MAX} = eV_S \Rightarrow K.E_{MAX}$ = eVs

$\therefore$ Vs = $\frac{K.E_{MAX}}{e}=\frac{2.745x10^{-19}}{1.6x10^{-19}}=1.715Volts$ .

**Question 3**: A photon of energy 10eV falls on molybdenum whose work function is 4.15eV. Find the stopping potential.

**SOLUTION**

hV – W$_o$ = eVs

10 – 4.15 = eVs $\Rightarrow$ eVs = 5.85eV

$\Rightarrow$Vs = 5.85volts.

***NOTE***

Newtonian mechanics, Maxwell's electromagnetic theory and thermodynamics guided the growth of science and engineering in the 17$^{th}$ – 19$^{th}$ centuries. The theories explained almost -all the scientific result of those times and it seemed nothing more could be added. The above theories which are successful in the realms of macroscopic world are regarded as classical physics.

In the classical physics, matter and fields are treated as entirely independent entities. Macroscopic particles move and interact according to Newton's laws. In classical physics

i) Physical quantities such as particle energy are continuous variables and can take any possible value e.g. P.E. is from mgh to zero.

ii) If the state of a particle is known at a particular instant, and if the forces acting on the particle at that instant are known, the state of the particle at any future instant can be exactly predicted e.g. finding V known U, using equations of motion

iii) A particle can be isolated from its environment and can be treated as an independent entity for the investigation. Thus the particle under investigation and the instrument measuring any of its parameters are mutually independent.

**According to classical wave theory**

- EM waves are generated by accelerated charges, if a charge oscillates with a constant frequency, V, it produces an EM waves of the same frequency V.

- The waves spread out continuously through space and the wave is not localized, but is distributed over the volume of the wave.
- The energy of the wave is not related to the frequency of the wave, it is proportional to the square of the amplitude A of the wave.
- The thermodynamic equilibrium of an assembly of neutral particles is governed by Maxwell-Boltzman statistics. A large number of particles can occupy a given energy state and the particles can have a continuous range of energies.

Recent phenomena development in physics, during the last decade of the $19^{th}$ century, such as photoelectric effect, x-rays, line-spectra etc, defied explanation on the basis of classical physics. In short, classical laws are not valid in the microscopic world which obeys the laws of quantum mechanics.

Max Planck introduced the concept of discontinuous emission and absorption of radiation by bodies but he treated the propagation of radiation through space as occurring in the form of a continuous waves as demanded by EM theory. Einstein refined the Planck's hypothesis and invested quantum with a clear and distinct identity. The energy quanta are named photons

Paticle Properties are attributed to the photon, these include:

- Energy – which is determined by its frequency and given by E = hV using $\omega$ = 2πv and $\hbar = \dfrac{h}{2\pi}$ this can be rewritten as E = $\hbar\,\omega$
- Velocity – photons always travel with the velocity, c – the speed of light
- Mass – the rest mass of a photon is zero, since a photon can never be at rest. Thus m = 0, however, as it travels with the velocity c, it will have a relativistic mass given by the mass -energy relation as

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} \qquad = \frac{\hbar\omega}{c^2}$$

- Linear momentum – p = mc = $\dfrac{h\nu}{c} \quad = \dfrac{\hbar\omega}{c} \quad = \hbar k \quad where \quad k = \dfrac{2\pi}{\lambda}$

- Angular momentum – also known as spin, which is an intrinsic characteristics of all micro particles. Photon has a spin of 1 unit i.e. $s = 1\hbar$
- Electrical nature – photons are electrically neutral. They cannot be influenced by electric and magnetic fields. They cannot ionize matter.

Einstein considered that the quantization of energy, which is evident in the emission and absorption processes, is retained as the energy propagates through space. He assumed that the light energy is not distributed evenly over the whole expanding wave front but rather remains concentrated in discrete quanta. Accordingly, a light beam is regarded as a stream of photons travelling with a velocity C.

An EM wave having a frequency $\nu$ contains identical photons, each having an energy $h\nu$. The higher the frequency of the wave, the higher is the energy content of each photon. Hence, as in the EM spectrum, the $\gamma$-ray and x-ray photons are far more energetic compared with the optical photons whereas the photons of radio waves are the most feeble.

An EM wave will have energy $h\nu$ if it contains only one photon; $2h\nu$ if it contains two photons, and so on.

***The Wave-Particle Duality of Light***

The photoelectric effect and the Compton effect established that light behaves as a flux of photons. On the other hand, the phenomena of interference, diffraction and polarization can be explained only when light is treated as a continuous wave. It suggests that on the one hand, light resembles a collection of particles having energy E and momentum p and on the other hand it may be regarded as a continuous EM wave of frequency $\nu$. Neither of these can separately explain all the experimental facts, the particle (corpuscular) nature and the wave nature appear to be mutually exclusive. A particle is precisely localized in space while a continuous wave cannot be attributed to a particular location in space. However, it is observed that light behaves as a stream of particles in some phenomena and as an advancing wave in some others. It implied that light behaves both as a stream of particles and as a continuous wave. Therefore, we say that light exhibits wave-particle duality.

The wave-particle dualism can better be understood if one considers the entire EM spectrum. At the lower frequency end are radio waves whose wavelengths are so large ($x10^2$m). The energy available at any point is insignificantly small and the particle nature cannot be observed. On the other hand, if ultra violet and x-rays on higher frequency side of the spectrum are considered, their wavelengths are so short ($\sim \overset{0}{A}$) that the wave energy is concentrated in a point of very small dimension and the wave properties are less noticeable compared to the particle properties. Thus at lower frequency, the wave behavior stands out and at higher frequency the particle nature dominates, the visible region represents the transition zone where both aspects can be observed.

It is seen that according to quantum theory, the energy and momentum of a photon is given by $E = h\nu$ and $p = h/\lambda$. The LHS parameters E and p are particle characteristics whereas the RHS parameters $\nu$ and $\lambda$ are characteristics of waves. These equations reflect the wave-particle dualism of light.

We earlier observed that the light intensity $I = |E|^2 = Nh\nu \Rightarrow A^2$ of a light wave at a point in space is proportional to the no of photons arriving at that point. In other words, the amplitude of a light wave determines the probability that a photon can be found at a particular point in space. Probability to observe photons is proportional to $|E|^2$. It is this expression that provides the ultimate connection between the wave behavior and the particle behavior.

**Atomic Model**

We all know that atoms of all elements are made up of two parts with equal and opposite charges in them, so that the atoms as a whole are electrically neutral. Positive and negative charges attract each other, according to Coulomb's force law. The first proposal regarding the possible arrangement of the positively and negatively charged parts in the atom was made by J.J. Thompson in 1907 in what is now known as Thompson Model of the atom.

According to Thompson, the entire positive charge of the atom is distributed within a sphere of atomic radius of the order of $\sim 10^{-10}$m. Within this sphere of positive charge, the negatively charged point electrons are embedded at certain definite points, much like the plums in a pudding. The electrons are however not at rest, but oscillate with definite frequency about their mean positions of rest. This model failed to explain some observations.

Rutherford in 1911 proposed what is now known as the Rutherford Model of the atom. He proposed that an atom consists of a very small sphere of radius of $\sim 10^{-14}$m, in which its positive charge and nearly all its mass are concentrated; this is the nucleus of the atom. The electrons revolve round this nucleus in circular orbits of radius of $10^{-10}$m much like the planets revolve round the sun in their respective orbits. The positive charge of the nucleus is equal to the total negative charge of the electrons. Just like the gravitational attraction of the sun provides the centripetal force $mv^2/r$ of the rotational motion of the planets in their orbits, so does the electrostatic attractive force of the nucleus on the electrons supplies the necessary centripetal force for their orbital rotation. However, this model has a serious flaw in it.

***Inadequacy of Classical Physics***

Though Rutherford Model showed an improvement over Thompson Model, yet both, based on classical electromagnetic theory, were unable to satisfactorily explain experimental observations.

The inadequacy of classical physics is well understood when it is applied to the Rutherford Model of the atom. According to the model, the atom is composed of a tiny, massive, positively charged nucleus, around which negatively charged electrons revolve. The atom is only mechanically stable as the Coulomb attractive force between the nucleus and the electrons is balanced by the centripetal force. However, according to EM theory, an accelerated charged particle must radiate energy in the form of EM radiation continuously. An election moving round the nucleus in circular orbit is subjected to a continuous acceleration of constant magnitude directed towards the nucleus. Therefore if an electron should radiate energy, it must do it continuously and as it does so, its own energy gets reduced. Due to the loss of energy by radiation, it would move along a spiral path of decreasing radius and gradually spiral into the nucleus leading to the ultimate collapse of the atom. In the process of the spiral motion, it would emit radiation of continuously increasing frequency and hence would give rise to a continuous spectrum. This means that atoms are unstable and exist only for a fraction of a second ($10^{-8}$s). Contrary to the above, atoms are known to be stable and they show no tendency to collapse. Moreover, atoms of any element emit radiation of discrete wavelengths which form the line spectrum of the element rather than a continuous spectrum suggested by classical theory. Therefore, Rutherford Model could not explain the stability of atoms, hence the failure of classical theory.

**Bohr Model**

In 1913, Niels Bohr carried out preliminary calculations on the Rutherford Model of the atom and arrived at the conclusion that the classical laws are not valid in the atomic world. He then proceeded fusing the Planck-Einstein quantum mechanical ideas with the purely mechanical model of Rutherford. Bohr introduced the following three postulates:

1. Electrons revolve about a nucleus only in certain special orbits called stationary orbit, though an infinite number of orbits are mechanically allowed. While moving in the permitted orbits, electrons do not emit or absorb EM radiations, though they are accelerated. Hence, the atom is stable.
2. The allowed electron orbits are those for which the angular momentum of the electrons about the nucleus is an integral multiple of ħ. The angular momentum of electron is

   $$mvr = n\hbar, \qquad L = n\frac{h}{2\pi} \qquad \therefore L = n\hbar$$

3. The atom radiates or absorbs energy when the electron jumps from one allowed orbit to another. The change in energy during the transition is given by

   $E_i - E_f = h\nu$ , Where $E_i$ and $E_f$ are the energies of initial and final stationary states.

Postulate 1 has offered a way out of the problem posed by classical theory while postulate 2 provides a method of identifying a non-radiative stationary state. This 2nd postulate implies the quantization of angular momentum and hence quantization of radii of the electron orbits. Postulate 3 is the extension of Planck-Einstein ideas to the atom.

Bohr's theory of hydrogen atom agreed well with all the experimental data known at that time.

The theory employed a semi-classical model where quantum postulates are introduced into a mechanical model. This formed its main drawback.

**Bohr Theory of Hydrogen Atom**

Based on the 3 postulates

From Postulate 1

$$F_e = M\frac{v^2}{r} \ \text{...............} \textit{Centripetal} \text{ force}$$

$$F_e = \frac{e^2}{4\pi \in_0 r^2} \equiv \frac{ke^2}{r^2} \text{.......} \textit{Coulomb's} \text{ force between the nucleus and electron}$$

For a stable orbit $F_c = F_e \Rightarrow \dfrac{mv^2}{r} = \dfrac{e^2}{4\pi \in_0 r^2}$

$$r = \frac{e^2}{4\pi \in_0 mv^2} \quad (**), also \text{ from } L = n\hbar \Rightarrow \quad mvr = n\hbar$$

$$r = \frac{n\hbar}{mr} \quad and \quad v^2 = \frac{n^2\hbar^2}{m^2r^2} \quad (*)$$

Subtitute (*) into (**) i.e $r = \frac{e^2.m^2r^2}{4\pi \in_0 m \quad .n^2\hbar^2}$ $\quad also, \quad \hbar = \frac{h}{2\pi}, \quad \hbar^2 = \frac{h^2}{4\pi^2}$

Substitute for $\hbar^2$, i.e $r = \frac{e^2.m^2r^2 \quad 4\pi^2}{4\pi \in_0 mn^2h^2}$

$$r = \frac{e^2.m\ r^2\pi}{\in_0 n^2h^2}$$

$$\therefore r_n = \frac{n^2h^2 \in_0}{\pi me^2}$$

This is the expression for $r_n$ of a Bohr orbit of quantum number n, which is the radius of the 1st and smallest allowed $r_1 \sim 0.529\overset{0}{A}$.

Hence $r_n = r_1.n^2$

$r_n$ = Bohr radius.

**Energy of the Electron in a Bohr Orbit**

The total energy of the electron in the atom is $E_{Tot} = E_k + E_p$

Thus from $F_c = F_e$

i.e. $\frac{m\ v^2}{r} = \frac{e^2}{4\pi \in_0 r^2}$

$$m\ v^2 = \frac{e^2}{4\pi \in_0 r}$$

$$\frac{1}{2}m\ v^2 = \frac{1}{2}\frac{e^2}{4\pi \in_0 r} \quad \Rightarrow \frac{e^2}{8\pi \in_0 r} \quad (*)$$

But $r = \frac{n^2h^2 \in_0{}^2}{\pi me^2}$ Substituting into (*)

$$\therefore \frac{1}{2} m\ v^2 = \frac{e^2 \pi m e^2}{8\pi \in_0 n^2 h^2 \in_0} \qquad = \frac{me^4}{8 \in_0^2 n^2 h^2} \equiv E_K$$

$E_T = E_K + E_P$

$$E_P = \frac{-e^2}{4\pi \in_0 r} \qquad \{\text{From } \partial w = F.\partial r \quad and \quad w = \int K \frac{q_1 q_2}{r^2} \partial r \quad \equiv \frac{-e^2}{4\pi \in_0 r}\}$$

and $E_K = \dfrac{e^2}{8\pi \in_0 r}$ then,

$$E_T = \frac{e^2}{8\pi \in_0 r} + \left( \frac{-e^2}{4\pi \in_0 r} \right)$$

$$E_T = \frac{e^2 - 2e^2}{8\pi \in_0 r}$$

$E_T \equiv E_n = \dfrac{-e^2}{8\pi \in_0 r}$ , substitute for r,

$$E_n = \frac{-e^2 \pi m e^2}{8\pi \in_0 n^2 h^2 \in_0} \Rightarrow \quad \frac{-me^4}{8h^2 \in_0{}^2} \quad . \frac{1}{n^2}$$

For n = 1, $E_1 = \dfrac{-me^4}{8h^2 \in_0{}^2} \Rightarrow \quad E_n = \dfrac{E_1}{n^2}$ .

The calculated value of $E_1$ in eV = -13.6 eV $\Rightarrow \quad E_n = \dfrac{-13.6}{n^2}$ .

**The energy of a photon emitted by hydrogen atom is given by the difference of two hydrogen energy levels: i.e $\Delta E_{n} = E_1 - E_2$.** That is ,

$$E_1 = \frac{-me^4}{8h^2 \in_0{}^2} \quad . \frac{1}{n_1{}^2}$$

$$E_2 = \frac{-me^4}{8h^2 \in_0{}^2} \quad . \frac{1}{n_2{}^2}$$

$$\Delta E_n = E_1 - E_2 \quad = \quad \frac{me^4}{8h^2 \in_0{}^2} \left[ \frac{1}{n_2{}^2} - \frac{1}{n_1{}^2} \right]$$

If $E_1 = E_i$ , Initial energy level and $E_2 = E_f$ , final energy level then,

$\Delta E_{n} = E_i - E_f \qquad \frac{me^4}{8h^2 \in_0{}^2}\left[\frac{1}{n_f{}^2} - \frac{1}{n_i{}^2}\right]$

And since $E = \frac{hc}{\lambda}$

$$\Delta\frac{1}{\lambda} = \frac{me^4}{8h^3 c \in_0{}^2}\left[\frac{1}{n_f{}^2} - \frac{1}{n_i{}^2}\right]$$

$$R = \frac{me^4}{8h^3 c \in_0{}^2} \equiv 1.097 x 10^7 \text{ m}^{-1}$$

**Spectral series of hydrogen atom**

In physics, the spectral lines of hydrogen correspond to particular jumps of the electron between energy levels. The simplest model of the hydrogen atom is given by the Bohr model. When an electron jumps from a higher energy to a lower one, a photon of a specific wavelength is emitted. The spectral lines are grouped into series according to their final destination ($n_f$).

1. **Lyman Series (Ultra violet, invisible)**

Are the series of lines emitted by transitions of the electron from outer orbits of quantum numbers $n_i$ = 2,3,4,5 … to the first orbit of quantum number $n_f$ = 1, the wavelength of the series is given by:

$\frac{1}{\lambda} = R_H\left[\frac{1}{1^2} - \frac{1}{n_i{}^2}\right]$, where $n_i$ = 2,3,4,5 ......

2. **Balmer Series (Visible)**

These series of lines are emitted by transitions of the electron from outer orbits of quantum numbers $n_i$ = 3,4,5 … to the second orbit of quantum number $n_f$ = 2. Thus the wavelength of the $H_\alpha, H_\beta, H_\gamma, H_\delta$ and lines are given by:

$$\frac{1}{\lambda} = R_H\left[\frac{1}{2^2} - \frac{1}{{n_i}^2}\right]$$ ,where $n_i$ = 3,4,5...

$$\frac{1}{\lambda_\alpha} = R_H\left[\frac{1}{2^2} - \frac{1}{3^2}\right], \frac{1}{\lambda_\beta} = R_H\left[\frac{1}{2^2} - \frac{1}{4^2}\right], \frac{1}{\lambda_\gamma} = R_H\left[\frac{1}{2^2} - \frac{1}{5^2}\right] \text{ and } \frac{1}{\lambda_\delta} = R_H\left[\frac{1}{2^2} - \frac{1}{6^2}\right]$$

**3. Paschen Series (in IR region, invisible)**

For this series $n_f$ = 3 and $n_i$ = 4, 5, 6......

**4. Bracket Series (in IR region, invisible)**

For this series $n_f$ = 4 and $n_i$ = 5, 6,7,8.......

**5. Pfund Series (in IR region, invisible)**

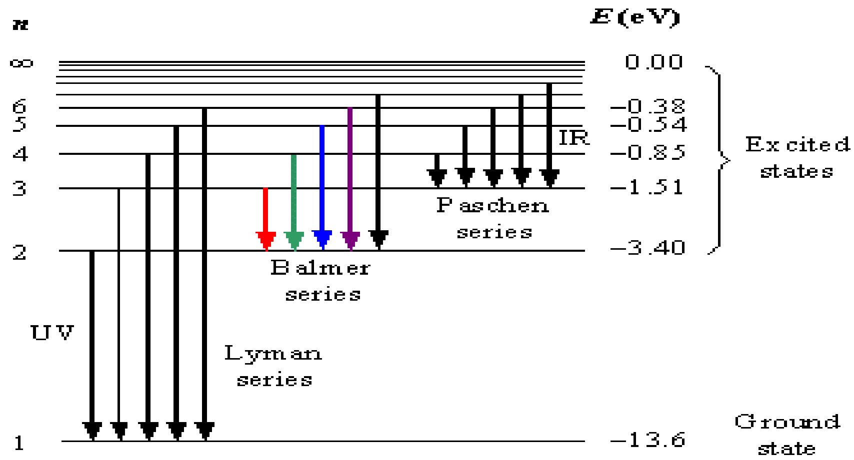
For this series $n_f$ = 5 and $n_i$ = 6,7,8.......

**Energy Level Diagrams**

An atom has a series of well-defined and discrete allowed energy levels $E_1$, $E_2$, … any other values of energy or intermediate energy levels are forbidden. The allowed energy levels of hydrogen are shown below. The ground state energy of the elctron is $E_1$ = -13.6 eV, and the valuess of energy in the excited states of the electron is $E_n = \frac{-13.6}{n^2}$ for different values of n plotted on the vertical axis against $E_n$. As n becomes larger, the energy levels become progressively closer; for n = ∞, $E_\infty$ = 0.

The allowed energy levels correspond to stationary orbits of election in the atom. The sequential representation of energy levels is known as an energy level diagram.

The energy level diagram for hydrogen is the simplest while for other complex atoms, the diagrams are complicated. The diagrams are characteristics of atoms.




Energy levels of the hydrogen atom with some of the transitions between them that give rise to the spectral lines indicated.

The ground state is the normal unexcited state of the atom. The atom can reside in the ground state for an indefinite time. An atom can get excited to a higher state only by absorbing energy. The energy may be supplied through inelastic collision with other atoms, through passing of an electric discharge through the gas or through other suitable means. The difference between the energy of an excited state and that of the ground state is called the excitation energy. An atom cannot stay in an excited state for long (not more than $10^{-8}$s) and spontaneously jumps to the ground state. This happens because an electron excited to a higher orbit is compelled to return to the ground state due to coulomb attraction exerted by the positive nucleus. The energy levels in the diagram are not evenly spaced; they get closer and closer at the upper end of the diagram. This happens because the energies of the allowed states are proportional to $\frac{1}{n^2}$ and as n increases, $\Delta E$ between successive levels rapidly decreases. The upper end of the spectrum, bounded by zero energy level for which $n = \infty$ is called the ionization level of the atom. When an electron reaches the ionization level, it gets separated completely from the nucleus and the atom is said to be ionized. The energy needed to remove an electron from an atom in its ground state is called its ionization energy. In the case of hydrogen, it is 13.6eV.

An atom absorbs a quantum that would take it to a higher allowed state. It cannot absorb a quantum that would take it to a forbidden level. It absorbs a whole quantum or nothing.

The absorption and emission processes are easily visualized by means of the energy-level diagram. It is customary to indicate a transition between two energy states by a vertical arrow. An upward arrow represents an upward transition of the electron consequent to absorption of energy in the form of a photon. The electron need not always jump up or down one energy level at a time; it can slip intermediate levels and go to any allowed level. The energy absorbed by an electron in jumping from an inner orbit to a particular outer orbit is the same emitted in the form of a photon in jumping down from that particular outer orbit to the same inner orbit.

Well established selection rules govern the transition by excited atoms to lower levels. All transitions are not allowed. In some atoms, an electron cannot jump from some of the excited levels to the ground state by emitting a photon. Therefore, it remains in the excited state indefinitely. Such excited states are known as metastable states.

The terms energy levels of electrons and energy levels of atoms are used synonymously. Actually, when an atom is said to be excited it means that one of the electrons in the atom has jumped to a higher state.

In a discharge tube at a given instant of time a very large number of hydrogen atoms are excited to a different higher energy states. Hence, electrons in these atoms will be making all possible types of quantum jumps form outer orbits to inner orbits, so that all possible spectral lines are emitted continuously. The intensity of any line depends on the number of electrons making transitions between the particular orbits in a group of atoms. If the outer orbit has a high quantum number, the intensity of the line emitted will be low because the probability of an atom getting excited to the energy state of high quantum number is low.

**Limitations of Bohr's Theory**

Bohr's theory explains quite successfully the spectrum of hydrogen and hydrogen-like ions such as singly ionized helium and doubly ionized lithium, but the theory has a number of limitations, a few of which are:

1. It cannot explain the experimental observation that many spectral lines extend over a finite wavelength range consisting of a small number of lines close together.
2. The theory cannot explain the difference in the intensities of certain spectral lines in the spectrum of an element i.e. the theory cannot explain the difference in the probabilities of occurrence f certain transitions of electrons between energy levels.
3. The spectral of complex atoms having two or more electrons each cannot be explained on the basis of the theory.

4. In the theory, the nucleus is considered stationary with the electron revolving around it, but both the nucleus and the orbital electron revolve around a common centre of mass.
5. The theory does not provide any clue about the forces of interaction between atoms which form a stable substance.
6. It could not explain why angular momentum is quantized and atoms make transition

**Election Shells**

The simple model of hydrogen was extended to other elements by Bohr and Stones. Each atom is composed of a positively charged nucleus and a number of electrons revolving in allowed orbits around it. The number of electrons in an atom is given by the atomic number Z. The orbits to which electrons are confined are the orbits characterized by different values of the quantum number n. The orbits are called electron shells. They are named K, L, M, N,... When an atom is built, electrons are added one after the other, filling one shell and then another. A shell is filled when $\boldsymbol{2n^2}$ electrons occupy it. Thus the K shell is filled when it has 2 electrons; the L shell is filled when 8 electrons occupy it and so on.

| Shell | Quantum Number (n) | Capacity ($2n^2$) |
|---|---|---|
| K | 1 | 2 |
| L | 2 | 8 |
| M | 3 | 18 |
| N | 4 | 32 |

There are however certain departures from the above order in case of heavier elements and are indicative of the chemical behavior of the heavy elements.

**QUESTION 1**: The energy of a particular state of an atom is 5.36eV and the energy of another state is 3.45eV. Find the wavelength of the light emitted when the atom makes a transition from one state to the other.

**Solution:** $E_1 = 5.36eV$ $E_2 = 3.45eV$

Applying $\Delta E = E_1 - E_2 = h\nu$ i.e. $E_1 - E_2 = \frac{hc}{\lambda}$, $\sin ce$ $c = \nu\lambda$

Then, $\lambda = \frac{hc}{E_1 - E_2}$, $i.e$ $\lambda = \frac{6.626x10^{-34}\, x3x10^{8}}{(5.36-3.45)(1.602x10^{-19})}$ $\Rightarrow \lambda = 6496\,\overset{o}{A}$

**QUESTION 2:** Calculate the energy of the electron in the second, third and fourth orbits given that the total energy of the electron in the first Bohr orbit of the hydrogen atom is – 13.6eV.

**Solution:** $E_1 = -13.6eV$

$$E_n = \frac{E_1}{n^2} = \frac{-13.6}{n^2},$$

when n = 2, $E_2 = \frac{E_1}{2^2} = \frac{-13.6}{4} \quad = -3.4eV$

when n = 3, $E_3 = \frac{E_1}{3^2} = \frac{-13.6}{9} \quad = -1.51eV$

when n = 4, $E_4 = \frac{E_1}{4^2} = \frac{-13.6}{16} \quad = -0.85eV$ .

**De-Broglie Hypothesis**

In 1924, de Broglie suggested that the wave particle dualism need not be a special feature of light alone but that material particles must also exhibit such dual behavior since nature exhibits symmetry, particles as well as waves exhibit properties of both particles and waves. This is the de-Broglies hypothesis.

He deduced the relation between the wave properties and particle property from energy expression based on Planck-Einstein Energy relation $E = h\nu$ and expression for momentum in classical theory P = E/C { since $E = mc^2$, therefore $m = E/c^2$. Also P = mc $\Rightarrow P = \frac{E.c}{c^2} = P = \frac{E}{c}$

}

And then had $\lambda = \frac{h}{P} \Rightarrow \lambda = \frac{h}{mv}$, (*) where m = mass and v = velocity. {from $E = h\nu$, and P = E/ c, then $P = h\nu/c$. Also, $\nu = c/\lambda$, therefore, $P = hc/c\lambda \Rightarrow P = h/\lambda$ and then $\lambda = h/P$ as shown.}

It could be seen from (*) that as m increases λ tends to zero.

Equation (*) is the wave – particle relation for photons which is called the de- broglie equation.

If a particle, say an electron is moving with a velocity V = $\sqrt{2eV_S / m}$ , then the wavelength is given by

$$\lambda = \frac{h}{\sqrt{2emV_S}} \quad \sin ce\{\frac{1}{2}mV^2 = eV_S \quad \therefore Vel = \sqrt{2eV_S / m} \quad and \quad \sin ce \quad \frac{h}{mv} \Rightarrow \frac{h}{m\sqrt{2eV_S / m}} \Rightarrow \frac{h}{\sqrt{2eV_S} m^2 / m} \quad \Rightarrow \lambda = \frac{h}{\sqrt{2emV_S}} \quad as \quad shown\}$$

The waves associated with moving particles are called matter waves or de Broglie waves. The wavelength of matter waves expressed above is called de Broglie wavelength from which it is seen that as the mass of the body increases, the wavelength tends to zero. The wavelength of macroscopic bodies is insignificant in comparison to the size of the bodies themselves. The waves associated with particles are not real 3-D waves in the way sound waves are, but are probability waves related to the probabilities of finding the particles in various places and with various properties.

**de Broglie model of the Atom**

de Broglie provided an explanation for the postulate regarding quantization of electron's angular momentum in Bohr model of the atom.

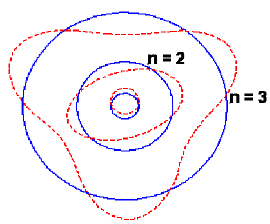
**Consequences of de Broglie's Concepts**

1. Quantization condition for the angular momentum of the electron in hydrogen atom.

In the Bohr Theory of the hydrogen atom, the quantization condition,

$L = \frac{nh}{2\pi}$ for the angular momentum L, of the electron, moving in a stationary circular orbit is only arbitrary.

As the electron travels round in one of its circular orbits, the associated matter waves propagate along the circumference again and again. A wave must meet itself after going round one full circumference. If it did not, the wave would be out of phase with itself after one orbit. After a large number of orbits, all possible phases would be obtained and the wave would be annihilated by destructive interference. It implies that the wave should produce a standing wave profile in the orbit to prevent the electron's energy from radiating away. A stationary wave pattern can form along the circumference ***if only an integral number of wavelengths fit into the orbit***, as in the classical case of a string fixed at both ends would vibrate with discrete frequencies whose value depend on the linear dimension of the string.




**The standing de Broglie waves set up in the first three Bohr orbits.**

Fig a

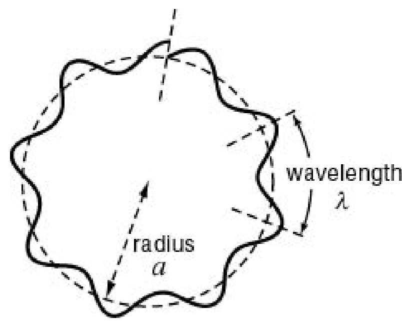



Fig b.

Figure a shows a standing de Broglie waves, while in figure b, the de Broglie wave does not connect smoothly, because the circumference is not an integral multiple of the wave length.

**De Broglie therefore suggested that:**

(i) The motion of the electron in a stationary circular orbit is represented by a standing matter wave of wavelength $\lambda$ given by the de Broglie relation $\lambda = h/mv$ where m is the mass of electron and v is its velocity in the orbit

(ii) The circular orbit contains an integral number of wavelengths.

Thus the condition for the formation of standing waves in an orbit is that

$2\pi r_n = n\lambda$, n = 1,2,3…

Where $r_n$ designates the radius of the orbit of quantum number n.

Combining the last two equations;

i.e $\lambda = \dfrac{h}{mv}$ *and* $2\pi r_n = n\lambda$

then $2\pi r_n = \frac{nh}{mv}$, therefore $mvr_n = \frac{nh}{2\pi} \Rightarrow$

$L_n = \frac{nh}{2\pi}$ *which is Bohr quantization condition*, $\Rightarrow$ $L_n = n\hbar$

$L_n$ is the angular momentum of electron in the $n^{th}$ orbit. de Broglie further showed that the quantization is a direct consequence of wave nature of electron.

The wavelength λ of the electron in the orbit in a ground state of hydrogen atom is given by

$\lambda = \frac{h}{e}\sqrt{\frac{4\pi \in_0 r_n}{m}}$ from λ = h/mv i.e

$$\Rightarrow = \frac{h}{m}\cdot\frac{1}{v} \quad \Rightarrow \frac{h}{m}\cdot\frac{\sqrt{4\pi \in_0 mr_n}}{e} \quad \text{since} \quad v = \frac{e}{\sqrt{4\pi \in_0 mr_n}} \quad \therefore \lambda = \frac{h}{e}\frac{\sqrt{4\pi \in_0 mr_n}}{m^2} \quad \Rightarrow \lambda = \frac{h}{e}\sqrt{\frac{4\pi \in_0 r_n}{m}}$$

Thus de Broglie hypothesis thus offered a new meaning to the quantum number n. It is the number of de Broglie wavelengths that fit into the circumference of Bohr allowed orbits.

We may now picture the electron in the atom in two ways: either as a particle moving in an orbit with a certain quantized value of mvr or as a standing de Broglie wave occupying a certain region around the nucleus.

## 2. Energies of a Particle in a Box

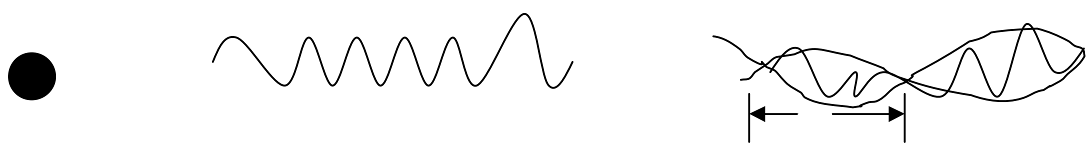
Let a particle of mass m be in linear motion between the opposite walls of a hollow rectangular box having rigid walls. The linear motion is perpendicular to the opposite walls. Let b be the distance between the walls. To obtain the expression for energy on the basis of de Broglie's hypothesis, the following assumptions are made:

(i) The linear motion of the particle between the walls is represented by a standing wave of wavelength λ given by λ = h/p where p is the linear momentum of the particle

(ii) The distance b between the walls is an integral number of half wavelengths i.e. b = nλ/2; n = 1,2,3… then $b = \frac{nh}{2P}$ *and* $P = \frac{nh}{2b}$ therefore the energy of the particle

$$E_n = \frac{P^2}{2m} is \quad now \quad \frac{n^2h^2}{2m.4b^2} \quad and \quad h = 2\pi\hbar \quad \therefore E_n = \frac{n^2 4\pi^2\hbar^2}{8mb^2} \Rightarrow E_n = \frac{n^2\pi^2\hbar^2}{2mb^2}$$

3. **Heisenberg's Uncertainty Principle:** Heisenberg's Uncertainty Principle is a direct consequence of de Broglie's hypothesis. It is known that a superposition of several waves having slightly different frequencies gives rise to a wave packet, such a wave possess both wave and particle properties.



A particle A wave Δx A wave packet

Since a wave packet therefore is of finite width/length, there will be an uncertainty in specifying the position of an electron in motion. At the same time the spectral distribution of the amplitude of a wave-packet covers a range of wave lengths, by the de broglie relationship this means that the momentum of the electron will also be uncertain. Therefore, in general, it is not possible to determine precisely and simultaneously, the position and the momentum of the electrum. Heisenberg, in 1972 summarized this in his Uncertainty principle thus: "It is not possible to make simultaneous measurements of the position and momentum of a particle to an unlimited accuracy".

Classically, i.e., in our macroscopic world, I can measure these two quantities to infinite precision (more or less). There is really no question where something is and what its momentum is. Consequently, I can tell with infinite precision where that particle will be at the next instant in time.

In the Quantum Mechanical world, the idea that we can know things exactly breaks down. More precisely, suppose a particle has momemtum p and a position x. In a Quantum Mechanical world, we would not be able simultaneously to measure both p and x precisely. There is an uncertainty associated with this pair of measurements, e.g., there is some Δp and

$\Delta x$, which we can never get rid of even in a perfect experiment!!!. The uncertainties in the knowledge of the coordinates x and the momentum $P_x$ of a particle is expressed by writing x $\pm$ x and $P_x \pm P_x$ respectively. i.e the particle is located between x - $\Delta x$ and x + $\Delta x$, and its momentum is between $P_x - \Delta p_x$ and $P_x + \Delta P_x$.. The uncertainty principle says that the product $\Delta x \, \Delta P_x$ will always be greater or equal to h.

$\Delta x \, \Delta P_x \geq h$.

Since $\Delta mv \approx h/\lambda$ and $\Delta x \geq \lambda$ the two can be combined: $\Delta x \Delta mv \geq h\lambda/\lambda$. Therefore, $\Delta x \Delta p \geq h$.

Note that sometimes Heisenberg's uncertainty principle is given as $\Delta x \Delta p_x \geq \frac{\hbar}{2}$.

{Where $\hbar = \frac{h}{2\pi}$ = 1.05457168 x $10^{-34}$ J.S = 6.58211916 x $10^{-16}$ eV.S.

ħ = differs by a fctor of $\frac{1}{2\pi}$ from h, owing to a difference respectively between energy and frequency units.}

The corresponding relations for the other components of position and momentum are $\Delta y \, \Delta p_y \geq h$ ; and $\Delta z \, \Delta p_z \geq h$. It means that it is physically impossible to know simultaneously the exact position and exact momentum of a particle. The uncertainty relation emphasizes that the momentum of a particle cannot be precisely specified without our loss of all knowledge of the position of the particle at that time; and vice versa.

The uncertainty principle also restricts the precision in the measurement of the particle's energy. If $\Delta E$ is the uncertainty in the energy E of the particle and $\Delta t$ is the uncertainty as to the instant of time at which the particle had the exact energy E, then, $\Delta E$ is related to $\Delta t$ through

$\Delta E . \Delta t \geq h$

**QUESTION 1** : The most rapidly moving valence electron in metallic sodium, at the absolute zero of temperature, has a K.E of 3ev. Show that its de Broglie wavelength is 7Å

**Solution:** K=3eV = 3x 1.6 x$10^{-19}$J = 4.8 x $10^{-19}$J

For the electron,

$M_o c^2$ = 9.11 x $10^{-31}$ (3 x $10^8)^2$ = 9.11 x 9 x $10^{-15}$) = 81.99 x $10^{-15}$J

= 81.99 x $10^{-15}$/(1.6x$10^{-19}$J) eV = 5.12 x $10^5$ eV

Therefore, the k.e of the electron is small compared with $m_o c^2$

hence the de Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2m_0 K}} \frac{6.626x10^{-34}}{\sqrt{2x9.11x10^{-34}}x4.810^{-19}} = \frac{6.626x10^{-9}}{9.35} \approx 7.1\overset{0}{A}$$

**Question 2**: find the wavelength of the waves associated with an electron having energy equal to 1 mev

Sol: K = 1meV = $10^6$ x 1.6 x $^{-19}$J= 1.6 x 10 $^{-13}$J

$M_o c^2$ for an electron is

$M_o c^2$ =9.11 x 10 $^{-31}$ (3 x $10^8)^2$ = 81. 99 x 10 $^{-15}$J = 5.12 x $10^5$ eV

Since k is comparable with $m_o c^2$, we use the relativistic expression for

λ: i.e. λ =

$$\lambda = \frac{hc}{\sqrt{K(K+2m_0c^2)}} = \frac{6.626x10^{-34}x3x10^8}{\sqrt{1.6x10^{-13}[1.6x10^{-13}+2(81.99x10^{-15})]}}$$

$$\frac{19.89x10^{-13}}{2.276} \approx 8.739x10^{-13}m \approx 0.008739\overset{0}{A}$$

**Question 3**: calculate the de Broglie wavelength of an election moving with $\frac{3}{5}c$.

SOLUTION: v = 3/5c or v/c = 3/5 0.6 now, λ = h/mv

Since v is comparable with c, m in the formula is the relativistic mass. Substituting

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{6.626x10^{-34}}{9.11x10^{-31}x0.6c}\sqrt{1-0.36} = 3.2326x10^{=12}m \Rightarrow \lambda = 0.0323\overset{0}{A}$$

**QUESTION 4 :**What is the de Broglie wavelength of a person with a mass of 50 kg jogging at 5 m/s ?

Solution:

Since λ = h / mv substitute the values for the know variables and solve for λ.

λ = 6.626 x $10^{-34}$ Js / (50 kg)(5 m/s)

$= 2.65 \times 10^{-36}$ m

For a large particle like the jogger there is NO WAY of seeing this wavelength, but we can apply the same principle to smaller particles such as electrons.

**QUESTION 5**:What is the de Broglie wavelength of an electron moving at $2.2 \times 10^{6}$ m/s?

Mass of electron is $9.1 \times 10^{-31}$ kg

$\lambda = h / mv$

$\lambda = 6.626 \times 10^{-34}$ Js/ ($9.1 \times 10^{-31}$ kg)($2.2 \times 10^{6}$ m/s)
= 0.332 nm

This is a scale at which noticeable effects such as diffraction patterns, and Doppler effects can be observed

**QUESTION 6:** Electrons experience a drop in energy of $6.409 \times 10^{-15}$ J, what is the wavelength of the electrons?

To calculate the velocity of the electrons the expression $E = \frac{1}{2} m_e v^2$ is used.

$$v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \times 6.409 \times 10^{-15} kgm^2s^{-2}}{9.110 \times 10^{-31} kg}} = \sqrt{1.407 \times 10^{16} m^2s^{-2}} = 1.186 \times 10^{8} ms^{-1}$$

Using de Broglie's equation calculate $\lambda$

$$\lambda = \frac{h}{m_e v} = \frac{6.6262 \times 10^{-34} Js}{9.110 \times 10^{-31} kg \times 1.186 \times 10^{8} ms^{-1}} = 0.06135 \times 10^{-10} \frac{kgm^2s^{-2}s}{kgms^{-1}} = 6.135 \times 10^{-12} m$$

**Schrodinger Wave Equation**

In 1926, Erwin Schrodinger developed further the de Broglie's ideas of the wave properties of matter.

In classical mechanics, a wave equation is a 2nd order diffrential equation in space and time. Solutions of this equation represent wave disturbances in a medium. Therefore, a wave equation is the usual basis of mathematical theory of a wave motion. For example, an electromagnetic wave, travelling in the x-direction is described by the wave equation.

$\frac{\partial^2 \in_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \in_y}{\partial t^2}$, ......(1), where $\varepsilon_y$ is the y- component of the electric intensity

A solution of the above equation is the familiar plane wave $E(x,t) = Ae^{i(kx-\omega t)}$ ..........(2)

Where $\omega = vk$ and v is the phase velocity

For a microparticle, ω and k may be replaced with E and P using Einstein and de Broglie relations respectively. Thus;

$\omega = \frac{E}{\hbar}$ *and* $K = \frac{P}{\hbar}$ {From E = hν, ν = E/h, but ω = 2πν avd ω = 2πE/h ⇒

$\omega = \frac{2\pi E}{2\pi\hbar}$. *And* From ω = vk, and ω = E/ħ ∴, vk = E/ħ,

vk = $\frac{h\nu}{\hbar}$. *but* $V = \nu\lambda$ $\therefore \nu\lambda k = \frac{h\nu}{\hbar}$. $\lambda k = \frac{h}{\hbar}$ $\Rightarrow \frac{hk}{P} = \frac{h}{\hbar}$ $\therefore \frac{k}{P} = \frac{1}{\hbar}$ $\therefore K = \frac{p}{\hbar}$ }

To get the expression for Schrodinger wave equation from the known wave equation,

Differentiate equation (*) wrt t

$\Psi(x,t) = Ae^{-i}(Et- Px)/\hbar$ ........(*)

$\frac{\partial\psi}{\partial t} = -\frac{iE\psi}{\hbar}$ ............................(**) *where* $\Psi(x,t) = Ae^{-i}(Et- Px)/\hbar$.

$$\frac{\partial^2\psi}{\partial t^2} = -\frac{E^2\psi}{\hbar^2}$$

{$From \frac{\partial\psi}{\partial t} = -\frac{iE}{\hbar} Ae^{-i(Et - Px)/\hbar}$ , $\frac{\partial^2\psi}{\partial t^2} = (-\frac{iE}{\hbar})(-\frac{iE}{\hbar})Ae^{-i(Et - Px)/\hbar}$ $\Rightarrow \frac{\partial^2\psi}{\partial t^2} = -\frac{E^2\psi}{\hbar^2}$

Also differentiate equation (*) wrt x

$$\frac{\partial\psi}{\partial x} = -\frac{iP\psi}{\hbar}$$

$\frac{\partial^2\psi}{\partial x^2} = -\frac{P^2\psi}{\hbar^2}$..........(*4)

{$From \frac{\partial\psi}{\partial x} = +\frac{iP}{\hbar} Ae^{-i(Et-Px)\hbar}$ , $\frac{\partial^2\psi}{\partial x^2} = (+\frac{iP}{\hbar})(-\frac{i.-.P}{\hbar})Ae^{-i(Et-Px)\hbar}$ $\Rightarrow$ $(+\frac{iP}{\hbar})(\frac{iP}{\hbar})Ae^{-i(Et-Px)\hbar}$ }

But generally, the total energy = P.E + K.E i.e $E_T$ = $E_T = -\frac{P^2}{2m} + v$

Multiply through by ψ

$$E_T\psi = \frac{P^2}{2m}\psi + v\psi \text{...........}(***)$$

From equation (**), make Eψ subject of the formular, multiply num and den. By i then substitute into (***).

$$E\psi = \frac{-\hbar}{i}.\frac{i}{i}\frac{\partial\psi}{\partial t} \qquad \Rightarrow E\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{P^2\psi}{2m} + v\psi$$

Substitute for $P^2\psi$ from equation *4 into (***).

$$P^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial x^2}$$

$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2\partial^2\psi}{2m\partial x^2} + v\psi$ ................(*5) This is the time dependent schrodinger wave equation in one dimension

In three dimensions, the time dependent schrodinger equation is

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + v\psi \quad = i\hbar\frac{\partial\psi}{\partial t} \text{................}(*6)$$

This equation can be written as $\frac{-\hbar^2}{2m}\nabla^2\psi + v\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad or$

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + v\right]\psi = i\hbar\frac{\partial\psi}{\partial t} \text{....................}(*7) \qquad where \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad ]$$

Now we use separation of variables technique to get expression of the equation of time and space separately.

So we can then write the wave function as a product to seperat variables. Since ψ(x,t) is a function of both space & time thus ψ(x,t) = ψ (x) φ(t) (separation of variable technique).

From (*5), $\frac{-\hbar^2\partial^2\psi}{2m\partial x^2} + v\psi = \quad i\hbar\frac{\partial\psi}{\partial t}$

Substituting $(\psi_{(x)}\varphi_{(t)})$ for ψ in equation (*5).

$$\frac{-\hbar^2\partial^2}{2m\partial x^2}(\psi_{(x)}\varphi_{(t)}) + v(\psi_{(x)}\varphi_{(t)}) = \quad i\hbar\frac{\partial}{\partial t}(\psi_{(x)}\varphi_{(t)}).$$

Differentiating wrt x, makes function of t constant and differentiating wrt t, makes the function of x a constant, so,

$$\frac{-\hbar^2\varphi_{(t)}}{2m}\frac{\partial^2\psi_{(x)}}{\partial x^2} + v(\psi_{(x)}\varphi_{(t)} = \quad \psi_{(x)} i\hbar\frac{\partial}{\partial t}\varphi_{(t)} \text{..........(*8)}$$

Divide through (*8) by $\psi_{(x)}\ \varphi_{(t)}$

$$\frac{-\hbar^2}{2m}\frac{1}{\psi_{(x)}}\frac{\partial^2\psi_{(x)}}{\partial x^2} + v = \quad \frac{i\hbar}{\varphi_{(t)}}\frac{\partial}{\partial t}\varphi_{(t)}$$

The RHS is a function of time only and the LHS is a function of space only so we can equate them to the same constant E

Therefore, LHS $\quad \frac{-\hbar^2}{2m}\frac{1}{\psi_{(x)}}\frac{\partial^2\psi_{(x)}}{\partial x^2} + v = E\text{......(*9)}$

and RHS, $\quad \frac{i\hbar}{\varphi_{(t)}}\frac{\partial\varphi_{(t)}}{\partial t} = E\text{...........(*10)}$

Multiplying (*9) through by ψ(x).

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi_{(x)}}{\partial x^2} + v\psi(x) = E.\psi(x)\text{.....(*11)}$$

Multiplying (*10) through by φ(t).

$$i\hbar\frac{\partial\varphi_{(t)}}{\partial t} = E\varphi_{(t)}$$

Rewrite (*11) as, $\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+v).\psi(x)=E.\psi(x)$ Time independent schrodinger equation

$\Rightarrow$ H.$\psi(x)$ = E .$\psi(x)$ ..........................H = Halmitonian.

For 3D, we may re- write (*11)

$$\frac{-\hbar^2}{2m}\nabla^2\psi+v\psi=E.\psi \text{....................(*13)} \quad or \quad [\frac{-\hbar^2}{2m}\nabla^2+v]\psi=E.\psi$$

Note that $\psi \Rightarrow \psi(x,y,z)$.

**Periodic system of elements**

The electrons in an atom occupy energy states or orbital's characterized by the quantum numbers n, l, and $m_l$. A set of these numbers define the states of the electrons. The states (or orbital's) are organized into shells and subshells. The possible orbital's corresponding to a particular value of n are said to constitute a shell. All the electrons in an atom which have the same principal quantum number n belong to the same electron shell. The electrons having n = 1 are said to form the k – shell of the atom, electrons in the n = 2 state form the L – shell, and so on.

| | | | | | | |
|---|---|---|---|---|---|---|
| Principal quantum number, n | 1 | 2 | 3 | 4 | 5 | 6 |
| Designation of the electron shell | K | L | M | N | O | P |
| Max. Capacity of shell, $2n^2$ | 2 | 8 | 18 | 32 | 50 | 72 |

Shells are built from subshells which accommodate electrons of the same value of orbital quantum number l. thus, electrons that share a certain value of l in a shell are said to occupy the same subshell. The number of subshells in a shell is equal to the value of n. Thus a K – shell has only one subshell corresponding to l=0, an L – shell has two subshells and so on. In atomic physics, states with a particular value of l have a particular name, a state with l =0 is called s – state; a state with l = 1 is called a p- state and so on.

| | | | | | | |
|---|---|---|---|---|---|---|
| Orbital quantum number, l | 0 | 1 | 2 | 3 | 4 | 5 |
| Designation of the state or subshell | s | p | d | f | g | h |

For a given set of value of n and l, there are (2l + 1) possible values of $m_l$. It means that for a given value of n and l, there are (2l + 1) orbitals or electronic states.

Thus if l = 0, there is only one electronic state, if l = 1, there are three electronic states etc.

The electronic states correspond to the different orientations of the orbitals. An energy state described by the three quantum numbers, n, l, and $m_l$ can be occupied by only two electrons which have opposite spin directions. The electron spin is characterized by the quantum number $m_s = \pm \frac{1}{2}$. The state of the electron described by the four quantum numbers n, l, $m_l$ and $m_s$ is called the *quantum state*. Thus each energy state consists of two quantum states. Note that $m_l$ is the orbital magnetic quantum number, $m_l = 0, \pm 1, \pm 2, \pm 3, \ldots \pm l$.

It may be noted that an energy level is not equivalent to an electronic state. An energy level is determined by the value of the principal quantum number n, and such a level corresponds to $n^2$ electronic states. Thus for n = 2, there are $2^2 = 4$ different electronic states. The energy of an electron is mainly determined by the value of n but to some extent by the quantum numbers l and $m_l$, also.

The occupancy of various shells and subshells in a complex atom is governed by three basic laws.

1. Minimum energy condition: Electrons tend to occupy the lowest available energy state such that their total energy is a minimum. They go to higher energy states only when the lower states are not vacant.
2. Pauli’s exclusion principle: An orbital in an atom described by the quantum numbers n, l and $m_l$ can be occupied by only electrons having opposite spin directions.
3. Hund’s rule: The order of filling of the orbitals of a subshell obey Hund;s rule which states that the total spin number of the electrons of a shell must be maximum. It means that the orbital’s of a subshell are filled first with one electron each, and then with the second electron successively. e.g. in Nitrogen atom, there are three electrons in the 2p sub – shell occupying 3 orbital’s $P_x$, $P_y$, $P_z$. the sequence of energy states in order of increasing energy of the orbital’s of a multielectron atom is in the following order.

   1s < 2s < 2p< 3s< 3p<4s<3d<4p< 5s< 4d< 5p< 6s< 4f< 5d< 6p

The electronic configuration of an atom is described using $nl^x$ notation where x denotes the number of electrons occupying the subshell.

1. Sodium, Na: Z = 11 ----- $1s^2$, $2s^2$, $2p^6$, 3s
2. Nitrogen, N: Z = 7 ------ $1s^2$, $2s^2$, $2p^3$
3. Potassium, K: Z = 19 ----- $1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^6$, 4s
4. Polomium, $P_o$: Z = 84 ------ $1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^6$, $4s^2$, $3d^{10}$, $4p^6$, $5s^2$, $4d^{10}$, $5p^6$, $6s^2$, $4f^{14}$, $5d^{10}$, $6p^4$.

A shell or subshell that contains its full quota of electrons is said to be closed or Completely filled. Thus, the K – shell and L – shell of Na atom are closed while the M – shell is partially filled. The 3s subshell of the M – shell contains only one electron, and it is half – filled.

The completely filled shells and sub – shell of atoms are stable and are not readily disturbed. The electrons in the outer shell will be in a position to bond or interact with holes/similar electrons in adjacent atoms. The number of such electrons determine the valency of the atom and the chemical behavior of the element.

A knowledge of electron configuration helps in understanding certain physical properties. For example, in the element potassium, the 4s level contains one electron while the 3d states are still unoccupied. The situation of partially filled 3d states exist until in element Copper when the 3d subshell is closed. The group of elements for which an inner subshell is not closed is called transition elements.

Transition of electron from one level to another gives rise to spectra lines which could be due to emission or absorption of energy. Emission spectra are the result of excitation; they are bright lines on dark background. Absorption spectra are the result of transition from higher to lower energy levels, they are dark line on bright background.