

PHS 312 LECTURE NOTES

ANALYTICAL MECHANICS II

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Course contents

Newtonian mechanics of system of particles, D'Alembert's principle,; Degree of freedom, generalized coordinates and Lagrange's formulation of mechanics; simple applications. The calculus of variations and the action principle. Hamiltonian mechanics. Invariance and conservation laws. Small oscillations and normal modes

Suggested references:

1. Classical mechanics by Golstein, published by McGrawhill
2. Advanced Engineering Mathematics by E. Kreyseic
3. A shorter intermediate mechanics by D. Humphrey
4. A textbook of dynamics by F. Chorton, published by Ellis Horwood ltd.
5. Advanced Mathematics by M.R. Spiegel (Schaum series)

ANGULAR MOMENTUM AND TORQUE FOR A PARTICLE

The angular momentum, L , for a single particle about a point O is defined as

$$L = r \wedge p = mr \wedge v \text{ -----1.0}$$

where r is the position vector of the particle with respect to O and $p=mv$ is the linear momentum.

The magnitude of L in Eq. 1.0 is given as

$$|L| = |r||p|\sin \theta \text{ -----1.1}$$

where θ is the smaller angle between the positive directions of r and p .

The tendency for a force to produce rotation is proportional to the magnitude of the force and to the distance from the point of application to the center of rotation. The quantitative measure of this rotation tendency is the **TORQUE** produced by the force. The torque is the moment of the force and can be expressed as

$$\tau = r \wedge f \text{ -----1.3}$$

TWO-PARTICLE SYSTEM

Let us consider two particles with masses m_1 and m_2 located at positions identified by the vectors r_1 and r_2 respectively in some inertia frame. The equation of motion between the two particles can be obtained by solving the coupled equations that represent the Newton's second law of the form:

$$F_{12} = m_1 \frac{d^2 r_1}{dt^2} \text{ -----1.4}$$

$$F_{21} = m_2 \frac{d^2 r_2}{dt^2} \text{ -----1.5}$$

The center of mass (C.M) position vector that related the position vectors r_1 and r_2 to C.M position vector R is given as

$$R = \frac{1}{M} \sum_i m_i r_i \quad \text{-----1.6}$$

where $i=1, 2$ and $M= m_1 + m_2$

By using the definition of relative coordinate ($r=r_{12}=r_2-r_1$) in Eq. 1.6, the expressions for r_1 and r_2 can be given as

$$r_1 = R - \frac{m_2}{M} r \quad \text{-----1.7}$$

$$r_2 = R + \frac{m_1}{M} r \quad \text{-----1.8}$$

MANY-PARTICLE SYSTEM

The most convenient way to describe systems that consists of many particles is to identify the center of mass (C.M) coordinate R and to specify the position of each particle with respect to center of mass. Thus, the particle labeled i has a position vector r_i in some inertia frame and a coordinate r_{ci} with respect to C.M and r_i is given as

$$r_i = R + r_{ci} \quad \text{-----1.9}$$

The total force for a many particle system is given as

$$F_t = F_i^e + \sum_{\substack{i=1 \\ i \neq j}}^n F_{ij} = \frac{dp_i}{dt} \quad \text{-----1.10}$$

where the index j runs from 1 to n but excluding $i=j$, thereby eliminating unphysical self -interaction force F_{ii} .

The total kinetic energy of a system of particles is given as

$$T = \frac{1}{2} MV^2 + \frac{1}{2} \sum m_i v_{ci}^2 \quad \text{-----1.11}$$

where $V = \frac{dR}{dt}$ and $v_{ci} = \frac{dr_{ci}}{dt}$.

The total angular momentum, L , of a system of particles is the sum of the individual angular momenta of the particles relative to the same origin i.e.

$$L = \sum_i l_i = \sum_i r_i \times p_i \text{ -----1.12}$$

By applying particle coordinates, the total angular momentum of a system of particles is given as

$$L = R \wedge P + \sum_i r_{ci} \wedge p_{ci} \text{ -----1.13}$$