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Flow and Natural Convection Heat Transfer in a Power Law Fluid Past a Vertical Plate with Heat Generation

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Abstract: The paper examines the flow and convection heat transfer in a pseudoplastic power law fluid past a vertical plate with heat generation. The governing non – linear partial differential equations describing the flow and heat transfer problem are transformed into non – linear ordinary differential equation, using similarity transformation, and the resulting problem is solved numerically using Runge – Kutta shooting method. The problem is studied for power law exponents between 0 and 1. And the analysis of results obtained showed that the heat generation parameter have significant influence on the flow and heat transfer.

Keywords: Natural convection; power law fluid;pseudoplastic fluid; mass transfer; power law exponent

1 Introduction

The study of heat and mass transfer in a non – Newtonian power law fluid obeying the Ostwald – de Waele rheological model,

$$\tau_{yx} = -m \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \tag{1}$$

has been attracting the interest of researchers and scientist in the recent time due to its applications in food, polymer, petrol-chemical, geothermal, rubber, paint and biological industries. The two parameter rheological equation (1) is also known as the power law model. When n=1, the equation represents a Newtonian fluid with a dynamic coefficient of viscosity m. Therefore, deviation of n from a unity indicates the degree of deviation from Newtonian behavior. For $n < 1$, the fluid is pseudoplastic and for $n > 1$, the fluid is dilatant. n, is power law exponent and m is the consistency coefficient.

Chung [2] examined the nonlinear stability of steady flow and temperature distribution of a Newtonian fluid in a channel heated from below and the viscosity is a function of temperature.

Hassanien et al. [3] investigated the flow and heat transfer in a power law fluid over a non-isothermal stretching sheet. They presented a boundary layer analysis for the problem of flow and heat transfer from a power law fluid to a continuous stretching sheet with variable wall temperature. They performed parametric studies to investigate the effect of non-Newtonian flow index, generalized Prandtl number, power law surface temperature and surface mass transfer. Their result showed that friction factor and heat transfer depend strongly on the flow parameter.

Howell et al. [4] examined momentum and heat transfer on a continuously moving surface in a power law fluid .They examined the momentum and heat transfer occurring in the laminar boundary layer on a continuously moving and stretching two dimensional surface in non Newtonian fluid. Their results in clued situation when then velocity is nonlinear and when the surface is stretched linearly.

Ibrahim et al [5] investigated the method of similarity reduction for problems of radiative and magnetic field effect on free convection and mass transfer flow past a semi-infinite flat plate. They obtained new

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similarity reductions and found an analytical solution for the uniform magnetic field by using lie group method. They also presented the numerical results for the non-uniform magnetic field.

Makinde [6] examined the free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. The plate is maintained at a uniform temperature with uniform species concentration and the fluid is considered to be gray, absorbing – emitting. The coupled non-linear momentum, energy and concentration equation governing the problem is obtained and made similar by introducing a time dependent length scale. The similarity equations are then solved numerically by using superposition method.

Sivasankaran et al [7] investigated the natural convection heat and mass transfer fluid past an inclined semi – infinite porous surface with heat generation using Lie group analysis. Their result revealed that the velocity and temperature of the fluid increases with the heat generation parameter. And also, the velocity of the fluid increases with the porosity parameter and temperature and concentration decreases with increase in the porosity parameter.

Uzun [8] presented the finite difference solution for laminar heat transfer of a non-Newtonian power law fluid in the thermal entrance region of arbitrary cross sectional ducts with constant wall temperature. In his study, the effects of axial heat conduction, viscous dissipation and thermal energy sources with the fluid were neglected.

Yu – shu and Karsten [9] review their previous work on the development of a three dimensional, fully implicit, integral finite difference simulation for simple and multi-phase flow of non-Newtonian fluids in porous fractured media. The methodology, architecture and numerical scheme of the model are based on a general multi-phase, multi-component fluid and heat flow simulator and presented a new discussion on the numerical scheme used in the treatment of non-Newtonian properties and several bench mark problems for model verification.

Yurusoy and Pakdemirli [10] examine the exact solution of boundary layer equations of a non-Newtonian fluid over a stretching sheet by the method of lie group analysis and they found that the boundary layer thickness increases when the non-Newtonian behavior increases. They also compared the results with that of Newtonian fluid.

In this paper, we examine the flow and convection heat transfer in a pseudoplastic power law fluid past a vertical plate with heat generation. The problem is studied for power law exponents between 0 and 1. And the analysis of results obtained showed that the heat generation parameter have significant influence on the flow and heat transfer.

2 Mathematical Formulation

Consider a two dimensional steady flow and natural convection heat transfer in a pseudoplastic heat generating power law fluid over a semi – infinite vertical plate. The flow is assumed to be in the x – direction, which is taken along the vertical plate in the upward direction and the y – axis is taken to be the normal to the plate. The surface of the plate is maintained at a uniform constant temperature T_w which is higher than the corresponding value of T_∞, sufficiently far away from the flat surface. It is also assumed that the free stream velocity parallel to the plate is constant. The appropriate governing equations of Continuity, Momentum and Energy are given by;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\upsilon\frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right)^{n}+g\beta(T-T_\infty) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho c}(T-T_\infty) \tag{4}$$

where u, v are the velocity components in the x – and y – directions respectively, $\upsilon=\frac{m}{\rho}$, m is the flow index, ρ is the density, g is the acceleration due to gravity, β is the coefficient of volume expansion, T, T_w and T_∞ are the temperature of the fluid inside the boundary layer, the plate, and the fluid temperature in the free stream, respectively, k is the thermal conductivity, c is the specific heat capacity, Q is the heat generation constant. And the appropriate boundary conditions are;

$$\left.\begin{array}{l} u = U, v = v_w(x), T = T_w \text{ at } y = 0 \\ u \to 0, T \to T_\infty \text{ as } y \to \infty \end{array}\right\} \tag{5}$$

3 Method of solution

Introduce the stream function formulation,

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

the continuity equation (2) is automatically satisfied.

Define a similarity variable,

$$\eta = \frac{Ay}{x^{\frac{1}{2n-1}}} \tag{7}$$

Such that

$$\psi = Uf(\eta) \tag{8}$$

And

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{9}$$

Therefore equations (3) and (4) together with the boundary and initial conditions (5) become,

$$\upsilon n U^{n-2} A^{2n-1} (-f'')^{n-1} f''' - \frac{1}{2n-1}(f')^2 - Gr_n \theta = 0 \tag{10}$$

$$\frac{\upsilon}{\Pr_n}(2n-1)\theta'' + He\theta = 0 \tag{11}$$

$$\left.\begin{array}{l} f = f_w, f' = 1, \theta = 1 \text{ at } \eta = 0 \\ f' = 0, \theta = 0 \text{ at } \eta \to \infty \end{array}\right\} \tag{12}$$

$f' = 0, \quad \theta = 0$ as $\eta \to \infty$

And the dimensionless parameters introduced in equations (10) and (11) are as defined below;

$Gr_n = \frac{x^{\frac{2n+1}{2n-1}} g\beta(T_w - T_\infty)}{U^2 A^2}$ is the Grashof number,

$\Pr_n = \frac{\upsilon \rho c U}{kAx^{\frac{2n-2}{2n-1}}}$ is the Prandtl number,

$He = \frac{Q(2n-1)x^{\frac{2n}{2n-1}}}{\rho c U A}$ is the heat generation parameter.

4 Numerical Solution

Resolve equations (10) and (11) into a system of first order differential equations. Let,

$$x_1 = \eta, x_2 = f, x_3 = f', x_4 = f'', x_5 = \theta, x_6 = \theta' \tag{13}$$

Taking the derivative of the system of equations (13), then equations (10) together with equation (11) become,

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ x_5' \\ x_6' \end{pmatrix} = \begin{pmatrix} 1 \\ x_3 \\ x_4 \\ \frac{x_3^2 + (2n-1)Gr_n x_5}{\upsilon n U^{n-2} A^{2n-1} (2n-1)(-x_4)^{n-1}} \\ x_6 \\ \frac{-\Pr_n x_5 He}{\upsilon(2n-1)} \end{pmatrix} \tag{14}$$

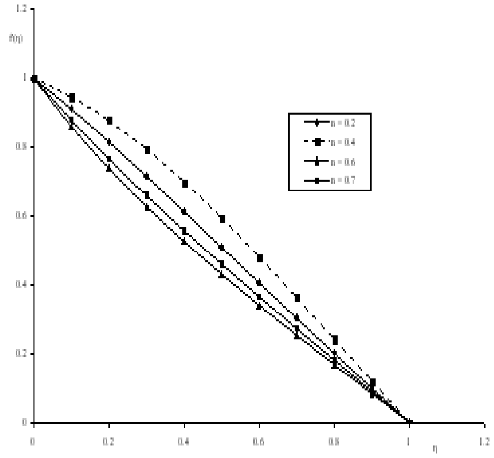

Fig1:Temperature profile for different values of the power law exponent n and for He = 0.2 and Grn = 0.9

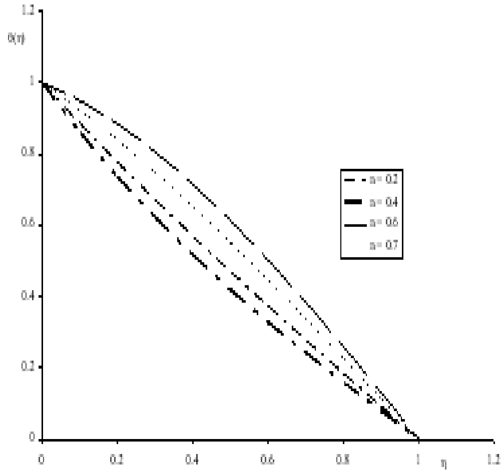

Fig2:Temperature profile for different values of the power law exponent n and for He = 0.2

And the initial conditions in (12) become,

$$x_1(0) = 0, x_2(0) = 0, x_3(0) = 1, x_4(0) = -\Gamma, x_5(0) = 1, x_6(0) = -\gamma \quad (15)$$

where Γ and γ are constants to be determined ? Problem (14) together with the initial conditions (15) is solved numerically using Runge – Kutta shooting method.

In a shooting method, the missing (unspecified) initial condition at the initial point of the interval is assumed, and the differential equation is then integrated numerically as an initial valued problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy. The numerical results are presented in table 1 and as velocity and temperature profiles in figures 1 – 4.

Table 1: Numerical result

n	He	Gr_n	$\theta'(0)$	$f''(0)$
0.2	0.2	0.9	-1.1630	-0.8633
0.4	0.2	0.9	-1.45985	-0.4808
0.4	0.5	0.9	-2.02355	-0.4662
0.4	0.8	0.9	-2.49065	-0.4558
0.4	1.5	0.9	-3.36493	-0.4400
0.4	3.5	0.9	-5.12438	-0.41945
0.6	0.2	0.9	-0.43155	-1.5173
0.7	0.2	0.9	-0.7327	-1.2677
0.7	0.5	0.9	-0.2652	-1.2766
0.7	0.6	0.9	-0.0862	-1.2802

Table 2: Numerical result

n	He	Gr_n	C_f	Nu
0.2	0.2	0.9	0.038841173217	1.1630
0.4	0.2	0.9	0.029843226489	1.45985
0.6	0.2	0.9	0.051369207540	0.43155
0.7	0.2	0.9	0.047224953021	0.7327

5 Skin – friction Coefficient and Nusselt number.

The local skin – friction coefficient and local Nusselt number which indicates the physical wall shear stress and rate of heat transfer respectively are parameters of engineering interest for the present problem.

The local wall shear stress is defined as

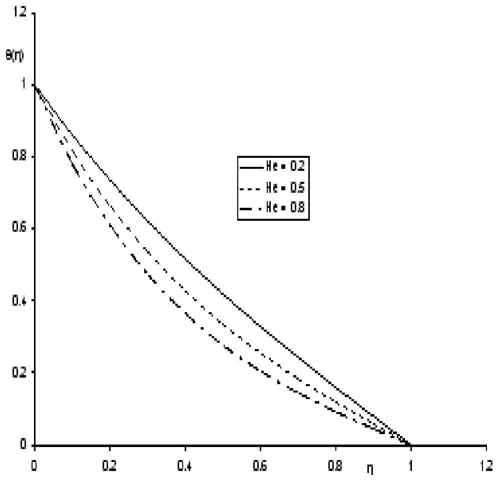

Fig3:Temperature profile for various values of He with power law exponent *n = 0.4*

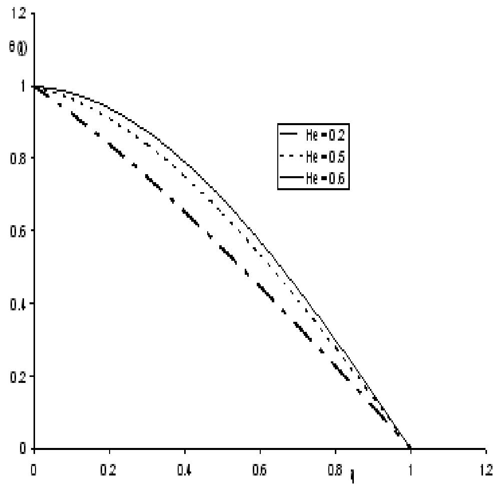

Fig4:Temperature profile for various values of He with power law exponent *n = 0.7*

$$\tau_w = \left(-m \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right)_{y=0} \tag{16}$$

And the skin-friction coefficient, C_f is given by,

$$C_f = \frac{2\tau_w}{\rho U^2} \text{ or } \frac{1}{2} C_f Re = \left(-f''(0)\right)^n \tag{17}$$

where Re = $\frac{\rho U^{2-n} x^{\frac{2n}{n-1}}}{m A^{2n}}$ is the Reynolds number

The heat flux, q_w at the wall is given by,

$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} \tag{18}$$

And the Nusselt number is given by

$$Nu = \frac{x^{\frac{1}{2n-1}} q_w}{k \Delta T} = -\theta'(0) \tag{19}$$

where $\Delta T = T_w - T_\infty$, the skin friction coefficient and Nusselt number are obtained numerically and the result is presented in tables 2, 3 and 4.

6 Discussion of result

It is interesting to note from the mathematical equation (10) and (11) that the fluid flow will experience a constant fluid velocity and zero temperature for the pseudoplastic power law fluid with the power law exponent $n = 0.5$. Thus, it is important to investigate the flow and heat transfer for the pseudoplastic power law fluids with the power law exponents $0 < n < 1/2$ and $1/2 < n < 1$. In the investigation, the dimensionless parameter Gr_n which is a measure of the buoyancy forces due to temperature is taken to be 0.9 and this correspond to pure forced convection. The value of the heat generation parameter is varied to observe its effects on the flow and heat transfer problem. For the pseudoplastic fluid with the power law exponent's $0 < n < 1/2$, heat generation parameter is varied between $0.2 \leq He \leq 3.5$ and for the pseudoplastic fluid with the power law exponent's $1/2 < n < 1$ heat generation parameter is varied between $0.2 \leq He \leq 0.6$.

The velocity profiles for different values of the power law exponent $n; n = 0.2, 0.4, 0.64$ and 0.7 with the heat generation parameter $He = 0.2$ and the Grashof number $Gr_n = 0.9$ are as shown in figure 1. It is observed that the velocity experience maximum for the pseudoplastic power law fluids with the power law exponents between $0 < n < {}^1/_2$ and the value of the maximum velocity increases with increase in the value the power law exponents. Also, the velocity experience minimum velocity for the pseudoplastic power law fluids with the power law exponents between ${}^1/_2 < n < 1$ and the value of the minimum velocity decreases with decrease in the value the power law exponents

The temperature profile for different values of the power law exponent $n; n = 0.2, 0.4, 0.6$ and 0.7 with the heat generation parameter $He = 0.2$ and the Grashof number $Gr_n = 0.9$ is as shown in figure 2. It is observed that the fluid temperature experience a minimum for the pseudoplastic fluid with the power law exponent between 0 and ${}^1/_2$ and experience a maximum for the pseudoplastic fluids with the power law exponent between ${}^1/_2 \, and 1$ .it is also interesting to note that the value of the minimum temperature decreases with increase in the power law exponent between 0 and ${}^1/_2$ and also the value of the maximum temperature increase with decrease in the value of the power law exponent between ${}^1/_2$ and 1 .

Fig. 3 shows the temperature profile for various values of heat generation parameter $He; He = 0.2, 0.5$ and 0.8 with the power law exponent $n = 0.4$. It is observed that the fluid temperature decrease with increase in the heat generation parameter and the rate at which the temperature goes to zero is fast with a low value of the heat generation parameter.

Fig. 4 shows the temperature profile for different values of the heat generation parameter $He; He = 0.2, 0.5 and 0.6$ with the power law exponent $n = 0.7$. It is observed that the fluid temperature increases with in the heat generation parameter.

The important physical quantities, the local shear stress and local rate of heat transfer are respectively measured in terms of the local skin friction, C_f and the local Nusselt number, Nu and the numerical is as shown in Table 2, 3 and 4.

Table 3: Numerical result

n	He	Gr_n	C_f	Nu
0.4	0.2	0.9	0.029843226489	1.45985
0.4	0.5	0.9	0.029477381285	2.02355
0.4	0.8	0.9	0.029212566879	2.49065
0.4	1.5	0.9	0.028803221349	3.36493
0.4	3.5	0.9	0.028257391637	5.12438

Table 4: Numerical result

n	He	Gr_n	C_f	Nu
0.7	0.2	0.9	0.047224953021	0.7327
0.7	0.5	0.9	0.047456792230	0.2652
0.7	0.6	0.9	0.047550432048	0.0862

Table 2 shows the skin friction C_f and the local Nusselt number, Nu for different values of the power law exponent $n; n = 0.2, 0.4, 0.6 and 0.7$. And the table shows that the skin friction C_f decreases with increase in the power law exponent for the pseudoplastic power law fluids with the power law exponents between $0 < n < {}^1/_2$ and the local Nusselt number, Nu increases with increase in the power law exponent. But, for the pseudoplastic power law fluids with the power law exponents between ${}^1/_2 < n < 1$, the skin friction C_f decreases with increase in the power law exponents and the local Nusselt number, Nu increases with increase in the power law exponents. Also the skin friction is minimal for the pseudoplastic power law fluids with the power law exponents between $0 < n < {}^1/_2$ and the local Nusselt number Nu is greatest for the pseudoplastic power law fluids with the power law exponents between $0 < n < {}^1/_2$. Because of the minimal value of the skin friction C_f and the greatest heat transfer rate, the pseudoplastic power law fluids with the power law exponents between $0 < n < {}^1/_2$ are better working pseudoplastic power law fluids in flow and heat transfer.

Table3 shows the skin friction C_f and the local Nusselt number, Nu with the power law exponent $n = 0.4$ for various values of the heat generation parameter He ; $He = 0.2, 0.5, 0.8, 1.5$ and 3.5 .

It is observed that for the power law exponent $n = 0.4$, the skin friction decreases with increase in the heat generation coefficient and the Nusselt number increases with increase in the heat generation parameter. Thus, high value of the heat generation parameter enhances high heat transfer.

Table3 shows the skin friction C_f and the local Nusselt number, Nu with the power law exponent $n = 0.4$ for various values of the heat generation parameter He ; $He = 0.2, 0.5$ and 0.6 .

It is observed that for the pseudoplastic fluid with the power law exponent $n = 0.7$, the skin friction increases with increase in the heat generation parameter and the Nusselt number decreases with increase in the heat generation parameter. Hence, low value of the heat generation parameter enhances high heat transfer.

7 Conclusion

In conclusion, it is obvious from the above discussions that the heat generation parameter has a significant influence on the flow and heat transfer and the result also shown that a pseudoplastic power law fluid with the power law exponent $0 < n < {}^1/_2$ gives a higher heat transfer coefficient than the pseudoplastic power law fluid with power law exponent ${}^1/_2 < n < 1$ .And due to the minimal value of the skin friction and greatest heat transfer rates, pseudoplastic power law fluid with the power law exponent $0 < n < {}^1/_2$ seem to be better working pseudoplastic power law fluid in flow and heat transfer. This work finds many practical applications in petroleum drilling, manufacturing of foods, production of polymers and slurries. More importantly, the boundary layer concept of non – Newtonian power law fluid has application in the reduction of frictional drags in many engineering process.

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