University of Agriculture, Abeokuta,
Department of Mathematics
2009/2010 First Semester Examination   July 2010
MTS211 - Abstract Algebra

INSTRUCTION: Answer Question 5 OR 6 and any other three Questions   Time: $1\frac{3}{4}$ hrs

1. (a) Define the following terms
   i. A greatest common divisor of integers a and b
   ii. A least common multiple of the set $\{a_i\}, i = 1, 2, ..., n$ and $a_1, a_2, ..., a_n$ are non zero integers

   (b) Prove that if a, b be two non-zero integers,then $d = (a, b)$ exist .Moreover prove that $d = ua + vb$ for some integers u, v.

   (c) If d is $g.c.d$ of $(1824, 760)$,find the integers u, v such that $d = ua + vb$.

2. (a) i. State the Division algorithm theorem for integers.
   ii. State the unique factorization theorem.

   (b) Proof that there are infinitely many primes.

   (c) Let $S = \{1, 2, 3, 4, 5\}$,Let a mapping $f : S \to S$ be defined as $f(1) = 1, f(2) = 3$, $f(3) = 3, f(4) = 4, f(5) = 2$.Let g be an injective mapping on S given by $g(1) = 4$, $g(3) = 3, g(5) = 5$ .If $gof = fog$,find the values of $g(2)$ and $g(4)$.

3. (a) i. Define an indexing set
   ii. Define a cartesian product or product set

   (b) Let $\Omega = \{a, b, c\}, S_a = \{1, 2, 3, 6, 8, 10\}, S_b = \{2, 4, 6, 7, 9\}, S_c = \{4, 11, 6, 1, 3, 18\}$.Compute the set sum $S_a \bigvee S_b \bigvee S_c$.How many elements are in the set.Compare their number with those in $S_a \bigcup S_b \bigcup S_c$.

   (c) Using the first principle of mathematical induction,show that 17 divides $(3 \times 5^{2n+1} + 2^{3n+1})$ for any $n \in \mathbf{N}$.

4. (a) i. Define an equivalence relation τ on a set S.
   ii. Define an equivalence class and quotient set.

   (b) Let $S = \mathbf{Z}$, $a, b \in S$.Define $a\tau b$ by 5 divides $(a - b)$.Show that τ is an equivalence relation and determine the equivalence class.

   (c) Consider $(\mathbf{R}, *, \bigotimes)$ where $a * b = ab$ and $a \bigotimes b = a + b + ab$ for all $a, b \in \mathbf{R}$.
   i. is $*$ distributive over $\bigotimes$
   ii. is $\bigotimes$ distributive over $*$.

5. (a) i. Define a homomorphism of a group
   ii. Define an isomorphism and an epimorphism of groups

   (b) prove that a homomorphic image of a group is also a group.

   (c) Prove that the homomorphism $\phi : G \to G'$ is injective if and only if $ker\phi$ consists of the identity element alone.

6. (a) i. What is a group?
   ii. What is a subgroup of a group?
   iii. What is a cyclic group?

   (b) show that the set of all 2×2 singular matrices is a group under multiplication o

   (c) Show that the order of a subgroup of a finite group G divides the order of the g