University of Agriculture, Abeokuta,
Department of Mathematics
2009/2010 Second Semester Examination October 2010
MTS212 - Linear Algebra

INSTRUCTION: Answer any four Questions Time: $1\frac{3}{4}$ hrs

1. (a) Define a vector space V and subspace U of V.
   (b) Let U and W be subspaces of a finite-dimensional vector space V over a field F. Show that:
      i. $U + W$ is a subspace of V, where $U + W = \{\{u + w\} | u \in U, w \in W\}$ is called the linear sum of U and W
      ii. $U \cap W$ is a subspace of V

2. (a) Define linearly independent and basis for a vector space V
   (b) Establish the linear dependent or independent of vectors $v_1 = (1, 0, -1, 2), v_2 = (1, 3, 1, 6), v_3 = (1, 5, -1, 16), v_4 = (4, 1, 0, 2)$ in $\Re^4$ and find the dimension of the space spanned by these vectors

3. (a) Define a linear transformation T and the null-space of T
   (b) i. Let $M_{2,3}(F)$ denote the vector space of 2×3 matrices over a field F. Prove that $M_{2,3}(F)$ is isomorphic to the vector space F^6 over F
      ii. Find a basis and dimension for a subspace of $M_{2,3}(F)$ generated by $\begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 3 \end{pmatrix}, \begin{pmatrix} 3 & -1 & 1 \\ 1 & 0 & 2 \end{pmatrix}, \begin{pmatrix} 4 & 0 & 3 \\ 2 & -3 & 1 \end{pmatrix}$

4. (a) What do you understand by a non-homogeneous system of linear equations
   (b) Verify if the following system is consistent or not. Solve if possible over the field Q
   $3x + 2y - 7z - 3w = 1$
   $7x - 5y + 3z + 22w = 12$

5. (a) Define the solution space of the homogeneous system of linear equations
   (b) Find the value of λ for which the system of linear equations
   $(2 - \lambda)x + 2y + 3 = 0$
   $2x + (4 - \lambda)y + 7 = 0$
   $2x + 5y + (6 - \lambda) = 0$
   are consistent and find the values of x, y corresponding to each of the values of λ over the field $\Re$ of real numbers

6. (a) Given a linear transformation $T : V_n(F) \to V_n(F)$, define the eigenvalue and eigenvector of T
   (b) Find the characteristic polynomial for the matrix A over $\Re$

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

   and show by direct substitution that this matrix satisfies its characteristic equation. Find the characteristic roots, characteristic vectors and minimal polynomial of A