University of Agriculture, Abeokuta,

Department of Mathematics

2009/2010 Second Semester

B.Sc. Degree Examination October 2010

MTS342 - Mathematical Methods II

INSTRUCTION: Answer 2 questions from each section Time: $2\frac{1}{2}$ Hrs

SECTION A

Answer any 2 questions from this section.

1. (a) When is a set of functions $f_1(x), f_2(x), ..., f_n(x)$ said to

   i. linearly dependent on some interval $a \leq x \leq b$

   ii. linearly independent on some interval $a \leq x \leq b$

   iii. Define the Wronskian of $f_1(x), f_2(x), ..., f_n(x)$.

   iv. Prove or disprove that $f_1(x) = sin^2x$ and $f_2(x) = 1 - cos2x$ are linearly independent on $-\infty < x < \infty$.

   (b) Using the method of variation of parameters, solve the following equations;

   i. $y'' + y = secxtanx$

   ii. $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$

2. (a) Given that $u_1(x)$ is a homogeneous solution of the differential equation

$$y'' + a_1(x)y' + a_2(x)y = f(x) \qquad (1)$$

by substituting $y = k(x)u_1(x)$, show that

$$u_2(x) = \lambda u_1(x) \int \frac{dx}{\alpha u_1(x)^2} + \gamma u_1(x)$$

is also an independent solution of the homogeneous part of the differential equation (1), where $\alpha = e^{\int a_1(x)dx}$ and λ and γ are constants.

Hence or otherwise, derive the Abel's formula

$$W(u_1(x), u_2(x)) = \lambda e^{-\int a_1(x)dx}.$$

(b) i. Given that $u_1 = x$ is a solution of the equation $x^3y''' - 3x^2y'' + x(6-x^2)y' - (6-x^2)y = 0$. Show that $y = k(x)x$ will also be a solution provided that k' satisfies the equation $(k')'' - k' = 0$. Find the other solution.

ii. Given that $u_1 = x$ is a solution of the equation $x^2y'' - 3xy' + 3y = 0$. Find the second solution and the general solution.

(a) Given a differential equation

$$a_2(x)y'' + a_1(x)y' + a_1(x)y = 0 \qquad (2)$$

when is the point x_0 called:

i. an ordinary point of the differential equation (2)

ii. a singular point of the differential equation (2)

iii. a regular singular point of the differential equation (2)

iv. By using the Leibnitz-Maclaurin's method, find the power series solution of the equation $(1 - x^2)y'' - 5xy' - 3y = 0$.

(b) i. By using the Frobenius method and assuming a series solution of the type $y = \sum_{r=0}^{\infty} a_r x^{m+r}$, obtain the general solution of the Bessel's equation

$$x^2y'' + xy' + (x^2 - v^2)y = 0.$$

ii. The Rodrigues' formula for the Legendre polynomial is

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n.$$

Verify this formula for $n = 0, 1, 2, 3$.

SECTION B

Answer any 2 questions from this section.

4. (a) Define the gamma function of the variable x. Deduce the gamma functions of the following from the first principles

i. $\Gamma(x + 1)$

ii. $\Gamma(\frac{1}{2})$

(b) Given that $I_{m,n} = \frac{m-1}{m+n} I_{m-2,n}$ and $B(m, n) = 2\int_0^{\frac{\pi}{2}} sin^{2m-1}\theta cos^{2n-1}\theta d\theta$. Prove that

$$B(m,n) = \frac{(m-1)(n-1)}{(m+n-1)(m+n-2)} B(m-1, n-1).$$

(c) Evaluate the following

i. $B(4, 3)$

ii. $B(5, 3)$

5. Given the Legendre's equation

$$(1-x^2)y_2 - 2xy_1 + \alpha(\alpha+1)y = 0$$

where α is a real constant.

(a) By using the Frobenius method, obtain the solution

$$y = a_0\left[1 - \frac{(k+1)}{2!}x^2 + \frac{k(k-2)(k+1)(k+3)}{4!}x^4\right],$$

for $c = 0$.

(b) Derive the recurrence relation for $c = 1$ for the Legendre's equation.

(c) Using the results obtained in (a) and (b) above, obtain the following polynomials (i) $P_2(x)$ (ii) $P_3(x)$

6. (a) Evaluate the following integrals

i.

$$I = \int_0^1 x^5(1-x)^4 dx$$

ii.

$$I = \int_0^{\frac{\pi}{2}} [tan\theta]^{\frac{1}{2}}\, d\theta$$

(b) State the convolution theorem for two functions $f(t)$ and $g(t)$. If $f(t) = t$ and $g(t) = e^t$, find the convolution of f and g.

(c) Solve the equation $y'' + 3y' + 2y = 4x$, where $y(0) = y'(0) = 0$ using the laplace transform method.