UNIVERSITY OF AGRICULTURE ABEOKUTA
UNIVERSITY EXAMINATIONS
2009/2010
B.Sc. Degree Examination
MTS 441 (Ordinary Differential Equations)
2nd July, 2010 - 9a.m. - 12noon

<u>Instructions:</u> **Full marks will be awarded for complete and legible answers to THREE QUESTIONS.**

1(a) Given an n-th order ODE of the form:

$$F(t\,x, x', \cdots, x^{(n)}) = 0 \tag{1.1}$$

where $x^{(n)}$ is the n-th order derivative of x with respect to t and F a function defined on some subset of $I\!R^{n+2}$ between the variables $(t, x, x' \cdots x^{(n)})$ such that it is implicit in nature and may represent a collection of some differential equations. To avoid ambiguity and assuming that the relation is solvable for $x^{(n)}$,

(i) write (1.1) in the form where the right hand side

$$f : I \times \mathcal{D}(\subseteq I\!R^n)) \longrightarrow I\!R^{n+1}$$

with $I = (a, b)$

(ii) reduce the n-th order equation so obtained to an equivalent system of n first order equations

(1(b)(i) State conditions for the initial value problem

$$x' = f(t, x), \quad x(t_0) = x_0 \tag{1.2}$$

to have at most one solution $x(t)$. From your statement, indicate what condition(s) if relaxed would impair uniqueness of the solution $x(t)$ of (1.2)

(ii) With the help of the method of successive approximations (Picard's iteration technique) solve the initial value problem:

$$x' = x; \quad x \in I\!R, \quad x(0) = 1 \tag{1.3}$$

2(a) Consider the linear system

$$x' = A(t)x \tag{2.1}$$

where the elements of the $n \times n$ matrix A are defined and continuous on some interval $q < t < r$,

(i) what is meant by a fundamental matrix of (2.1).

Show that:

(ii) if Φ is fundamental for (2.1) on (q, r) so also is ΨC for any constant non-singular $n \times n$ matrix C,

(iii) every fundamental matrix has the form ΦC.

2(b) With reference to the periodic system

$$x' = A(t)x; A(t) = A(t+s); t \in I\!R \quad s \neq 0$$

show that if $\Psi(t)$ (with $\Psi(0) = I$, I being the identity matrix) is fundamental for (2.2), then

(i) $\Psi(t+s)$ is also fundamental for (2.2)

(ii) $\Psi(t+s) = \Psi(t)\Psi(s)$

3(a)(i) Solve the system

$$x' = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3.1}$$

(ii) Employ the transformation

$$v = \exp\left[-\frac{1}{2}\int_0^t p(s)ds\right] \tag{3.2}$$

in the linear second order homogeneous ODE

$$x'' + p(t)x' + q(t)x = 0 \tag{3.3}$$

to reduce the equation

$$t^2x'' + \alpha t x' + \beta x = 0 \tag{3.4}$$

to the normal form.

3(b) With reference to the equation

$$x'' + q(t)x = 0 \tag{3.5}$$

where $q(t)$ is a real-valued, continuous function on $t_0 \leq t < \infty$;

(i) what do you understand by an "oscillatory equation". Justify your answer with a suitable example.

(ii) Show by using either of Sturm's theorem on oscillation of equations (if applicable) or otherwise that the zeros of the two linearly independent solutions of

$$x'' + x = 0 \quad \left(x' = \frac{dx}{dt}\right). \tag{3.6}$$

4(a) By considering the non-autonomous system

$$x' = F(t, x) \tag{4.1}$$

explain clearly what you understand by the notions: stability, uniform stability and asymptotic stability of a solution $x(t)$ of (4.1)

4(b) Employ the Liapunov function

$$V(x_1, x_2) = 2x_1^2 + 3x_2^2 \tag{4.2}$$

to determine the stability of the trivial solution of the system

$$\begin{aligned} x_1' &= -6x_2 - \frac{1}{4}x_1x_2^2 \\ x_2' &= 4x_1 - \frac{1}{6}x_2 \end{aligned} \tag{4.3}$$

Dr. V.F. Payne

Endorsed