UNIVERSITY OF AGRICULTURE, ABEOKUTA.
UNIVERSITY EXAMINATIONS

B.Sc. Degree Examination
Mathematics

MTS 442 : Partial Differential Equations.

Tuesday, 12 October, 2010 **Time : 3 HOURS**

Write legible and correct Answers to any <u>THREE</u> QUESTIONS.

1(a) Let $a, b, c \in C^0(\Omega)$, Ω being a domain in $I\!R^3$. Give a concise description of how the integral surface of the quasilinear equation

$$a(t,x,u)u_t + b(t,x,u)u_x + c(t,x,u) = 0 \tag{1.1}$$

is related to the integral curves of the simultaneous equations

$$\frac{dt}{a} = \frac{dx}{b} = \frac{du}{b} \tag{1.2}$$

You may need to illustrate your answer with a geometrical interpretation.
What three distinct possibilities arise from the determination of an integral surface $\mathcal{S}$ of (1.1) passing through a prescribed space curve

$$\mathcal{C} : u(x,0) = h(x)? \tag{1.3}$$

Proceeding formally in view of the information obtained above solve the initial value problem

$$\left.\begin{aligned} uu_t + u_x &= 1 \\ t &= s \\ x &= s \quad 0 \le s \le 1 \\ u &= \tfrac{1}{2}s \end{aligned}\right\} \tag{1.4}$$

From the foregoing deduce the detailed and sufficient requirement which admits a unique solution for the problem

$$\left.\begin{aligned} au_t + bu_x + c &= 0 \\ \mathcal{C} : t &= t_0(s) \\ x &= x_0(s) \quad 0 \le s \le 1 \\ u &= u_0(s) \end{aligned}\right\} \tag{1.5}$$

1(b) With reference to the general first order partial differential equation

$$F(t,x,u,p,q) = 0 \tag{1.6}$$

where $p = u_t$, $q = u_x$, write down five characteristic differential equations associated with the PDE (1.6). What are meant by the following concepts associated with (1.6):

(i) characteristic strip

(ii) element of a strip

(iii) strip condition?
Use the consequence of concepts in 1(b) above to seek an integral surface for the equation

$$tp + xq = pq \tag{1.7}$$

which passes through

$$t = t_0 \,, x = 0 \,,\ u = \frac{1}{2}t_0. \tag{1.8}$$

2(a) Consider the initial value problem

$$\left.\begin{array}{rcl} u^2u_t + u_x & = & 0 \\ u(t,0) & = & t \end{array}\right\} \tag{2.1}$$

Derive the solution

$$(2.2) \qquad u(t,x) = \begin{cases} t, & x = 0 \\ \dfrac{\sqrt{(1+4tx)}-1}{2x}, & x \neq 0,\ 1+4tx > 0 \end{cases}$$

Do shocks ever develop? Show that $\lim_{x\to 0} = u(t,x) = t$.

2(b) Given the second order linear partial differential equation

$$a(x,t)u_{xx} + 2b(x,t)u_{tx} + c(x,t)u_{tt} = \Phi(x,t,u,u_x,u_t) \tag{2.3}$$

where $(t,x) \in \mathcal{D}$, a domain of $I\!R^2$

(i) write down the characteristics for (2.3)

(ii) describe a method of classifying the equation into types in $\mathcal{D}$.

(iii) show that type of (2.3) are invariant under a regular transformation.

(3(a) Find the complete and singular solutions of the equation

$$u = p^2 + q^2 \tag{3.1}$$

(Hint : You may consider (3.1) as of type $f(u,p,q) = 0$ and assume that $F(t+ax) = F(w)$ where a is an arbitrary constant).

3(b) Classify the following second order PDES into types showing these classification in the appropriate region of tx-plane :
(i) $2tu_{tx} + 2xu_{tt} = 0$.
(ii) $tu_{xx} + xu_{tt} = 0$.

4(a) Reduce via a regular transformation the equation

$$xu_{tt} + (t+x)u_{tx} + tu_{xx} = 0 \tag{4.1}$$

to canonical form, write down and give geometrical illustrations of the characteristics, then find the general solution.

4(b) State conditions under which the following Cauchy problem admits a unique and analytic solution of the form

$$u(t,x) = \sum_{m=0} \sum_{n=0} \frac{D_t^m D_x^n u(0,0) t^m x^n}{m!n!} \text{ in the neighbourhood of } (0,0)$$

:

$$\left. \begin{aligned} u_t &= u u_x \\ u(0,x) &= 1 + x^2 \end{aligned} \right\} \tag{4.2}$$

VFP, Sept. 2010