University of Agriculture, Abeokuta,

Department of Mathematics

2009/2010 First Semester Examination July 2010

MTS461 - General Topology

INSTRUCTION: Answer Any Four Questions Time: $2\frac{1}{2}$ Hours

1. (a) What is meant by saying that a collection τ of subsets of X is a topology for the set X? Let τ be a class of subsets of $\mathcal{N}$, where $\mathcal{N}$ is the set of natural numbers consisting of the empty set and all subsets of $\mathcal{N}$ of the form $E_n = \{n, n+1, n+2, n+3, ...\}$ with $n \in \mathcal{N}$.

   (i) Show that τ is a topology on $\mathcal{N}$.

   (ii) List the open sets containing the positive integer 7.

   (b) When is a topology on X said to

   (i) indiscrete (ii) discrete (iii) cofinite (iv) usual

   Let $f : X \to Y$ be a function from a non-empty set X into a topological space (Y, τ_Y). Show that $\tau_X = \{f^{-1}(G) : G \in \tau_Y\}$ is a topology on X.

2. (a) Let X be a topological space. When do we say that

   i. $p \in X$ is an accumulation point of a subset A of X

   ii. $\overline{A}$ is the closure of a subset A of X

   iii. $p \in A$ is an interior point of a subset A of X

   iv. $Ext(A)$ is the exterior of a subset A of X

   v. $b(A)$ is the boundary of a subset A of X.

   Show that $b(A \cup B) \subset b(A) \cup b(B)$

   (b) Show that the interior of a set A is the union of all open subsets of A. Furthermore, show that

   (i) $Int(A)$ is open (ii) $Int(A)$ is the largest open subset of A

   (iii) A is open if and only if $A = Int(A)$

3. (a) Let A be a subset of a topological space (X, τ). Show that the relative topology $\tau_A = \{A \cap G : G \in \tau\}$ is a topotogy on A.

Let $X = \{a, b, c, d, e\}$ and let $\tau = \{X, \Phi, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}\}$ be a topology on X. List the members of the relative topology τ_A on $A = \{a, b, d, e\}$.

Show that every subspace of a discrete topological space is also a discrete topological space.

(b) Let X be a topological space and let $p \in X$. When is a subset N of X called a neighbourhood of p?. Let $X = \{a, b, c, d, e\}$ and let

$\tau = \{X, \{\}, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}\}$ be a topology on X. Find the neigborhood system of

(i) the point e (ii) the point c.

Let N_p denote the neighbourhood system of p. Show that

i. N_p is not empty and $p \in N$ for every $N \in N_p$

ii. $N_1 \cap N_2 \in N_p$ for every $N_1, N_2 \in N_p$

iii. If $N_1 \in N_p$ and $N_1 \subset N_2$ then $N_2 \in N_p$.

4. (a) Let (X, τ) be a topological space when do we say that

i. A class $\mathcal{B}$ of subsets of X is a base for the topology τ on X?

ii. A class $\mathcal{S}$ of open subsets of X is a subbase for the topology τ of X?

iii. A class $\mathcal{B}_p$ of open sets containing $p \in X$ is a local base at p?

Let A be a subset of X. Show that the class $\mathcal{S}_A = \{A \cap S : S \in \mathcal{S}\}$ is a subbase for the relative topology τ_A on A.

(b) i. Let $\mathcal{B}$ be a base for a topology τ on X and let $\mathcal{B}^*$ be a class of open sets containing $\mathcal{B}$. Show that $\mathcal{B}^*$ is also a base for τ

ii. Show that every point p in a discrete space X has a finite local base.

5. (a) Let X be a topological space. When is X said to be

(i) T_1 space (ii) T_2 space (iii) Regular space (iv) Normal space (v) Tychonoff space

Show that every metric space is a Hausdorff space.

(b) Let X be a topological space. When do we say that

i. X is a first countable space

ii. X is a second countable space.

Show that

iii. Every subspace of a first countable space is first countable

iv. Every subspace of a second countable space is second countable

6. (a) Let X and Y be topological spaces and let $f : X \to Y$ be a function. When is f said to be

(i) continuous at $p \in X$ (ii) continuous over X

(iii) sequentially continuous at $p \in X$ (iv) an open function

(v) a closed function (vi) a homeomorphism

For a function f on a topological space X to a topological space Y. Show that the following conditions are equivalent

(i) The mapping f is continuous

(ii) Inverse images of all closed subsets of Y are closed in X

(b) Let X be a topological space. When do we say that

i. X is locally compact

ii. a subset A of X is countably compact

iii. a subset A of X is compact

iv. a subset A of X is disconnected?

Let $X = \{a, b, c, d, e\}$ with the topology $\tau = \{\Phi, X, \{c\}, \{c, d, e\}, \{a, b, c\}\}$. Prove or disprove that $A = \{a, d, e\} \subset X$ is disconnected.