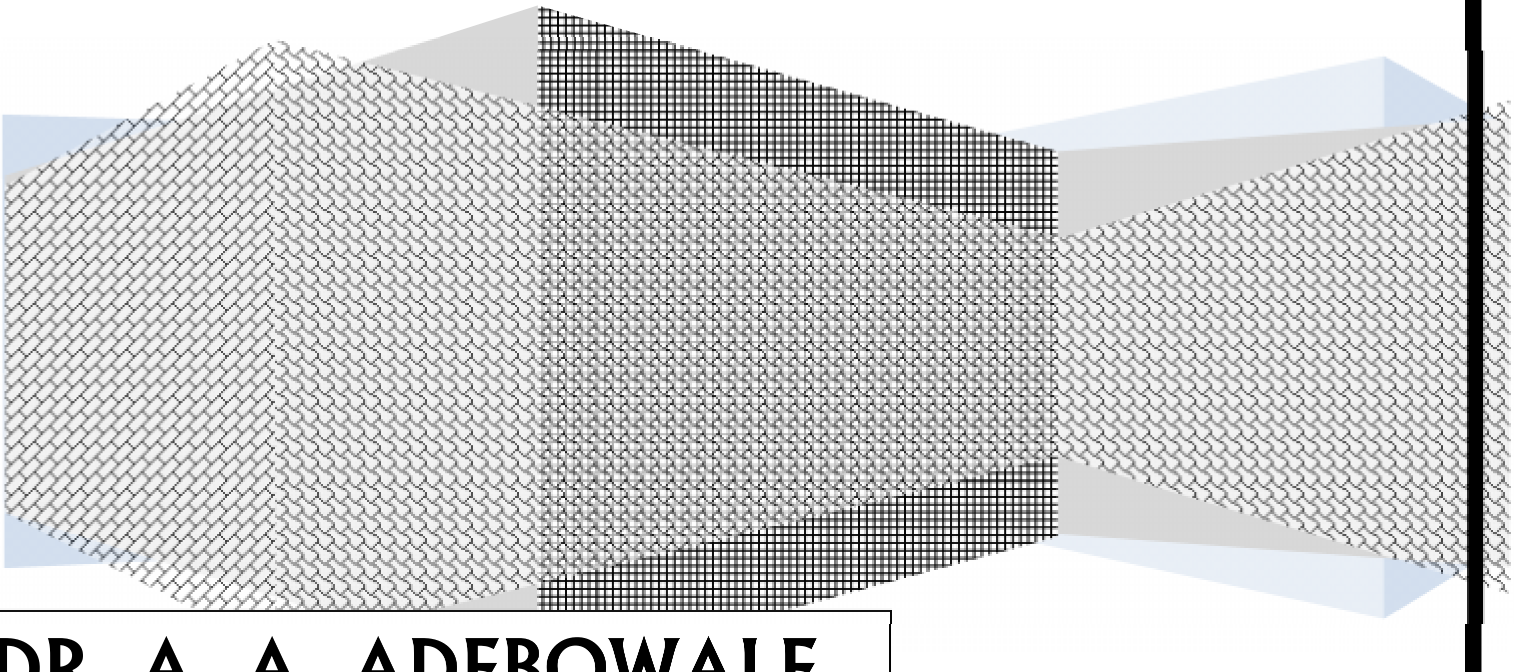
FOOD RHEOLOGY

(FST 310)



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INTRODUCTION

Rheology is the science of flow and deformation of matter and describes the interrelation between force, deformation and time. The term comes from Greek ***rheos*** meaning to flow. Rheology is applicable to all materials, from gases to solids. The science of rheology is only about 70 years of age. It was founded by two scientists meeting in the late 1920s and finding out they have the same need for describing fluid flow properties. The scientists were Professor Marcus Reiner and Professor Eugene Bingham. The Greek philosopher Heraclitus described rheology as ***panta rei*** - everything flows. Translated into rheological terms by Marcus Reiner; this means everything will flow if you just wait long enough.

Fluid rheology is used to describe the consistency of different products, normally by the two components: viscosity and elasticity. By viscosity is usually meant resistance to flow or thickness and by elasticity usually stickiness or structure. Rheological relationships help us to understand the fluids we are working with so that we can either know how they are behaving or force them to behave according to our needs. Once a correlation has been developed between rheological data and product behaviour, the procedure can then be reversed and rheological data may be used to predict performance and behaviour.

Food rheology

Described as the study of the deformation and flow of the raw materials, the intermediate products and the final products of the food industry. The rheological property of a food system is dependent on the composition or the ingredients of the system. There are numerous areas where rheological data are needed in the food industry:

a. Process engineering calculations involving a wide range of equipment such as pipelines, pumps, extruders, mixers, coaters, heat exchangers, homogenizers, and on line viscometers;
b. Determining ingredient functionality in product development;
c. Intermediate or final product quality control;
d. Shelf life testing;
e. Characterizing ingredients and final products, as well as for predicting product performance and consumer acceptance.
f. Evaluation of food texture by correlation to sensory data;
g. Analysis of rheological equations of state or constituent equations.
h. They are also a way to predict and control a host of product properties, end use performance and material behaviour as well as sensory analysis and quality control of foods.

Why Make Rheological Measurements?

Anyone beginning the process of learning to think “Rheologically” must first ask the question, “Why should I make a rheological measurement?” The answer lies in the experiences of thousands of people who have made such measurements, showing that, much useful behavioural and predictive information for various products can be obtained, as well as knowledge of the effects of processing, formulation changes, aging phenomena, etc. A frequent reason for the measurement of rheological properties can be found in the area of quality control, where raw materials must be consistent from batch to batch. For this purpose, flow behaviour is an indirect measure of product consistency and quality. Another reason for making flow behaviour studies is that a direct assessment of process-ability can be obtained. For example, a high viscosity liquid requires more power to pump than a low viscosity one. Knowing its rheological behaviour, therefore, is useful when designing pumping and piping systems. It has been suggested that rheology is the most sensitive method for material characterization because flow behaviour is responsive to properties such as molecular weight and molecular weight distribution. Rheological measurements are also useful in following the course of a chemical reaction. Such measurements can be employed as a quality check during production or to monitor and/or control a process. Rheological measurements allow the study of chemical, mechanical, and thermal treatments, the effects of additives, or the course of a curing reaction.

Thinking Rheologically

To begin, consider the question, “Can some rheological parameter be employed to correlate with an aspect of the product or process?” To determine this, an instinct must be developed for the kinds of chemical and physical phenomena which affect the rheological response. For the moment, assume this information is known and several possibilities have been identified. The next step is to gather preliminary rheological data to determine what type of flow behaviour is characteristic of the system under consideration. At the most basic level, this involves making measurements with whichever viscometer is available and drawing some conclusions based on the descriptions of flow behaviour types. Once the type of flow behaviour has been identified, more can be understood about the way components of the system interact.

THREE SCHOOLS OF THOUGHT ON RHEOLOGICAL MEASUREMENT

From experience, there are basically three schools of thought on the use of viscometers in rheological applications. They are presented here and you can then decide which you fall into, remembering that there is no “right” one and that each has its merits and demerits.

The Pragmatic School

This first school of thought is the most pragmatic. The person who adheres to this school cares only that the viscometer generates numbers that tell something useful about a product or process. This person has little or no concern about rheological theory and measurement parameters expressed in absolute terms. Quality control and plant production applications are typical of this category.

The Theoretical School

The second school of thought involves a more theoretical approach. Those adhering to this school know that some types of viscometers will not directly yield defined shear rates and absolute viscosities for non-Newtonian fluids. However, these people often find that they can develop correlations of “dial viscosity” with important product or process parameters. Many people follow this school of thought. The applications of rheology literature is replete with statements along the line of “I know the data isn’t academically defined, but I keep this fact in mind and treat the multi-point rheology information as if it were.” In many cases, this produces eminently satisfying results and eliminates the necessity of buying a highly sophisticated and very expensive piece of rheological equipment.

The Academic School

The third school of thought is quite academic in nature. People adhering to this school require that all measurement parameters particularly shear rate and shear stress, be defined and known. They need equipment with defined geometries such as cone and plate or coaxial cylinders. Examples from the Brookfield line would be the Wells-Brookfield Cone/ Plate and CAP Viscometers and the UL adapter, Small Sample Adapter, Thermosel, Din Adapter and Spiral Adapter accessories, as well as the R/S and PVS Rheometers. With this equipment the shear rate is defined and accurate absolute viscosities are obtained directly.

RHEOLOGICAL PARAMETERS

Viscosity

Simply put viscosity is the resistance to deformation and flow. It is the measure of the internal friction of a fluid. This friction becomes apparent when a layer of fluid is made to move in relation to another layer. Different fluids deform at different rates under the same shear stress. Fluid with a high viscosity such as syrup, deforms more slowly than fluid with a low viscosity such as water. The greater the friction, the greater the amount of force required to cause this movement, which is called "shear". Shearing occurs whenever the fluid is physically moved or distributed, as in pouring, spreading, spraying, mixing, etc. Highly viscous fluids, therefore, require more force to move than less viscous materials.

Isaac Newton defined viscosity by considering the model represented in the Figure 1. Two parallel planes of fluid of equal area "A" are separated by a distance "dx" and are moving in the same direction at different velocities "V_1" and "V_2." Newton assumed that the force required in maintaining this difference in speed was proportional to the difference in speed through the liquid, or the velocity gradient. To express this, Newton wrote:

$$F/A = \eta dv/dx \qquad 1$$

Where η is a constant for a given material and is called its "viscosity'. The velocity gradient, dv/dx, is a measure of the change in speed at which the intermediate layers move with respect to each other. It describes the shearing the liquid experiences and is thus called "shear rate (γ)." Its unit of measure is called the "reciprocal second"(s^{-1}). The term F/A indicates the force per unit area required to produce the shearing action. It is therefore referred to as "shear stress" (τ) its unit of measurement is "dynes per square centimeter" ($dynes/cm^2$).

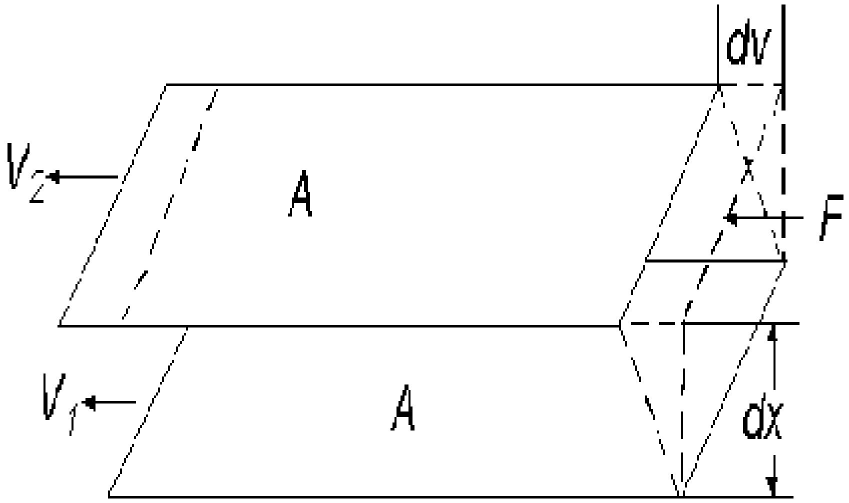


Figure 1: Isaac Newton’s model for viscosity

Using these simplified terms, viscosity may be defined mathematically by the formula:

$$\eta = \text{viscosity} = \frac{F'}{S} = \frac{\text{shear stress}}{\text{shear rate}} \tag{2}$$

The fundamental unit of viscosity measurement is the "poise”. A material requiring a shear stress of one dyne per square centimeter to produce a shear rate of one reciprocal second has a viscosity of one poise, or 100 centipoise. Viscosity measurements are also expressed in "Pascal-seconds" (Pa•s) or "milli-Pascal-seconds" (mPa•s); these are units of the International System (SI) and are sometimes used in preference to the Metric designations. One Pascal-second is equal to ten poise; one milli-Pascal-second is equal to one centipoise. Newton assumed that all materials have, at a given temperature, a viscosity that is independent of the shear rate. In other words, twice the force would move the fluid twice as fast. As we shall see later, Newton was only partly right.

Newtonian fluids

The type of flow behavior Newton assumed for all fluids is called "Newtonian." It is, however, only one of several types of flow behavior encountered in fluid flow. A Newtonian fluid is represented graphically in Figure 2. Graph A shows that the relationship between shear stress (F’) and shear rate (S) is a straight line. Graph B shows that the fluid’s viscosity remains constant as the shear rate is varied. Typical Newtonian fluids include water and thin motor oils.

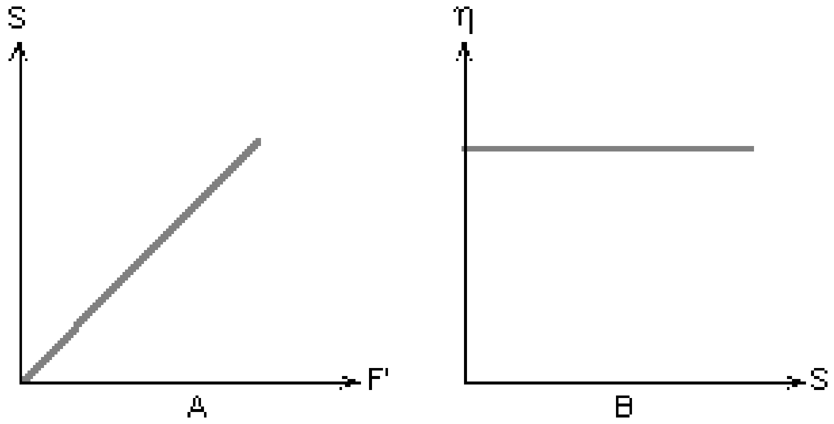


Figure 2: Graphical representation of Newtonian Fluids (A, shear stress/shear rate; B, viscosity/shear rate)

What this means in practice is that at a given temperature the viscosity of a Newtonian fluid will remain constant regardless of which viscometer model, spindle size or speed that is used to measure it. Newtonian fluids are obviously the easiest fluids to measure - just grab your viscometer and take readings. However, they are not as common as the much more complex group of fluids, the non-Newtonians.

Non-newtonian fluids

A non-Newtonian fluid is broadly defined as one for which the relationship shear stress/shear rate is not a constant. In other words, when the shear rate is varied, the shear stress does not vary in the same proportion (or even necessarily in the same direction). The viscosity of such fluids will therefore change as the shear rate is varied. Thus, the experimental parameters of viscometer model, spindle and speed all have an effect on the measured viscosity of a non-Newtonian fluid. This measured viscosity is called the "apparent viscosity" of the fluid and is accurate only when explicit experimental parameters are furnished and adhered to. Non-Newtonian flow can be envisioned by thinking of any fluid as a mixture of molecules with different shapes and sizes. As they pass by each other, (as happens during flow), their size, shape, and cohesiveness will determine how much force is required to move them. At each specific rate of shear, the alignment may be different and more or less force may be required to maintain motion. There are several types of non-Newtonian flow behavior, characterized by the way a fluid’s viscosity changes in response to variations in shear rate. The most common types of non-Newtonian fluids encountered in food process engineering include:

1. Psuedoplastic: This type of fluid will display a decreasing viscosity with an increasing shear rate, as shown in figure 3. Probably the most common of the non-Newtonian fluids, pseudo-plastics include starch paste, paints, emulsions, and dispersions of many types. This type of flow behavior is sometimes called "shear-thinning."

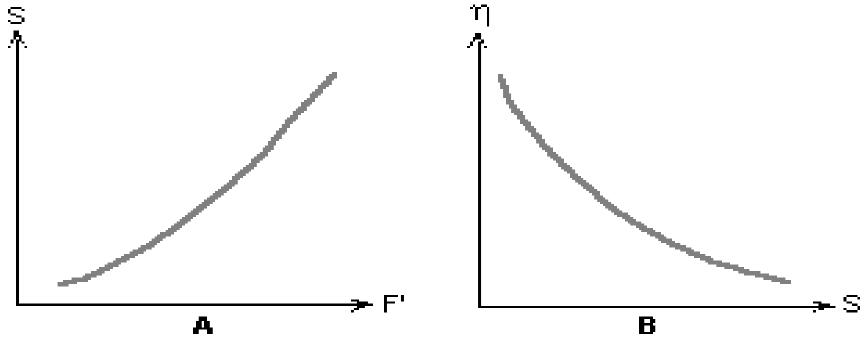


Figure 3: Graphical representation of Psuedoplastic Fluids

2. Dilatant: Increasing viscosity with an increase in shear rate characterizes the dilatant fluid; (Figure 4). Although rarer than pseudoplasticity, dilatancy is frequently observed in fluids containing high levels of deflocculated solids, such as candy compounds, corn starch in water, and sand/water mixtures. Dilatancy is also referred to as "shear-thickening" flow behavior.

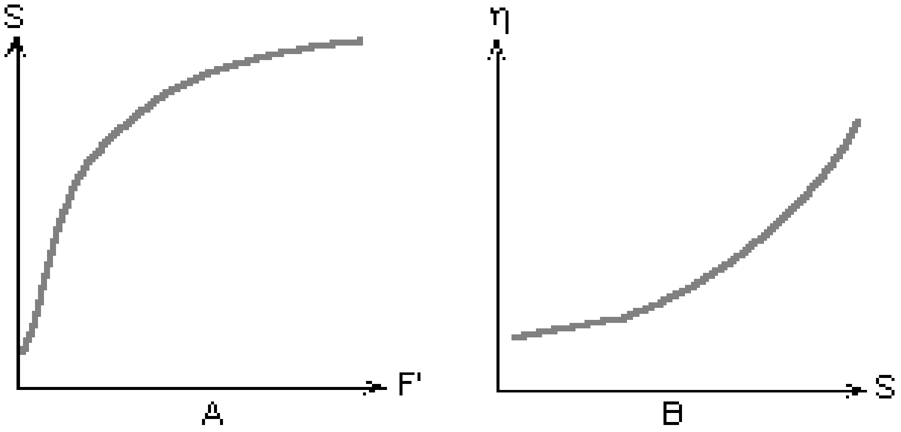


Figure 4: Graphical representation of Dilatant Fluids

3. Plastic: This type of fluid will behave as a solid under static conditions. A certain amount of force must be applied to the fluid before any flow is induced; this force is called the "yield value”. Tomato ketchup is a good example of this type of fluid; its yield value will often make it refuse to pour from the bottle until the bottle is shaken or struck, allowing the ketchup to gush freely. Once the yield value is exceeded and flow begins, plastic fluids may display Newtonian, pseudoplastic, or dilatant flow characteristics. This is depicted in Figure 5.

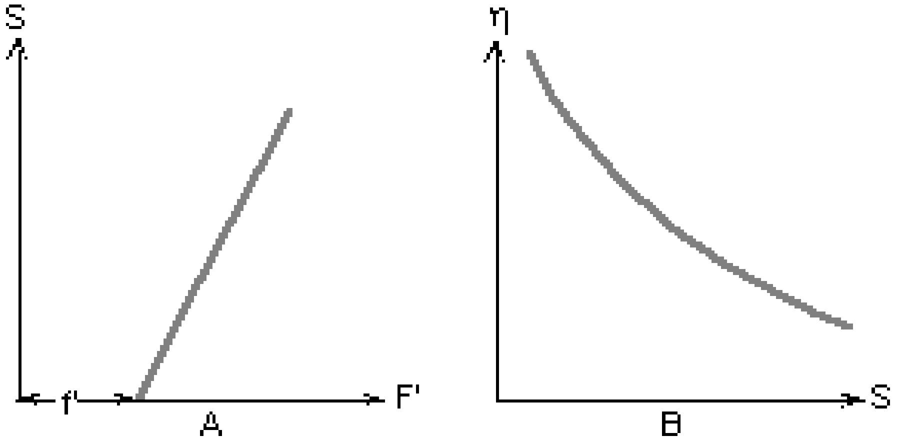


Figure 5: Graphical representation of Plastic Fluids

4. Thixotropy and rheopexy

Some fluids will display a change in viscosity with time under conditions of constant shear rate. There are two categories:

i. Thixotropy: As shown in the Figure 6(A), a thixotropic fluid undergoes a decrease in viscosity with time, while it is subjected to constant shear.

ii. Rheopexy: This is essentially the opposite of thixotropic behavior, in that the fluid's viscosity increases with time as it is sheared at a constant rate (figure 6(B)).

Both thixotropy and rheopexy may occur in combination with any of the previously discussed flow behaviors, or only at certain shear rates. The time element is extremely variable; under conditions of constant shear, some fluids will reach their final viscosity value in a few seconds, while others may take up to several days. Rheopectic fluids are rarely encountered. Thixotropy, however, is frequently observed in materials such as greases, heavy printing inks, and paints.

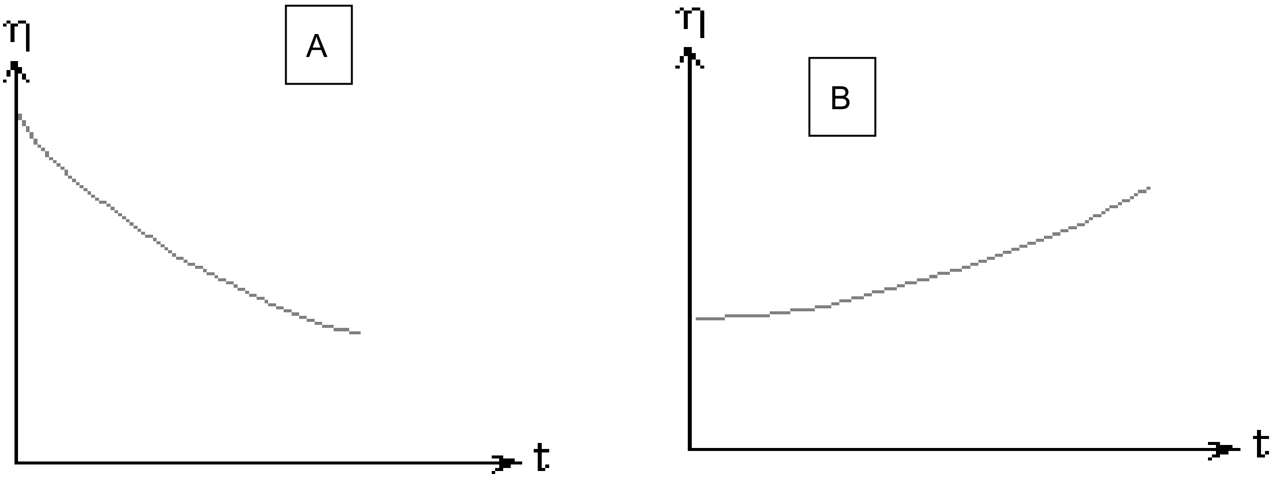


Figure 6: Graphical representation of (A) Thixotropic Fluids (B) Rheopexy fluids

Laminar and turbulent flow

The very definition of viscosity implies the existence of what is called "laminar flow": the movement of one layer of fluid past another with no transfer of matter from one to the other. Depending on a number of factors, there is a certain maximum speed at which one layer of fluid can move in relatiion to another, beyond which an actual transfer of mass occurs. This is called "turbulence." Molecules or larger particles jump from one layer to another and dissipate a substantial amount of energy in the process. The net result is that a larger energy input is required to maintain this turbulent flow than a laminar flow at the same velocity. The increased energy input is manifested as an apparently greater shear stress than would be observed under laminar flow conditions at the same shear rate. This results in an erroneously high viscosity reading. The point at which laminar flow evolves into turbulent flow depends on other factors besides the velocity at which the layers move. A material's viscosity and specific gravity as well as the geometry of the Viscometer spindle and sample container all influence the point at which this transition occurs. Care should be taken to distinguish between turbulent flow conditions and dilatant flow behaviour. In general, dilatant materials will show a steadily increasing viscosity with increasing shear rate; turbulent flow is characterized by a relatively sudden and substantial increase in viscosity above a certain shear rate. The material's flow behavior may be Newtonian or non-Newtonian below this point.

FACTORS AFFECTING RHEOLOGICAL PARAMETERS

Viscosity data often function as a "window" through which other characteristics of a material may be observed. Viscosity is more easily measured than some of the properties that affect it, making it a valuable tool for material characterization. Some of the factors that affects viscosity are highlighted in this section.

Temperature

One of the most obvious factors that can have an effect on the rheological behaviour of a material is temperature. Some materials are quite sensitive to temperature, and a relatively small variation will result in a significant change in viscosity. Others are relatively insensitive. Consideration of the effect of temperature on viscosity is essential in the evaluation of materials that will be subjected to temperature variations in use or processing, such as starch paste, vegetable oils, etc.

Shear rate

Non-Newtonian fluids tend to be the rule rather than the exception in the real world, making an appreciation of the effects of shear rate a necessity for anyone engaged in the practical application of rheological data. It would, for example, be disastrous to try to pump a dilatant fluid through a system, only to have it go solid inside the pump, bringing the whole process to an abrupt halt. While this is an extreme example, the importance of shear rate effects should not be underestimated. When a material is to be subjected to a variety of shear rates in processing or use, it is essential to know its viscosity at the projected shear rates. If these are not known, an estimate should be made. Viscosity measurements should then be made at shear rates as close as possible to the estimated values.

It is frequently impossible to approximate projected shear rate values during measurement due to those values falling outside the shear rate range of the Viscometer. In this case, it is necessary to make measurements at several shear rates and extrapolate the data to the projected values. This is not the most accurate method for acquiring this information, but it is often the only alternative available, especially when the projected shear rates are very high. In fact, it is always advisable to make viscosity measurements at several shear rates to detect rheological behavior that may have an effect on processing or use. Where shear rate values are unknown or not important, a simple plot of viscosity versus viscometer speeds (rpm) will often suffice. Examples of materials that are subjected to, and are affected by, wide variations in shear rate during processing and use are: certain food products, paints, cosmetics, liquid latex, coatings, and blood in the human circulatory system.

Measuring conditions

The condition of a material during measurement of its viscosity can have a considerable effect on the results of such measurement. It is therefore important to be aware of, and to control as much as possible, the environment of any sample you are testing. Variables such as viscometer model, spindle/speed combination, sample container size, absence or presence of the guard leg, sample temperature, sample preparation technique, etc., all affect not only the accuracy of viscometer measurements, but the actual viscosity of the material that is being measured.

Another factor which may affect viscosity measurements is the homogeneity of the sample. It is usually desirable to have a homogeneous sample so that more consistent results may be obtained. Sometimes, however, tendency of a material to separate into non-homogeneous layers is the characteristic of most interest. Care must be taken in such instances not to disturb that which you wish to study by mixing or shaking the sample.

Time

The time elapsed under conditions of shear obviously affects thixotropic and rheopectic (time- dependent) materials. But changes in the viscosity of many materials can occur over time even though the material is not being sheared. Aging phenomena must be considered when selecting and preparing samples for viscosity measurement. Consider also the fact that many materials will undergo changes in viscosity during the process of a chemical reaction, so that a viscosity measurement made at one time in the reaction may differ significantly from one made at another time.

Pressure

Variations in pressure may cause dissolved gases to form bubbles; entrained gases to change size as well as distribution, and in some cases, turbulence. Pressure is not experienced as often as other parameters. Pressure compresses fluids, and thus, increases intermolecular resistance. Liquids are compressible under the influence of very high pressures - similar to gases but to a much lesser extent. Increases of pressure tend to increase the viscosity. As an example: The flow properties of highly concentrated slurries (above 70-80% by volume of particles) where there is insufficient liquid to fill completely all the voids between the particles results in a three-phase mixture (i.e. solids, liquids, and usually air). Due to the presence of air, the mixture is compressible, and therefore, the more you compress it, the greater the resistance to flow.

Previous history

What has happened to a sample prior to a viscosity measurement can significantly affect the result, especially in fluids sensitive to heat or aging. Thus, storage conditions and sample preparation techniques must be designed to minimize their effect on subsequent viscosity tests. Thixotropic materials in particular are sensitive to prior history, as their viscosity will be affected by stirring, mixing, pouring, or any other activity which produces shear in the sample.

Composition and additives

The composition of a material is a determining factor of its viscosity. When this composition is altered, either by changing the proportions of the component substances, or by the addition of other materials, a change in viscosity is quite likely.

RHEOLOGICAL INSTRUMENTS FOR FLUIDS

Common instruments, capable of measuring fundamental rheological properties of fluid and semi-solid foods, may be placed into two general categories (Figure 7): rotational type and tube type. Most are commercially available. Costs vary tremendously from the inexpensive glass capillary viscometer to a very expensive rotational instrument capable of measuring dynamic properties and normal stress differences. Solid foods are often tested in compression (between parallel plates), tension, or torsion. Instruments which measure rheological properties are called rheometers. Viscometer is a more limiting term referring to devices that only measure viscosity. Rotational instruments may be operated in the steady shear (constant angular velocity) or oscillatory (dynamic) mode.

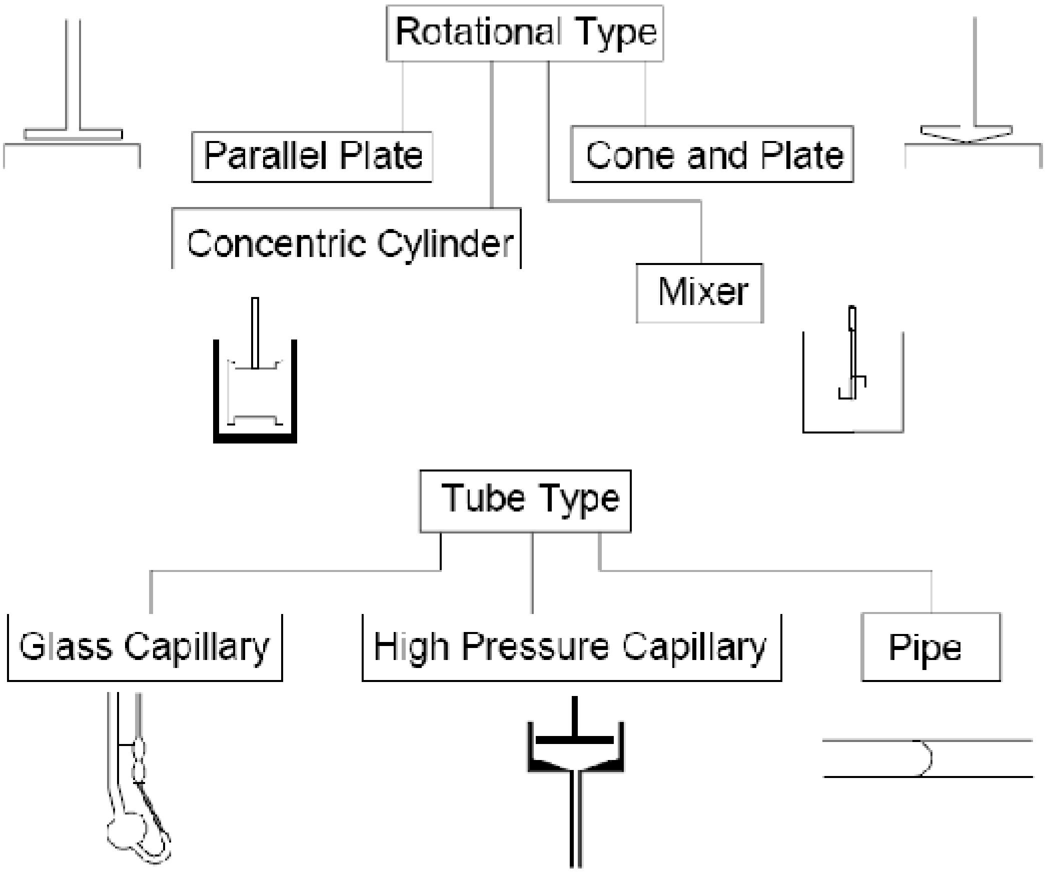


Figure 7: Common rheological instruments divided into two major categories: rotational and tube type.

Some rotational instruments function in the controlled stress mode facilitating the analysis of materials at very low shear rates, and the investigation of yield stresses. This information is needed to understand the internal structure of materials. The controlled rate mode is most useful in obtaining data required in process engineering calculations. Rotational systems are generally used to investigate time-dependent behavior because tube systems only allow one pass of the material through the apparatus.

There are advantages and disadvantages associated with each instrument. For instance, gravity operated glass capillaries, are only suitable for Newtonian fluids because the shear rate varies during discharge. Cone and plate systems are limited to moderate shear rates but calculations (for small cone angles) are simple. Pipe and mixer viscometers can handle much larger particles than cone and plate, or parallel plate, devices. Problems associated with slip and degradation in structurally sensitive materials are minimized with mixer viscometers. High pressure capillaries operate at high shear rates but generally involve a significant end pressure correction.

VISCOMENTRY

Tube Viscometry

Tube viscometers are very useful in collecting rheological data. These instruments may be placed into three basic categories: glass capillaries (Figure 8), often called U-tube viscometers because of their resemblance to the letter U; high pressure capillaries (Figure 9); and pipe viscometers (Figure 10). All establish a pressure difference to create flow. The major difference between a capillary and a pipe viscometer is the diameter of the tube. Although there is no clearly defined size at which a tube should be called a capillary or a pipe, some guidelines can be offered. Diameters in commercial capillary instruments typically range from 0.1 to 4 mm with a variation in entrance angles of 15 to 90 degrees. Pipe viscometers are usually built "on-site" so size varies widely. Some may be as small as 7mm in diameter but values greater than 12 mm (typically 12 to 32mm) are not uncommon in food applications.

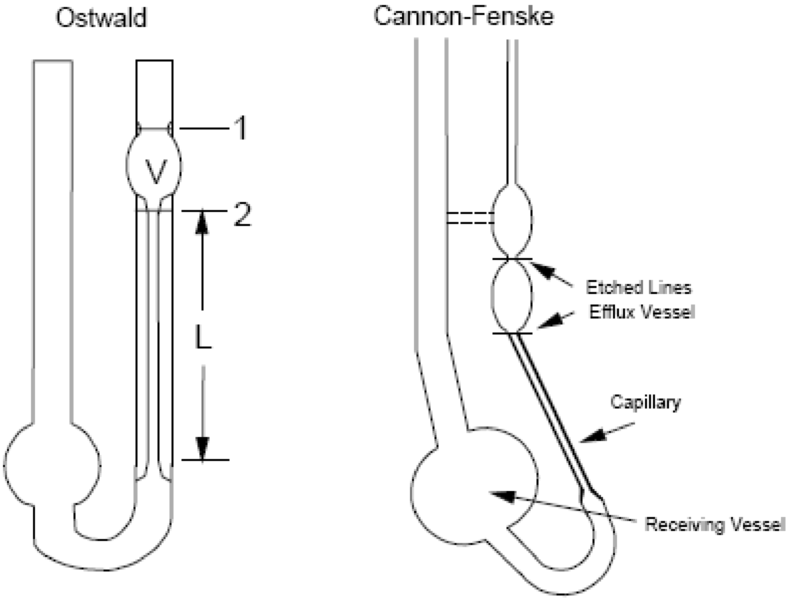


Figure 8: Ostwald and Canon-Fenske glass capillary (U-tube) viscometers

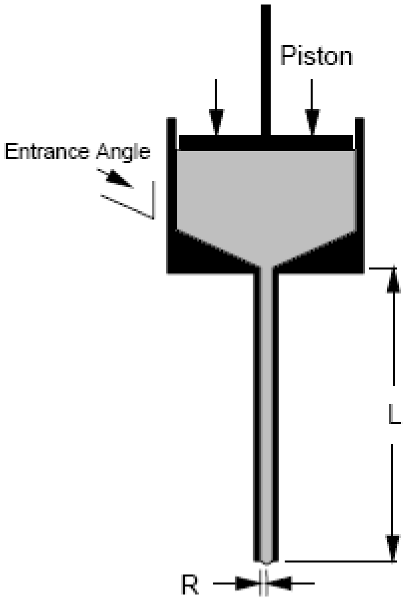


Figure 9: High pressure capillary viscometer.

In typical operation, the U-tube viscometer is filled by inverting it into a sample and sucking (into the side with the capillary) fluid into the fixed sample bulb. The viscometer is turned upright, then placed in a temperature controlled bath and allowed to reach thermal equilibrium. After a certain period of time, usually 5 to 10 minutes, the fluid is allowed to flow down through the capillary. A stopwatch is started when fluid passes the upper etched line and stopped when the fluid surface passes the lower etched line. The resulting time is considered the efflux time for fluid discharge from the bulb and fluid viscosity is calculated from this value. Raw data for tube viscometers are pressure drop and volumetric flow rate. The pressure drop is determined from pressure transducers or, in the case of U-tube viscometers, the height of fluid above a reference point. In high pressure capillaries, flow rates are calculated from the assumption that volumetric flow in the piston (or barrel) and the capillary are equivalent. Volumetric flow rate may be determined from the mass flow rate measured in pipe systems using a mass flow meter or a weight tank. Density is required for this calculation because the volumetric flow rate equals the mass flow rate divided by the density.

Rotational Viscometry

Rotational viscometers have proven to be very useful in evaluating time-dependent fluid behaviour because (unlike tube viscometers) they easily allow materials to be subjected to alternate periods of shear and rest. Traditional rotational viscometers include cone and plate, parallel plate, and concentric cylinder units operated under steady shear conditions. They may also be capable of operating in an oscillatory mode. Cone and plate systems are sometimes capable of determining normal stress differences. Concentric cylinder systems have been used in research to evaluate these differences; however, commercial instruments of this type are not available.

VISCOMETERS FOR PROCESS CONTROL AND MONITORING

The goal of this section is to provide an overview of the primary measurement concepts and issues involved in the use of viscometers for continuous process control or monitoring. All process viscometers used for food products must conform to appropriate sanitary standards and accepted practices such as the 3-A and Egg 3-A standards published by the International Association of Milk, Food and Environmental Sanitarians. Viscometers discussed here are typical of industrially available units which are generally acceptable for use in the food industry.

In-line or On-line Installation. Viscometers for process control or monitoring may be installed using various measurement schemes. "In-line" systems are installed directly in the process line, usually in a pipe. These systems are subject to process variations, such as changes in sample temperature, which may significantly influence sensor output. "On-line" units make measurements on a product side stream, also called a by-pass loop, taken from the main process flow line. One advantage of this type of system is that sample variables (including flow rate, temperature, and pressure) may be controlled during testing. A third type of process control viscometer is the immersion system designed for use in process vessels, particularly mixing tanks. The above units provide alternatives to "off-line" measurements where a small sample is removed from the process line and evaluated in a standard laboratory instrument.

Practical Considerations in Selecting a Process Control Viscometer.

It is important to have a good understanding of the fluid under consideration before selecting a process control viscometer. The influence of temperature, ingredient formulation, and processing conditions on flow behaviour must be ascertained before quality control or set-points can be accurately established. Temperature has such a strong influence on rheological behaviour, hence, it is usually necessary to either carefully control it when conducting measurements or compensate for it when making final calculations. Process control viscometers are generally designed for Newtonian fluids where the viscosity is not a function of shear rate. To evaluate non-Newtonian fluids, multiple data points, taken at different shear rates are required. Hence, operational shear rates must be established and matched to the capabilities of the process control viscometer. Process control viscometers may provide a strictly empirical parameter, an apparent viscosity, or a flow curve if samples are deformed at multiple shear rates. This information must be correlated to specific processing factors, such as the amount of cocoa butter in chocolate or the amount of water in tomato paste, before a control scheme can be initiated. If an on-line instrument is used for quality control, then quality must be carefully defined and directly related to the rheological property being measured. Also, the acceptable variation in quality must be known to establish the proper limits (or set-points) required in developing a control strategy. All process control viscometers must be calibrated regularly and carefully observed to ensure satisfactory long term performance.

Empirical Measurement Methods for Foods

The food industry uses many empirical instruments (Table 1) to measure the flow behavior of food products. These devices are not used to determine fundamental rheological properties, but results may find diverse applications: quality control, correlation to sensory data, or even serve as official standards of identity. Food engineers may find it necessary to replace empirical devices, like the Bostwick Consistometer used for pureed foods, with more fundamental instruments to achieve engineering objectives related to process control. All the instruments discussed in this section are used for food products. It is important to recognize the fact that numerous foods are so complex; it is not practical, and in many cases not possible, to measure their fundamental rheological properties. Carrots, peanuts, peas, or beans (for example) are non-homogeneous, nonisotropic materials with complex geometries. Only empirical testing devices, capable of measuring composite material behavior, provide a suitable means of characterizing these foods. Empirical instruments are a valuable and well established part of the food industry. Since they do not measure fundamental properties, they may appropriately be called indexers.

Dough Testing Equipment (Farinograph, Mixograph, Extensigraph, Alveograph).

Dough, a combination of cereal (usually wheat) flour, water, yeast, salt, and other ingredients, is probably the most complex material facing the food rheologist; hence, it is not surprising that many empirical instruments have been developed to evaluate the flow behaviour of dough. The instruments may be divided into two major groups: those which measure the power input during dough development caused by a mixing action, and those which subject prepared (developed) dough to an extensional deformation. The following discussion will describe the major instruments found in each group.

One of the most widely used dough mixers is the **Farinograph.** This instrument combines dough ingredients using two Z-shaped mixing blades that rotate, at different speeds, in opposite directions. Mixing is initiated with dry flour and water is added from a titrating buret during testing. A dynamometer is used to record torque on the drive shaft of the mixing blades. Output is given as a farinogram: a plot of an instrument-dependent parameter proportional to torque, expressed as a Brabender unit (BU, also called consistency), versus time. The shape of the farinogram is interpreted in terms of factors related to flour quality and the behaviour of the dough in the bakery: dough development time, stability, mixing tolerance, and degree of softening. The amount of water required to give a consistency of 500 BU to a14% moisture content (wet basis) flour is also an important flour parameter, known as the farinograph water adsorption, determined using the Farinograph.

Table 1: Typical Empirical Testing Instruments and Applications for Food Products

Device	Common Application
Adams Consistometer	consistency of semi fluid food purees
Armour Tenderometer	beef tenderness
Baker Compressimeter	staleness of bread
Ballauf Pressure Tester	puncture testing of fruit and vegetables
BBIRA Biscuit Texture Meter	hardness of cookies and crackers
Bloom Gelometer	puncture test of gelatins and gelatin jellies
Bostwick Consistometer	flow of baby foods and similar purees
Chatillon Pressure Tester	puncture testing of fruit and vegetables
Effi-Gi Pressure Tester	puncture testing of fruit and vegetables
Extensigraph	behaviour of wheat dough
Farinograph	baking quality of wheat flour
FMC Pea Tenderometer	quality and maturity of fresh green peas
FTC Texture Test System	attachments for many foods
GF Texturometer	attachments for many foods
Haugh Meter	egg quality
Hilker-Guthrie Plummet	firmness of cultured cream
Instron Universal Testing Machine	attachments for many foods
Kramer Shear Press	tenderness of peas and other particulate foods
Magness-Taylor Pressure Tester	puncture testing of fruit and vegetables
Marine Colloids Gel Tester	puncture test marine extract gels
Mixograph	baking quality of wheat flour
Ottawa Pea Tenderometer	quality and maturity of fresh green peas
Ottawa Texture Measuring System	attachments for many foods
Pabst Texture Tester	firmness of particulate foods
Penetrometer	firmness of butter and margarine
Plint Cheese Curd Torsiometer	setting of cheese curd
Resistograph	baking quality of wheat flour
Ridgelimiter	stiffness of pectin and fruit jellies
Stevens Compression Response Analyzer	attachments for many foods
Succulometer	maturity and quality of fresh sweet corn
SURDD Hardness Tester	hardness of fats and waxes
Torry Brown Homogenizer	toughness of fish
USDA Consistometer	consistency of semifluid food purees
Van Dorran Pressure Tester	puncture testing of butter
Warner-Bratzler Shear	toughness of meat

An alternative to the Farinograph is the **Mixograph** which involves a planetary rotation of vertical pins (lowered into the dough) about stationary vertical pins attached to the mixing bowl. Torque is recorded while mixing a fixed amount of flour and water. Results are given in terms of a mixogram which is interpreted in a manner analogous to that discussed for the farinogram.

The **Extensigraph** generally conducts tests on doughs prepared in the Farinograph. A special molding device shapes the dough into a cylindrical specimen which is placed horizontally into a support system. The ends are clamped firmly in place leaving the middle section free for testing. A hook contacts the middle of the sample and stretches it while moving downward at a constant speed. The force on the sample, caused by the downward motion of the hook, is recorded. Results are given as an extensogram, a plot of force (in Brabender units) versus time and extension, which provides valuable quality control information for the dough. It is important to note that materials with similar farinograms may have very different extensograms. The effects of oxidizing agents and enzymes on dough behaviour, for example, can often be evaluated with extensograms.

The **Alveograph**, also called the Chopin Extensigraph, measures dough behaviour when subjecting it to an extensional deformation. In this instrument, a circular disk is cut from a sheet of dough and clamped, around its circumference, to the base plate of the test apparatus. Air flowing through the base plate causes the dough to expand into a spherically shaped bubble which eventually ruptures, completing the test. The air pressure in the bubble over time is recorded and plotted as an alveogram. In routine testing, the maximum height, overall length, and the area under the curve are the primary parameters taken off the alveogram.

Cone Penetrometer

Stiff materials --like butter, peanut butter, or margarine -- are often assessed for "spreadability" using cone penetration data. This instrument consists of a weighted cone that is positioned vertically over the flat surface of the test sample. Cone angles of 20 or 45 degrees are typical. In standard testing, the cone is released into the sample and the depth of penetration, after a fixed period of time, is measured. Results may be presented in terms of a yield value which is directly proportional to the weight of the cone assembly and inversely proportional to the depth of penetration. Operating cone penetrometers with a constant downward speed, instead of a constant weight, is also an effective method of obtaining experimental data.

Visco-Amylograph

The Visco-Amylograph was designed to evaluate the behavior of starch solutions during gelatinization. It consists of a rotating bowl with eight vertical pins and a matching, suspended element, with seven vertical pins. Torque is recorded on the upper element during rotation of the bowl. The system includes a thermoregulator which allows the sample to be heated (the standard rate is 1.5°C per minute) during testing. When an aqueous suspension of starch is heated above the gelatinization temperature, the fluid thickens dramatically. A complete amylograph test usually involves four distinct thermal periods while the bowl is rotated at a constant speed: heating, holding, cooling, and holding. Results are presented as an amylogram which is a plot of torque (given as viscosity in Brabender units) versus time. Amylograms have proven useful in evaluating the quality of starch and its behavior as a thickening agent in many food systems.

Rapid Visco Analyser

This instrument generates data similar to that provided by the Visco-Amylograph. Small samples, typically 3 to 4 grams of starch in water, are heated in a small mixing vessel with a pitched paddle impeller. Samples are subjected to user program changes in temperature (heating, holding, and cooling) intended to match processing conditions found in a particular application. Torque, or instrument viscosity, are measured over time while the sample is agitated and programmed temperature changes are executed. This instrument, originally intended to evaluate the quality of Australian wheat, may be used to examine the quality of a wide variety of food starches.

Brookfield Rotating Disks and T-Bars

Brookfield Engineering Laboratories (Stoughton, MA) manufactures a rotational instrument (called the Brookfield Viscometer) that is extensively used in the food industry. The most common sensors are flat disks (spindles) attached to the instrument with a vertical shaft. Disks are available in various sizes and may be rotated at different speeds. Torque required to maintain constant rotation is measured. Disk sensors can be very useful in obtaining a relative index of food thickness for the purpose of comparing products or making quality control judgments. In addition to disks, T-shaped bars are made for the same purpose. Instruments, equipped with the T-bars, can also be attached to the Brookfield Helipath Stand which allows the entire instrument to be lowered during testing. This causes the sensor to take a spiral path through the sample while torque data are obtained. The Helipath Stand is typically used for thick pastes and gels where a rotating disk would be difficult to insert or create a channeling effect during measurement.

Falling Ball Viscometer

This type of viscometer involves a vertical tube where a ball is allowed to fall, under the influence of gravity, through a Newtonian fluid. Viscosity is calculated on the basis of the time taken to fall a fixed distance. If the vessel diameter is 10 times the ball diameter, wall effects can be neglected. The rising bubble viscometer represents another application of the falling ball concept. In this case, a bubble of air is allowed to ascend through a column of sample. Rising time over a set distance is correlated to Newtonian viscosity.

Brabender-FMC Consistometer

This unit was originally designed to evaluate cream style corn but it has also been used for ketchup, tomato paste, baby food, and similar products. It is designed to lower a thin, rectangular shaped, paddle into a sample held in a stainless steel cup. The cup is rotated at a single speed of 78 rpm. This motion creates a torque on the paddle that is read from a dial located on the top of the instrument. Paddles are available in various sizes: 5.08 cm (2 inch) by 3.56 cm (1.4 inch) is typical.

Simple Compression

Biological materials may be evaluated in terms of a bioyield point and a rupture point. A curve such as the one illustrated in Figure 8 is typical for solid foods, like fruits and vegetables, when a cylindrical sample is tested in simple compression. The initial portion of the curve (a-b) is a straight line up to the linear limit (b). Young's modulus may be calculated from the stress and strain at that point. The slope of the initial portion of the curve is often taken as an index of firmness. When stress and strain cannot be calculated, data may be simply plotted in terms of force and deformation. The bioyield point (c) is related to a failure in the microstructure of the material associated with an initial disruption of cellular structure. The rupture point (d) of the material, defined by and, correlates to the macroscopic failure in the sample. With more brittle materials the rupture point may be very close to the bioyield point: These points may be widely separated in tough materials.

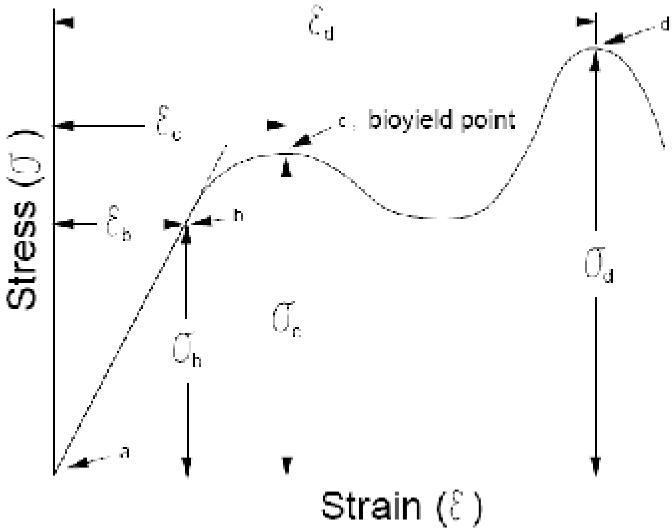


Figure 10: Generalized compression curve for a biological solid

TEXTURE PROFILE ANALYSIS (TPA)

Texture refers to the human sensation of food derived from its rheological behavior during mastication and swallowing. Obtaining a quantitative description of texture using instrumental data is very complicated because no instrument can duplicate human capabilities. From an engineering perspective, the mouth can be considered an intricate mechanical system and chemical reactor that can crush, wet, enzymatically degrade, pressurize, heat or cool, pump, chemically sample for taste, and sense force and temperature. In addition, this "eating machine" has a sophisticated feedback control system. Initially there is open loop, feed forward control to set primary parameters: size of mouth opening, surface election for first bite (incisors or molars), etc. Once the food is in the mouth, there is an adaptive feed back control system with a variable gain -high with unfamiliar foods, low with everyday foods- that depends on bolus development during mastication. This process is influenced by many factors: volume of the oral cavity, rate of addition of saliva, chemical composition of saliva, chemical and physical interaction of saliva and the food, rate of chewing, total number of chews, surface area in contact with the food, movement of the lips and cheeks, dynamic volume of the oral cavity during mastication, residence time of the bolus, initial volume of the bolus, and partial fluid removal (by swallowing) during chewing. Given the above, it is not surprising that little progress has been made in correlating fundamental rheological properties to the human perception of texture. There has been limited success with some fluid and solid foods. Overall, there are two methods to evaluate food texture: sensory and instrumental. The sensory method of developing a texture profile utilizes a human taste panel and provides the ultimate test which, as discussed above, cannot be completely duplicated by any instrumental procedure. Instrumental methods, however, are much less costly and time consuming than sensory tests. Moreover, they often correlate to critical sensory attributes which allow some measure of consumer acceptability. It is, however, rare for them to stand alone as a complete test. In any event, they can certainly be very valuable when used in conjunction with sensory panels. Generating and interpreting texture profile information, with instrumental or sensory means, is called Texture Profile Analysis.

MODELING RHEOLOGICAL BEHAVIOR OF FLUIDS

Modeling provides a means of representing a large quantity of rheological data in terms of a simple mathematical expression. Many forms of the equations are possible and one master model, suitable for all situations, does not exist. The equations covered here are acceptable for a large number of practical problems involving homogeneous materials which do not experience a phase change over the range of conditions under consideration. The analysis of viscometer data may be enhanced through the use of mathematical models. Various models for approximation of rheological data have been presented. These models are:

The power law (de Waele, 1923)

$$\sigma = K(\dot{\gamma})^n \qquad 3$$

Linearizing equation 3 we have:

$$\ln \sigma = \ln K + n \ln \dot{\gamma} \qquad 4$$

A plot of ln σ against ln $\dot{\gamma}$ gives a straight line graph with slope = n and intercept = ln K. The consistency index (K) is an indication of the viscous nature of the food.

The Herschel–Bulkley model

$$\sigma = \sigma_0 + K(\dot{\gamma})^n \qquad 5$$

and

The Casson model

$$\sigma^{\frac{1}{2}} = \sigma_0^{\frac{1}{2}} + (\eta_c \dot{\gamma})^{\frac{1}{2}} \tag{6}$$

The apparent viscosities for power-law and Herschel-Bulkley model fluids are given by equations (7) and (8), respectively:

$$\eta = K(\dot{\gamma})^{n-1} \tag{7}$$

$$\eta = \sigma_0(\dot{\gamma})^{-1} + K(\dot{\gamma})^{n-1} \tag{8}$$

Where σ= shear stress (Pa), $\dot{\gamma}$ = shear rate (s^{-1}), η = apparent viscosity (Pa.s),
K = consistency factor, σ_0 = yield stress on the material and n is the flow behaviour index.

There is some clear similarity between these models and indeed the Herschel–Bulkley model reduces to the power law if the yield $n = \frac{\sigma}{\dot{\gamma}}$ stress is zero. One drawback with these models comes with their prediction of apparent viscosity. For low values of shear rate, these all give large values for η. For this reason, the Cross model has been proposed

$$\sigma = \dot{\gamma}\left[\eta_\infty + \frac{\eta_0 - \eta_\infty}{1 + K\dot{\gamma}^n}\right] \tag{9}$$

Where n_o = viscosity at zero shear, n_∞ = viscosity at infinite shear, and K and n are constants similar to those for the power law and Herschel–Bulkley.

The models should be utilized as a tool to better understand and interpret viscometer data. The utilization of rheological models normally requires viscosity data collection under defined conditions of shear rate and shear stress.

Predicting the influence of temperature and concentration upon rheological properties

It is well-known that temperature can dramatically influence the viscosity of a fluid. This makes it imperative to be able to predict rheological properties as a function of temperature in order to control a process or product properties. Knowledge of rheological properties of foods and their variation with temperature and concentration have been globally important for industrialization of food technology for quality, understanding the texture, process engineering application, correlation with sensory evaluation, designing of transport system, equipment design (heat exchanger and evaporator), deciding pump capacity and power requirement for mixing. Fluid foods are subjected to different temperature and concentration during processing, storage, transportation, marketing, consumption. The effect of temperature on the apparent viscosity or consistency coefficient at a specific shear rate is generally expressed by the Arrhenius-type equation:

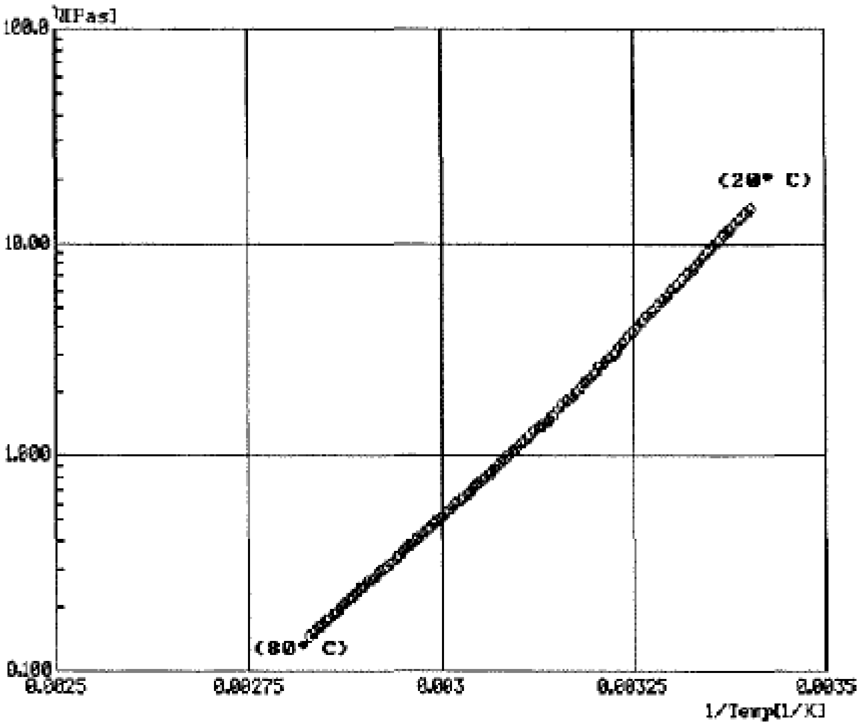
$$\eta = A \exp (E/RT) \qquad 10$$

Where, η is the viscosity at a particular shear rate, E is the Activation energy of flow (kJ/mol.K), R is the Universal gas constant (8.314 kJ/mol.K), T is the Absolute temperature (K), and A is a fitting parameter (Constant)

In order to linearize this expression, the logarithm of both sides should be taken:

$$\ln \eta = E/RT + \ln A \qquad 11$$

If In η is plotted as a function of 1/T, where the temperature is expressed in degrees Kelvin (K); the slope will equal E/R and the y-intercept InA. An example of this procedure is plotted in the graph below:



It is interesting to note that this temperature sweep was carried out from 20° C to 80°C and the viscosity dropped by two orders of magnitude. This is a striking example of how sensitive a fluid’s viscosity can be relative to the system temperature.

The effect of concentration on apparent viscosity or consistency is generally described by an exponential or a power type of relationship.

$$K = {}_{a}X^{b} \qquad 12$$

Where a and b are constant, X is concentration.

FLUIDS

There are two aspects of fluid mechanics which make it different to solid mechanics:

1. The nature of a fluid is much different to that of a solid.
2. In fluids we usually deal with *continuous* streams of fluid without a beginning or end. In solids we only consider individual elements.

We normally recognise three states of matter: solid; liquid and gas. However, liquid and gas are both fluids: in contrast to solids they lack the ability to resist deformation. Because a fluid cannot resist the deformation force, it moves, it *flows* under the action of the force. Its shape will change continuously as long as the force is applied. A solid can resist a deformation force while at rest, this force may cause some displacement but the solid does not continue to move indefinitely. A Fluid is a substance which deforms continuously, or flows, when subjected to shearing forces and conversely this definition implies the very important point that: If a fluid is at rest there are no shearing forces acting.

Ideal Fluid

This is a fluid which is assumed to have no viscosity, which is not real, it does not exist. This is a useful concept when theoretical solutions are being considered - it does help achieve some practically useful solutions.

Properties of Fluids

The properties outlines below are general properties of fluids which are of interest in food process engineering.

Density

The density of a substance is the quantity of matter contained in a unit volume of the substance. It can be expressed in three different ways.

a. **Mass Density:** Mass Density, ρ is defined as the mass of substance per unit volume.

Units: Kilograms per cubic metre, kg/m^3 (or kgm^{-3})

Typical values: Water = 1000 kgm^{-3} , Mercury = 13546 kgm^{-3} Air = 1.23 kgm^{-3} , Paraffin Oil = 800 kgm^{-3} (at pressure =1.013 x 10^{-5} Nm-^{2} and Temperature = 288.15 K).

b. **Specific Weight:** Specific Weight ω, (sometimes ý, and sometimes known as *specific gravity*) is defined as the weight per unit volume or the force exerted by gravity, g, upon a unit volume of the substance.

$$\omega = \rho g$$

13

Units: Newton’s per cubic metre, Nm-^{3})

Typical values:

Water =9814 Nm-^{3} , Mercury = 132943 Nm-^{3} , Air =12.07 Nm-^{3} , Paraffin Oil =7851 Nm-^{3}

c. **Relative Density:** Relative Density, σ is defined as the ratio of mass density of a substance to some standard mass density. For solids and liquids this standard mass density is the maximum mass density for water (which occurs at 4°C) at atmospheric pressure.

$$\sigma = \frac{\sigma_{substance}}{\sigma_{H_2O(at\,4^\circ c)}} \tag{14}$$

Units: None, since a ratio is a pure number.
Typical values: Water = 1, Mercury = 13.5, Paraffin Oil =0.8.

d. **Viscosity:** Viscosity, μ, is the property of a fluid, due to cohesion and interaction between molecules, which offers resistance to sheer deformation. Different fluids deform at different rates under the same shear stress. Fluid with a high viscosity such as syrup, deforms more slowly than fluid with a low viscosity such as water. All fluids are viscous, "Newtonian Fluids" obey the linear relationship given by Newton's law of viscosity,

$$\tau = \mu \frac{du}{dy}, \tag{15}$$

Units Nm^{-2} ; kg $m^{-1}s^{-2}$

FLUID FLOW

The measurement of flow rate in a liquid transport system is an essential component of the operation. Knowledge of flow rate and/or liquid velocity is important in design calculations. In addition, periodic measurements during actual operations are required to ensure that system components are performing in an expected manner. Consider the flow in a pipe in which water is flowing. At the pipe wall the velocity of the water will be zero. The velocity will increase as we move toward the centre of the pipe. This change in velocity across the direction of flow is known as velocity profile and shown graphically in the figure (11) below:

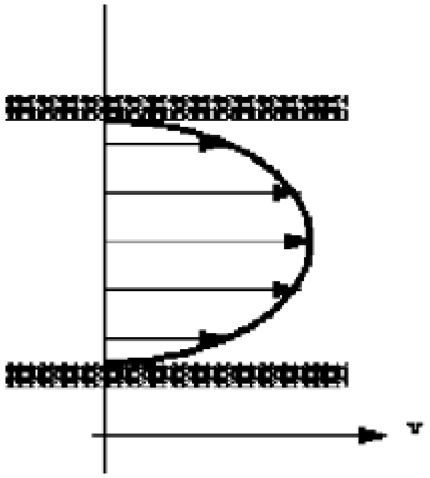


Figure 11: Velocity profile in a pipe

Because particles of fluid next to each other are moving with different velocities there **are** shear forces in the moving fluid i.e. shear forces are **normally** present in a moving fluid. On the other hand, if a fluid is a long way from the boundary and all the particles are travelling with the same velocity, the velocity profile would look something like this:

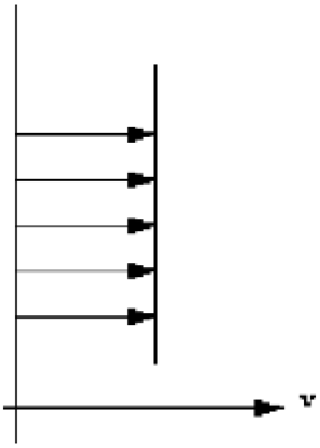
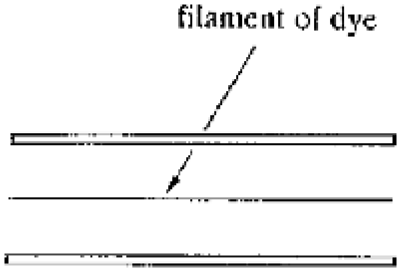


Figure 12: Velocity profile in uniform flow

LAMINAR AND TURBULENT FLOW

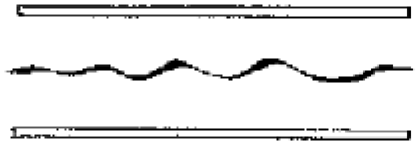
If we were to take a pipe of free flowing water and inject a dye into the middle of the stream, what would we expect to happen?

This



Laminar (viscous)

this

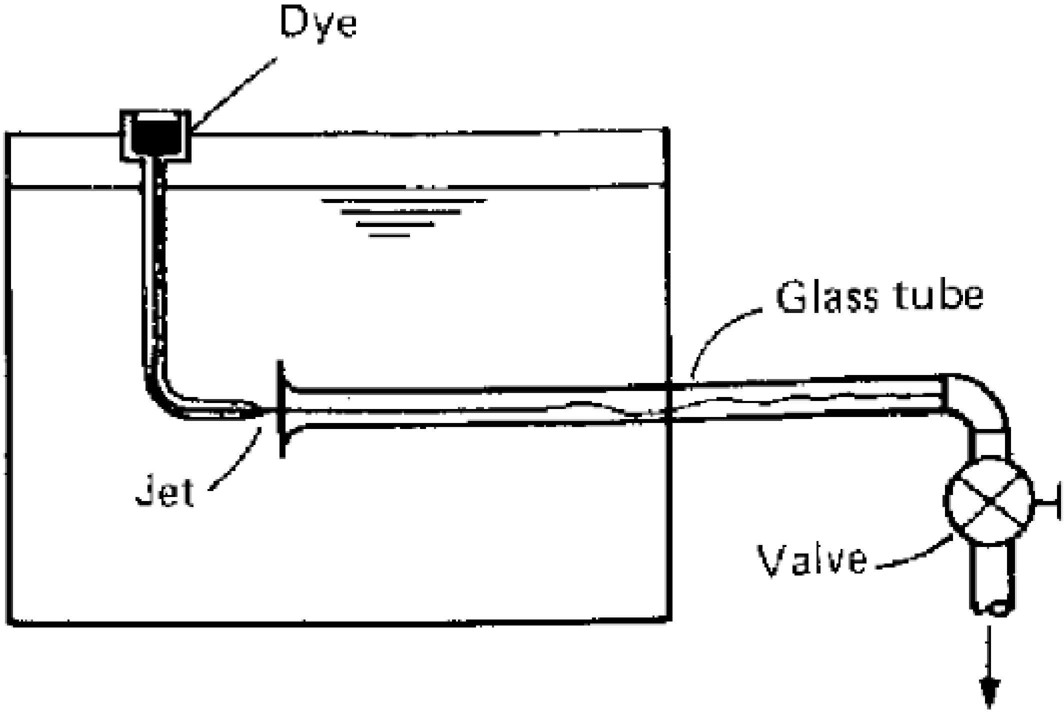


Transitional

or this

Turbulent

Actually both would happen - but for different flow rates. The top occurs when the fluid is flowing fast and the lower when it is flowing slowly. The top situation is known as **laminar** flow and the lower as **turbulent** flow. In laminar flow the motion of the particles of fluid is very orderly with all particles moving in straight lines parallel to the pipe walls. The phenomenon was first investigated in the 1880s by Osbourne Reynolds in an experiment which has become a classic in fluid mechanics.



He used a tank arranged as above with a pipe taking water from the centre into which he injected a dye through a needle. After many experiments he saw that this expression

$$\frac{\rho u d}{\mu}$$

(where ρ = density, *u* = mean velocity, *d* = diameter and μ = viscosity) would help predict the change in flow type. If the value is less than about 2000 then flow is laminar, if greater than 4000 then turbulent and in between these then the flow is in the transition zone.

Laminar flow: Re < 2000
Transitional flow: 2000 < Re < 4000
Turbulent flow: Re > 4000

What are the units of this Reynolds number? We can fill in the equation with SI units:

$$\rho = kg/m^3, \quad u = m/s, \qquad d = m$$
$$\mu = Ns/m^2 = kg/ms$$

$$\mathrm{Re} = \frac{\rho u d}{\mu} = \frac{kg}{m^3}\frac{m}{s}\frac{m}{1}\frac{m}{kg} = 1$$

i.e. it has **no units**. A quantity that has no units is known as a **non-dimensional** (or dimensionless) quantity. Thus the Reynolds number, Re, is a non-dimensional number.

FLUID DYNAMICS

This section discusses the analysis of fluid in motion - fluid dynamics. The motion of fluids can be predicted in the same way as the motion of solids are predicted using the fundamental laws of physics together with the physical properties of the fluid. It is not difficult to envisage a very complex fluid flow. Spray behind a car; waves on beaches; hurricanes and tornadoes or any other atmospheric phenomenon are all example of highly complex fluid flows which can be analysed with varying degrees of success (in some cases hardly at all). There are many common situations which are easily analysed.

Uniform Flow, Steady Flow

It is possible - and useful - to classify the type of flow which is being examined into small number of groups. If we look at a fluid flowing under normal circumstances - a river for example - the conditions at one point will vary from those at another point (e.g. different velocity) we have non-uniform flow. If the conditions at one point vary as time passes then we have unsteady flow. Under some circumstances the flow will not be as changeable as this. The following terms describe the states which are used to classify fluid flow:

Uniform flow: If the flow velocity is the same magnitude and direction at every point in the fluid it is said to be *uniform*.

Non-uniform: If at a given instant, the velocity is not the same at every point, the flow is *non-uniform*. In practice, by this definition, every fluid that flows near a solid boundary will be non-uniform – as the fluid at the boundary must take the speed of the boundary, usually zero. However if the size and shape of the of the cross-section of the stream of fluid is constant the flow is considered *uniform*.

Steady: A steady flow is one in which the conditions (velocity, pressure and cross-section) may differ from point to point but do not change with time.

***Unsteady**:* If at any point in the fluid, the conditions change with time, the flow is described as *unsteady*. In practise there is always slight variations in velocity and pressure, but if the average values are constant, the flow is considered *steady*.

Combining the above we can classify any flow in to one of four type:

1. *Steady uniform flow*. Conditions do not change with position in the stream or with time. An example is the flow of water in a pipe of constant diameter at constant velocity.
2. *Steady non-uniform flow.* Conditions change from point to point in the stream but do not change with time. An example is flow in a tapering pipe with constant velocity at the inlet - velocity will change as you move along the length of the pipe toward the exit.
3. *Unsteady uniform flow*. At a given instant in time the conditions at every point are the same, but will change with time. An example is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.
4. *Unsteady non-uniform flow.* Every condition of the flow may change from point to point and with time at every point. For example waves in a channel.

If you imaging the flow in each of the above classes you may imagine that one class is more complex than another. And this is the case - *steady uniform flow* is by far the most simple of the four.

Compressible or Incompressible

All fluids are compressible - even water - their density will change as pressure changes. Under steady conditions, and provided that the changes in pressure are small, it is usually possible to simplify analysis of the flow by assuming it is incompressible and has constant density. As you will appreciate, liquids are quite difficult to compress - so under most steady conditions they are treated as incompressible. In some unsteady conditions very high pressure differences can occur and it is necessary to take these into account - even for liquids. Gasses, on the contrary, are very easily compressed, it is essential in most cases to treat these as compressible, taking changes in pressure into account.

FLOW RATE.

Mass flow rate

If we want to measure the rate at which water is flowing along a pipe. A very simple way of doing this is to catch all the water coming out of the pipe in a bucket over a fixed time period. Measuring the weight of the water in the bucket and dividing this by the time taken to collect this water gives a rate of accumulation of mass. This is know as the *mass flow rate.*

For example an empty bucket weighs 2.0kg. After 7 seconds of collecting water the bucket weighs 8.0kg,

then:

$$\begin{aligned}\text{mass flow rate} = \dot{m} &= \frac{\text{mass of fluid in bucket}}{\text{time taken to collect the fluid}}\\ &= \frac{8.0-2.0}{7}\\ &= 0.857 kg/s \quad (kg\,s^{-1})\end{aligned}$$

Performing a similar calculation, if we know the mass flow is 1.7kg/s, how long will it take to fill a container with 8kg of fluid?

$$\begin{aligned}\text{time} &= \frac{\text{mass}}{\text{mass flow rate}}\\ &= \frac{8}{1.7}\\ &= 4.7s\end{aligned}$$

Volume flow rate - Discharge.

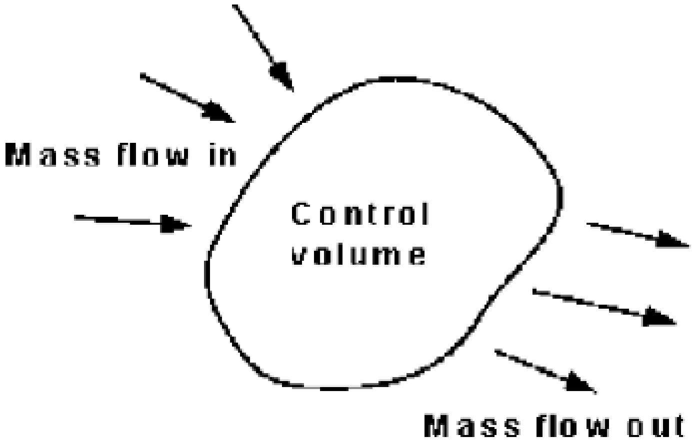
More commonly we need to know the volume flow rate - this is more commonly known as *discharge.* (It is also commonly, but inaccurately, simply called flow rate). The symbol normally used for discharge is *Q*. The discharge is the volume of fluid flowing per unit time. Multiplying this by the density of the fluid gives us the mass flow rate. Consequently, if the density of the fluid in the above example is 850 kgm^{-3} then:

$$\begin{aligned}\text{discharge, } Q &= \frac{\text{volume of fluid}}{\text{time}} \\ &= \frac{\text{mass of fluid}}{\text{density} \times \text{time}} \\ &= \frac{\text{mass flow rate}}{\text{density}} \\ &= \frac{0.857}{850} \\ &= 0.001008\, m^3/s \quad (m^3 s^{-1}) \\ &= 1.008 \times 10^{-3} m^3/s \\ &= 1.008\, l/s\end{aligned}$$

As has already been stressed, we must always use a consistent set of units when applying values to equations. It would make sense therefore to always quote the values in this consistent set. This set of units will be the SI units. Unfortunately, and this is the case above, these actual practical values are very small or very large (0.001008m^3/s is very small). These numbers are difficult to imagine physically. In these cases it is useful to use *derived units*, and in the case above the useful derived unit is the litre. (1 litre = 1.0 x10-^{3}m^3). So the solution becomes 1.008 l/s . It is far easier to imagine 1 litre than 1.0 x10-^{3} m^3. Units must always be checked, and converted if necessary to a consistent set before using in an equation.

CONTINUITY

Matter cannot be created nor destroyed - (it is simply changed into a different form of matter). This principle is known as the *conservation of mass* and we use it in the analysis of flowing fluids. The principle is applied to fixed volumes, known as control volumes (or surfaces), like that in the figure below:



An arbitrarily shaped control volume.

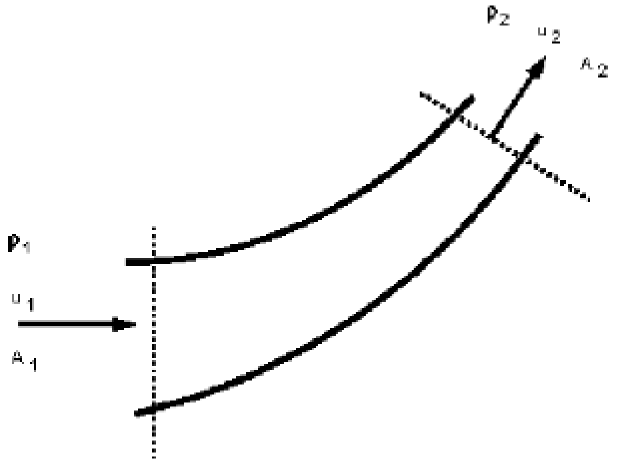
For any control volume the principle of conservation of mass says.

Mass entering per unit time = Mass leaving per unit time + Increase of mass in the control volume per unit time

For steady flow there is no increase in the mass within the control volume, so For steady flow

Mass entering per unit time = Mass leaving per unit time

This can be applied to a stream tube such as that shown below. No fluid flows across the boundary made by the streamlines so mass only enters and leaves through the two ends of this streamtube section.



A streamtube

We can then write
mass entering per unit time at end 1 = mass leaving per unit time at end 2

$$\rho_1 \delta A_1 u_1 = \rho_2 \delta A_2 u_2$$

16

Or for steady flow,

$$\rho_1 \delta A_1 u_1 = \rho_2 \delta A_2 u_2 = \text{ Constant} = \dot{m}$$

17

This (eqn 17) is the equation of continuity.

The flow of fluid through a real pipe (or any other vessel) will vary due to the presence of a wall - in this case we can use the mean velocity and write

$$\rho_1 A_1 u_{m1} = \rho_2 A_2 u_{m2} = \text{ Constant} = \dot{m}$$

18

When the fluid can be considered incompressible, i.e. the density does not change, $\rho_1 = \rho_2 = \rho$ so (dropping the *m* subscript)

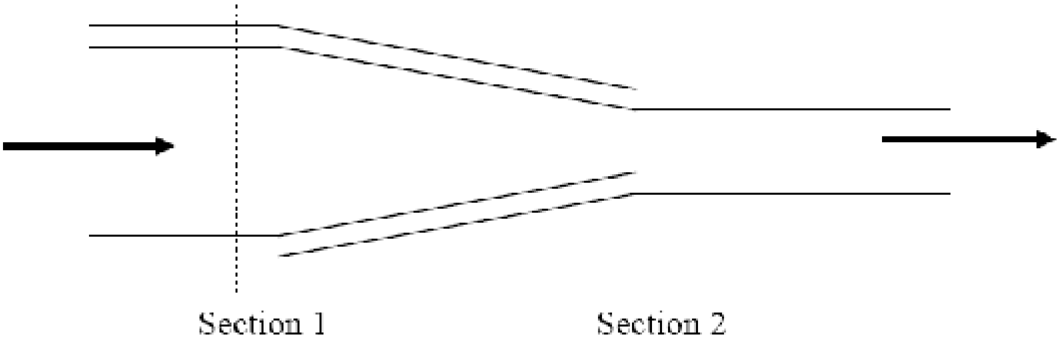
$$A_1 u_1 = A_2 u_2 = Q$$

19

This is the form of the continuity equation most often used. This equation is a very powerful tool in fluid mechanics.

Some applications of continuity equation

We can apply the principle of continuity to pipes with cross sections which change along their length. Consider the diagram below of a pipe with a contraction:



A liquid is flowing from left to right and the pipe is narrowing in the same direction. By the continuity principle, the mass flow rate must be the same at each section i.e. the mass going into the pipe is equal to the mass going out of the pipe. So we can write:

$$A_1 u_1 \rho_1 = A_2 u_2 \rho_2$$

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(with the sub-scripts 1 and 2 indicating the values at the two sections)
As we are considering a liquid, usually water, which is not very compressible, the density changes very little so we can say $\rho_1 = \rho_2 = \rho$. This also says that the volume flow rate is constant or that Discharge at section 1 = Discharge at section 2

$$Q_1 = Q_2$$
$$A_1 u_1 = A_2 u_2$$

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For example if the area $A_1 = 10 \times 10^{-3}$ m^2 and $A_2 = 3 \times 10^{-3} m^2$ and the upstream mean velocity, $U_1 = 2.1$ m/s , then the downstream mean velocity can be calculated by

$$u_2 = \frac{A_1 u_1}{A_2}$$
$$= 7.0 m / s$$

Notice how the downstream velocity only changes from the upstream by the ratio of the two areas of the pipe. As the area of the circular pipe is a function of the diameter we can reduce the calculation further,

$$u_2 = \frac{A_1}{A_2} u_1 = \frac{\pi d_1^2 / 4}{\pi d_2^2 / 4} u_1 = \frac{d_1^2}{d_2^2} u_1$$

$$= \left(\frac{d_1}{d_2} \right)^2 u_1$$

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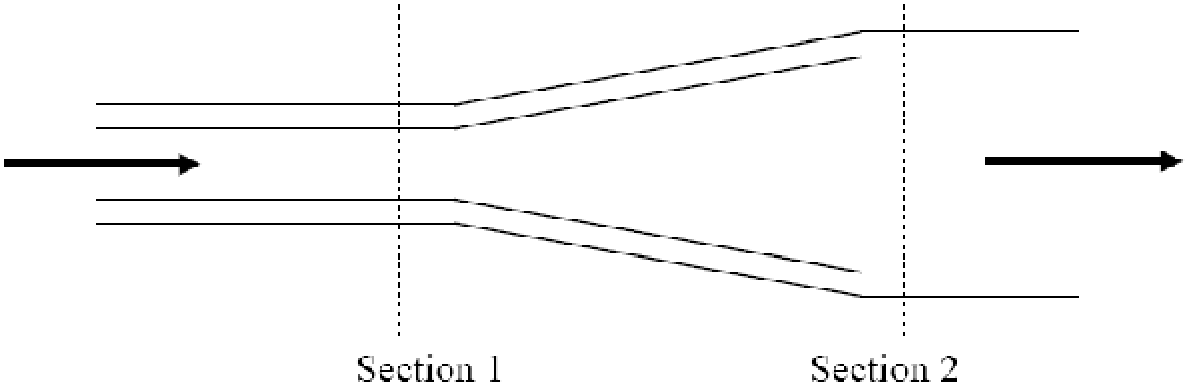
Now try this on a *diffuser,* a pipe which expands or diverges as in the figure below,

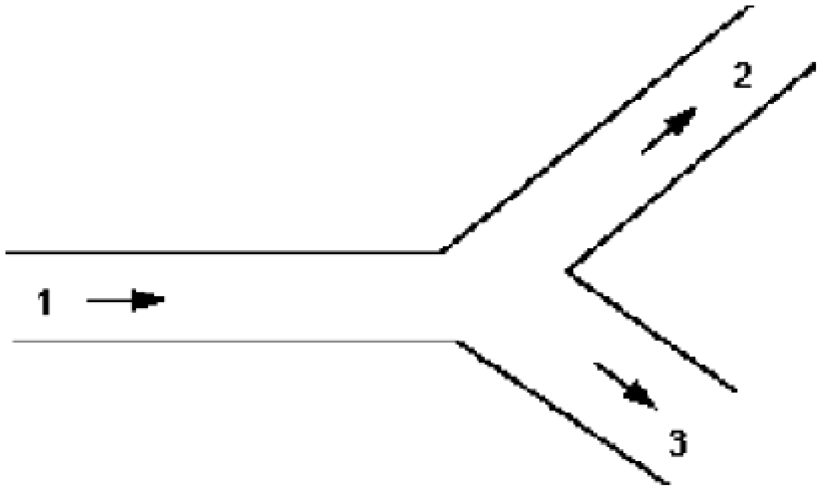


If the diameter at section 1 is d_1 = 30mm and at section 2 d_2 = 40 mm and the mean velocity at section 2 is u_2 = 3.0 m/s. The velocity entering the diffuser is given by,

$$u_1 = \left(\frac{40}{30} \right)^2 3.0$$

$$= 5.3 m / s$$

Another example of the use of the continuity principle is to determine the velocities in pipes coming from a junction.



Total mass flow into the junction = Total mass flow out of the junction

$$\rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3$$

23

When the flow is incompressible (e.g. if it is water) $\rho_1 = \rho_2 = \rho$

$$Q_1 = Q_2 + Q_3$$
$$A_1 u_1 = A_2 u_2 + A_3 u_3$$

If pipe 1 diameter = 50mm, mean velocity 2m/s, pipe 2 diameter 40mm takes 30% of total discharge and pipe 3 diameter is 60mm. What are the values of discharge and mean velocity in each pipe?

$$\begin{aligned} Q_1 = A_1 u_1 &= \left(\frac{\pi d^2}{4}\right) u \\ &= 0.00392 m^3 / s \end{aligned}$$

$$\begin{aligned} Q_2 &= 0.3 Q_1 = 0.001178 m^3 / s \\ Q_1 &= Q_2 + Q_3 \\ Q_3 &= Q_1 - 0.3 Q_1 = 0.7 Q_1 \\ &= 0.00275 m^3 / s \end{aligned}$$

$$\begin{aligned} &Q_2 = A_2 u_2 \\ &u_2 \quad 0\,936 m \quad s \end{aligned}$$

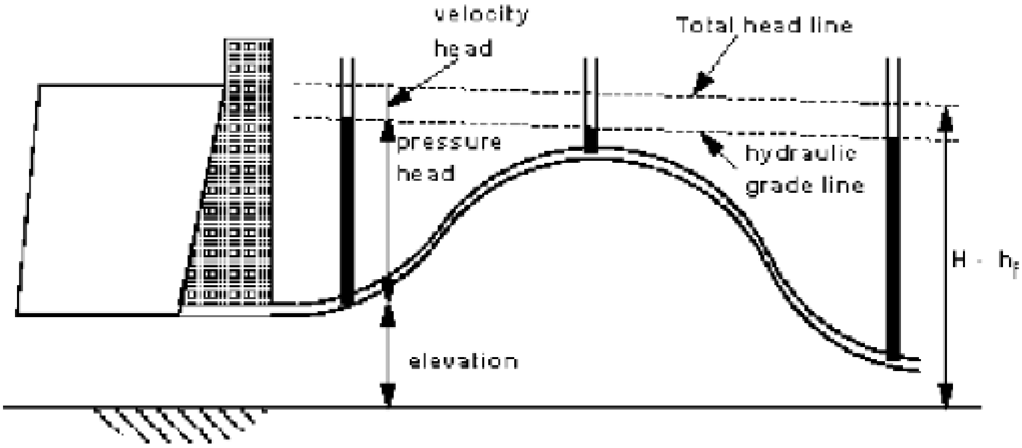
$$\begin{aligned} &Q_3 = A_3 u_3 \\ &u_3 \quad 0\,972 m \quad s \end{aligned}$$

FRICTIONAL LOSSES IN PIPES

Friction loss has several causes, including:

- The conditions of flow and the physical properties of the system.
- Movement of water molecules against each other.
- Movement of water molecules against the inside surface of a pipe, particularly if the inside surface is rough, textured, or otherwise not smooth
- Bends, kinks, and other sharp turns in hose or piping

In a real pipeline there are energy losses due to friction - these must be taken into account as they can be very significant. How would the pressure and hydraulic grade lines change with friction? Going back to the constant diameter pipe, we would have a pressure situation like this shown below:



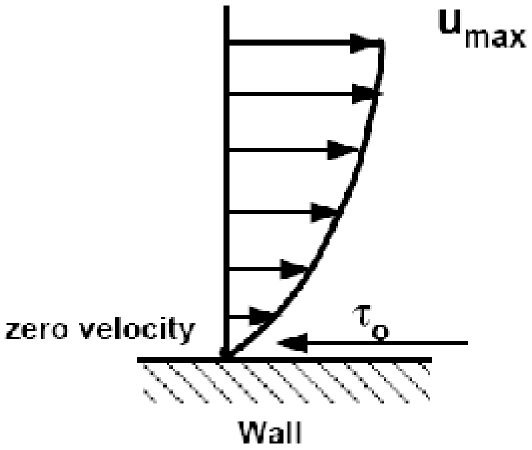
Hydraulic Grade line and Total head lines for a constant diameter pipe with friction

We are considering energy conservation, so if we allow for an amount of energy to be lost due to friction the total head will change. We have seen the equation for this before (MCB 205). But here it is again with the energy loss due to friction written as a head and given the symbol h_f. This is often know as the head loss due to friction.

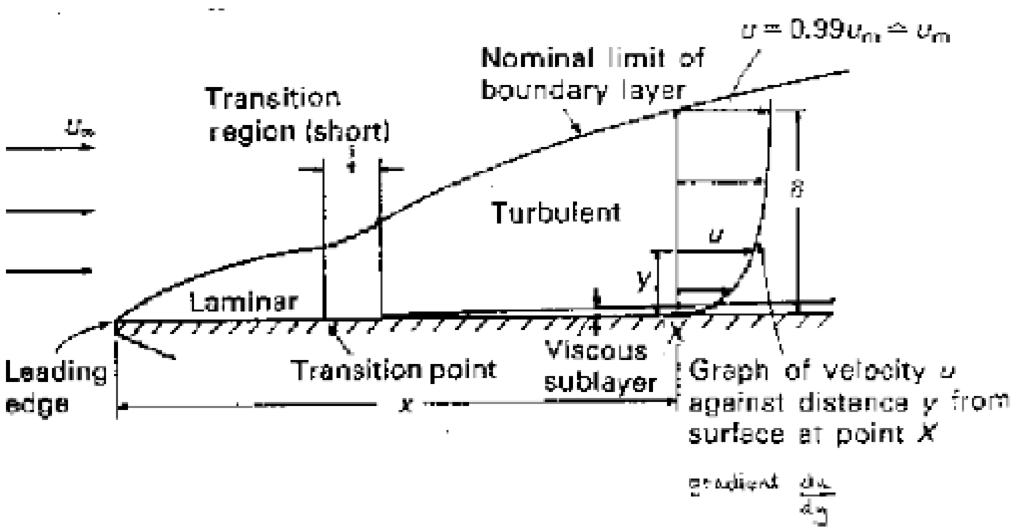
$$\frac{p_1}{\rho g}+\frac{u_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{u_2^2}{2g}+z_2+h_f$$

BOUNDARY LAYERS

When a fluid flows over a stationary surface, e.g. the bed of a river, or the wall of a pipe, the fluid touching the surface is brought to rest by the shear stress τ_0 at the wall. The velocity increases from the wall to a maximum in the main stream of the flow as shown below:



Looking at this two-dimensionally we get the above velocity profile from the wall to the centre of the flow. This profile doesn't just exit, it must build up gradually from the point where the fluid starts to flow past the surface e.g. when it enters a pipe. If we consider a flat plate in the middle of a fluid, we will look at the build up of the velocity profile as the fluid moves over the plate. Upstream the velocity profile is uniform, (free stream flow) a long way downstream we have the velocity profile. This is the known as **fully developed flow**. But how do we get to that state? This region, where there is a velocity profile in the flow due to the shear stress at the wall, we call the **boundary layer**. The stages of the formation of the boundary layer are shown in the figure below:



Boundary layer on flat plate (y scale greatly enlarged)

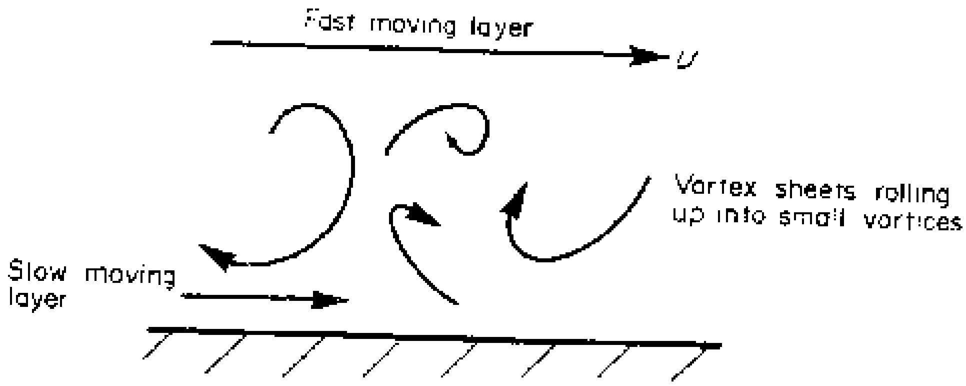
We define the thickness of this boundary layer as the distance from the wall to the point where the velocity is 99% of the "free stream" velocity, the velocity in the middle of the pipe or river, boundary layer thickness, d = distance from wall to point where u = 0.99 $u_{mainstream}$.
The value of d will increase with distance from the point where the fluid first starts to pass over the boundary. It increases to a maximum in fully developed flow. Correspondingly, the drag force D on the fluid due to shear stress τ_o at the wall increases from zero at the start of the plate to a maximum in the fully developed flow region where it remains constant. We can calculate the magnitude of the drag force by using the momentum equation. But this complex and not necessary for this course. Our interest in the boundary layer is that its presence

greatly affects the flow through or round an object. So here we will examine some of the phenomena associated with the boundary layer and discuss.

Formation of the boundary layer

Above we noted that the boundary layer grows from zero when a fluid starts to flow over a solid surface. As is passes over a greater length more fluid is slowed by friction between the fluid layers close to the boundary. Hence the thickness of the slower layer increases. The fluid near the top of the boundary layer is dragging the fluid nearer to the solid surface along. The mechanism for this dragging may be one of two types:

> The first type occurs when the normal viscous forces (the forces which hold the fluid together) are large enough to exert drag effects on the slower moving fluid close to the solid boundary. If the boundary layer is thin then the velocity gradient normal to the surface, (*du/dy),* is large so by Newton’s law of viscosity the shear stress, τ_0 *=(du/dy)*, is also large. The corresponding force may then be large enough to exert drag on the fluid close to the surface. As the boundary layer thickness becomes greater, so the velocity gradient become smaller and the shear stress decreases until it is no longer enough to drag the slow fluid near the surface along. If this viscous force was the only action then the fluid would come to a rest. It, of course, does not come to rest but the second mechanism comes into play. Up to this point the flow has been **laminar** and Newton’s law of viscosity has applied. This part of the boundary layer is known as the **laminar boundary layer.** The viscous shear stresses have held the fluid particles in a constant motion within layers. They become small as the boundary layer increases in thickness and the velocity gradient gets smaller. Eventually they are no longer able to hold the flow in layers and the fluid starts to rotate.



This causes the fluid motion to rapidly becomes turbulent. Fluid from the fast moving region moves to the slower zone transferring momentum and thus maintaining the fluid by the wall in motion. Conversely, slow moving fluid moves to the faster moving region slowing it down. The net effect is an increase in momentum in the boundary layer. We call the part of the boundary layer the **turbulent boundary layer**. At points very close to the boundary the velocity gradients become very large and the velocity gradients become very large with the viscous shear forces again becoming large enough to maintain the fluid in laminar motion. This region is known as the **laminar sub-layer**. This layer occurs within the turbulent zone and is next to the wall and very thin – a few hundredths of a mm.