

UNIVERSITY OF AGRICULTURE, ABEOKUTA

DEPARTMENT OF PHYSICS

PHS 352...Quantum Physics (3 units)

Module	Short-Description	Duration
1	Genesis of Quantum Physics	2 lectures
2	wave-particle duality	2 lectures
3	Basic principles of Quantum Physics	4 lectures
4	Commutator Relations in Quantum Physics	2 lectures

		10 lectures

References:

1. Mathews, P.T.: Introduction to Quantum Mechanics
2. Pauling, L and Wilson, E.B. : Introduction to Quantum Mechanics
3. R. Shankar ; Principles of Quantum Mechanics
4. A. Ghatak and S.Lokanathan ;Quantum Mechanics

Dr. A.O. Obawole

Module 1(Genesis of Quantum Physics)
(2 lectures)

1.1 Genesis of Quantum Physics

Brief description of experiments and theories of Quantum Physics summarized as follows:

- (a) Blackbody radiation...Planck
- (b) The photoelectric effect...Einstein
- (c) Quantum theory of atomic states...Bohr
- (d) The Davisson-Germer experiment...de Broglie hypothesis
- (e) The uncertainty principle....Heisenberg
- (f) Probability waves..Born
- (g) wave equation...Schrodinger
- (h) Exclusion principle...Pauli

Module 2(Wave-Particle Duality)
(2 lectures)

2.1 Wave-particle duality of electromagnetic radiation

- (a) Particle characteristics to explain:
 - (i) photoelectric effect,
 - (ii) Compton scattering
- (b) Wave characteristics to explain:
 - (i) Interference
 - (ii) diffraction experiments.

2.2 Wave-particle duality of matter

De-Broglie hypothesis

Module 3(Basic principles of Quantum Physics)
(4 lectures)

3.1 Basic Principles of Quantum Physics

There are 4 basic principles of Quantum Physics summarized as follows:

- (1) Observables and operators
- (2) Measurement in Quantum Physics
- (3) The state function and expectation values
- (4) Time development of the state function

Tutorial 1

1. Consider the operators $\hat{D}, \hat{I}, \hat{B}, \hat{P}, \hat{\Theta}$ defined below as :

$$\hat{D}\psi(x) = \frac{\partial}{\partial x}\psi(x), \hat{I}\psi(x) = \psi(x), \hat{B}\psi(x) = \frac{1}{3}\psi(x), \hat{P}\psi(x) = 3\psi^2 - 2\psi, \hat{\Theta}(x) = 0$$

For each of the operators listed above, construct the square, that is \hat{D}^2 .

2. Let X be the one-dimensional position operator, $X\psi = x\psi$, and let D be the derivative operator: $D\psi = \frac{d\psi}{dx}$. Calculate DX
3. The state $\psi(x)$ of a system of electrons of mass m in 1-D is given by

$$\psi(x) = Ae^{-2\pi x^2}.$$

- (a) Normalize to determine the value of A.
 - (b) What is the normalized state function?
 - (c) Calculate the average energy of the electrons in this normalized state.
4. The **Dirac delta** function $\delta(x-a)$ may be accurately expressed as an eigenfunction of \hat{x} in the coordinate representation.

Evaluate the following: (i) $\int dx \delta(x-2)$ (ii) $\int dx (x-4)\delta(x+3)$

(iii) $\int dx (\log_{10} x)\delta(x-0.01)$ (iv) $\int dx (e^{x+2})\delta(x+2)$

(v) $\int_0^{\infty} dx [\cos(3x) + 2e^{ix}](\delta(x-\pi) + \delta(x))$

5. Consider a free particle moving in 1-D space such that the states are given by $\phi_1(x) = Ae^{-ikx}$ and $\phi_2(x) = B \cos kx$, where A, B and k are constants.
- (a) What **momentum** is associated with the particle when in state $\phi_1(x)$?
 - (b) What **energy** is associated with the particle when in state $\phi_1(x)$?

- (c) What **momentum** is associated with the particle when in state $\phi_2(x)$?
- (d) What **energy** is associated with the particle when in state $\phi_2(x)$?
- (e) How do the states $\phi_1(x)$ and $\phi_2(x)$ differ with regard to measurements of **momentum** and **energy**?

6. The time-dependent state $\psi(x,t)$ of a 1-D system is given by:

$$\psi(x,t) = e^{i\beta t} (A \sin \alpha x + B i \cos \alpha x) .$$

If the potential energy is given by V_0 ,

- (a) determine whether $\psi(x,t)$ is an energy eigenfunction. **(5 marks)**
- (b) If so, calculate the measurable energy value in terms of α .**(10 marks)**
- (c) What is the measurable energy value in terms of β ? **(10 marks)**

Module 4 Commutator Relations in Quantum Physics
(2 Lectures)

Definition : The commutator between 2 operators A and B is :

$$\begin{aligned} & [A, B] \text{ such that :} \\ & [A, B] = AB - BA \end{aligned} \quad (2.1)$$

4.2 Property : If $[A, B] = -[B, A]$, the 2 operators A and B are said to commute with each other. i.e. A and B are *compatible*.

$$\text{Thus } AB = BA \quad (2.2)$$

$$\text{i.e. } [A, B] = 0 \quad (2.3)$$

$$\text{If } [A, B] \neq 0 \quad (2.4)$$

\Rightarrow A and B are *not compatible*

Tutorial 2

1. Prove that for the operators A,B and C, the following identities are valid :

$$\begin{aligned} \text{(i) } [A+B, C] &= [A, C] + [B, C] & \text{(ii) } [A, BC] &= [A, B]C + B[A, C] \\ \text{(iii) } [A, B+C] &= [A, B] + [A, C] & \text{(iv) } [AB, C] &= A[B, C] + [A, C]B \end{aligned}$$

2. One of the most important *commutators* in physics is that between the coordinate, \hat{x} , and the momentum, \hat{p} .

(i) Show that $[\hat{x}, \hat{p}] = i\hbar$

Hence, or otherwise, deduce that

(ii) $[\hat{x}^2, \hat{p}] = 2i\hbar\hat{x}$; (iii) $[\hat{x}, \hat{p}^2] = 2i\hbar\hat{p}$; (iv) $[\hat{H}, \hat{x}] = -\frac{i\hbar}{m}\hat{p}$;

(v) If g is an arbitrary function of x, show that $[\hat{p}, g] = -i\hbar\frac{dg}{dx}$