# CVE 304: Hydraulics II (2 Units)

- Simulation of complex flow fields using sources, sinks, uniform flows and doublets and combination of vortices.
- Steady and unsteady flows in open channels.
- Dimension analysis and similitude.
- Hydraulic modeling techniques.
- Pipe network analysis.
- Design of reticulation systems. Unsteady flow in pipes with special emphasis

- **Lecturer: Dr. O.S. Awokola**
- Lectures: Time Table
- Tutorial: *To be decided*
- Assignments 0-5%
- Midterm 25% (likely to be 2 tests or one plus snap tests)
- Final 70%
- **References:**
- (1) Fluid Mechanics: J.F. Douglas, J.M. Gasiorek & J.A. Swaffield
- (2) Fluid Mechanics, Victor L. Streeter, E. Benjamin
- (3) Fluid Mechanics With Engineering Applications, Robert L. Daugherty & Joseph B. Franzini
- This course is intended to provide the student with the knowledge of basic principles of Hydraulics
- On completion of this course the student should be able to:

- Understand the importance of uniform flow in open channel
- Understand the importance of non-uniform flow in open channel
- Understand the importance of unsteady flow.
- Use Chezy's and Mannings equation
- Explain the concept of dimensional analysis
- List the applications of dimensional analysis
- Solve problems using principles of dimensional analysis
- Define similitude
- Explain the geometric, kinematic and dynamic similarity
- Explain the application of principles of geometric, kinematic and dynamic similarity of Reynolds and Froude Model laws

**OPEN CHANNELS**

- DEFINITION: Every conduit in which a flowing liquid is confined only by the sides and bottom while the surface is free.
- TYPES:

a) Natural or artificial

b) Dug in the ground with or without protective linings

c) Made of pipes

d) Rectangular, Triangular, Trapezoidal, Circular, Semi-Circular or irregular shape

- An open channel can be defined also as a conduit in which the liquid flows with a free surface subjected to atmospheric pressure.

- The flow is caused by the slope of the channel and of the liquid surface.

FLOW REGIMES

$V = \frac{Q}{A}$

$Q = VA$

Q= Discharge $m^3 / s$

V= Velocity m/s

A= X-Section Area ( $m^2$ )

Variation of the above values w.r.t longitudinal distance (L) and time (T) define different flow regimes

1. STEADY FLOW: defined under pipe flow as condition in which flow characteristics at any point do not change with time $\frac{dV}{dt} = 0$ , $\frac{dy}{dt = 0}$

2. UNIFORM FLOW : refers to the condition in which the DEPTH, SLOPE VELOCITY and CROSS –SECTION remain constant over a given length of channel $\frac{dV}{dL} = 0$, $\frac{dy}{dL} = 0$

3. NON-UNIFORM FLOW/VARIED FLOW occurs when the depth of low occurs when the depth of flow changes along the length of the open channel $\frac{dy}{dL} \neq 0$. The velocity changes from cross-section to cross-section $\frac{dV}{dL} \neq 0$

OTHER COMBINATIONS ARE POSSIBLE

**UNIFORM STEADY FLOW**

The equations commonly used in calculating uniform, steady flow are

1. CHEZY EQUATION $V = C\sqrt{RS}$ V=average velocity (m/s), C=Chezy's coefficient, R=hydraulic radius=$\frac{A}{P}$, S= Bed slope of the channel.
2. MANNING'S EQUATION $V = \frac{1}{n} R^{2/3} S^{1/2}$ this the most widely used formula for open channel flow. Manning's n depends only on the roughness of the channel sides and bottom

**ENERGY PRINCIPLE IN OPEN CHANNEL FLOW**

- Need to examine/determine BERNOULLI EXPRESSION

$$H = \frac{P}{\gamma} + \frac{V^2}{2g} + Z$$

- Define the terms
- What is Hydraulic Grade Line

EXAMPLE:

The depths a short distance upstream and down stream of a sluice gate in horizontal channel are 2.4m and 0.6m respectively. The channel is of rectangular section and 3m wide find the discharge under the gate. ( to be discussed in class)

**<u>SPECIFIC ENERGY AND ALTERNATR DEPTHS</u>**

We can define the specific energy E as the energy referred to the channel bed as datum i.e.

$$E = y + \frac{V^2}{2g} \cdots\cdots 1$$

Consider a wide rectangular channel of width b

Let q denotes the flow per unit width of a wide rectangular channel

$$q = \frac{Q}{b} = vy = \frac{AV}{b}$$

$$V = \frac{Q}{A} = \frac{qb}{yb} = \frac{q}{y}$$

rewrite eqn 1

$$E = y + \frac{q^2}{2gy^2}$$

We can consider how E will vary with y for a given constant value of q.

We can construct a graph on the E-y plane

$$(E - y)y^2 = \frac{q^2}{2g} = \text{a constant}$$

$$q = y\sqrt{2g(E - y)}$$

THE GRAPH AS ILLUSTRATED IN THE CLASS AND IN THE TEXT.

NOTE:

- When two depths of flow are possible for a given E and q they are referred to as ALTERNATE DEPTHS.
- Alternatively we may say that the curve represents two possible REGIMES of flow i.e. slow and deep on the upper limb, fast and shallow on the lower limb meeting at the crest of the curve ( **at critical depth)**

**ANALYTICAL PROPERTIES OF CRITICAL FLOW**

Let us drive equations defining critical flow

$$E = y + \frac{q^2}{2gy^2}$$

E is minimum at critical depth

We obtain minimum by differentiation

$$\frac{dE}{dy} = 0 = 1 - \frac{q^2}{gy^3}$$

$$1 - \frac{q^2}{gy^3} = 0$$

$$\frac{q^2}{gy^3} = 1$$

$$q^2 = gy^3$$

i.e. $y_c = \sqrt[3]{\frac{q^2}{g}} = \left(\frac{q^2}{g}\right)^{1/3}$

$y_c =$ Critical depth

The depth for minimum energy is called critical depth

NOTE:

$$q = vy$$

$$q^2 = gy_c{}^3 = v^2 y^2$$

$$V_c = gy_c$$

$\frac{V^2{}_c}{2g} = \frac{y_c}{2}$ or $V_c = \sqrt{gy_c}$

$$E_c = y_c + \frac{V^2{}_c}{2g} = \frac{3}{2} y_c$$

$$y_c = \frac{2}{3} E_c$$

The above equations are established by considering the variation of E with y for a given q.

IT IS ALSO OF PRACTICAL INTEREST TO STUDY HOW q varies with y for a given E i.e constant E

We can find the maximum by rearranging the specific energy and differentiate

$$E = y + \frac{q^2}{2gy^2}$$

differentiate with respect to y

$$2q\frac{dq}{dy} = 4gyE - 6gy^2 = 0$$

$$6qy^2 = 4qyE$$

$$y = \frac{2}{3}E$$

We have therefore established another important property of critical flow:

- It shows not only minimum specific energy for a given q
- Also a maximum q for a given E

SUMMARY:

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$V_c^{\,2} = gy$$

$$V_c = \sqrt{gy_c}$$

$$\frac{V_c^{\,2}}{2g} = \frac{y_c}{2}$$

$$E_c = \frac{3}{2}y_c$$

$$y_c = \frac{2}{3}E_c$$

When $y > y_c$ subcritical flow exist

$y<y_c$ super critical flow exist

**EXAMPLE**

1. Water flows uniformly in a 2m wide rectangular channel at a depth of 45cm. The channel slope is 0.002 and n=0.014. Find the flow rate in cumecs.
2. At what depth will water flow in a 3m wide rectangular channel if n=0.017, S=0.00085 and Q=4cumecs.
3. A rectangular channel 10m wide carries 8cumecs when flowing 1m deep, what is the specific energy? Is the flow sub-critical or supercritical

**ASSIGNMENT**

A rectangular channel 3m wide carries $10m^3/s$ (a) Tabulate depth of flow against specific energy for depths 0.3 to 2.4m (b) Determine the minimum specific energy (c) What type of flow exists when the depth is 0.6m and when it is 2.4m? (d) When n=0.013 what slopes are necessary to maintain the depths I (c)

**CRITICAL DEPTH IN NON-RECTANGULAR CHANNELS**

The theory presented dealt with only channels of rectangular section but such channels are not common in practice.

In natural rivers the waterway may have a most irregular section.

The trapezoidal section being often preferred in the interest of economy and bank stability

$$E = y + \frac{V^2}{2g}$$

To explore the dependence of E on y we can no longer use the discharge per unit width q, since it has lost its specific meaning.

$E = y + \frac{Q^2}{2gA^2}$ where Q is the total discharge and A is the whole cross section.

We can find the condition of minimum specific energy by differentiation

$$\frac{dE}{dy}1 - \frac{Q^2 dA}{gA^3 dy}$$

$$\frac{dE}{dy} = 1 - \frac{Q^2 B}{gA^3} = 0$$

condition for E minimum or Critical flow or depth is

$$Q^2 B = gA^3$$

$$V_c^{\,2} A^2 B = gA^3$$

$$V^2{}_c = \frac{gA}{B}$$

$$V_c = \sqrt{\frac{gA}{B}}$$

**EXAMPLES**

1. An open channel conveying water is of trapezoidal cross section, the base width is 1.5m and side slopes are at 60 degrees to the horizontal. The channel is 1in 400 and the depth is constant at 1meter. Calculate the discharge in $m^3/s$. If the Chezy C is calculated from Basin relationship as shown below.

   $C = \frac{87}{1 + \frac{0.2}{\sqrt{R}}}$ $Q = AC\sqrt{RS_o}$ R= hydraulic Radius

2. A trapezoidal channel has a bottom width of 6m side slopes of 2H 1V is to carry a flow of $25m^3/s$. Calculate the critical depth and velocity.

3. A trapezoidal channel has a bottom width of 6m and side slopes of 2H:1V. When the depth of water is 1m, the flow is $10m^3/s$. What is the specific energy, is the flow sub critical or super critical?

**NON-UNIFORM FLOW**

There are two types of non-uniform flow:

(i) Gradually varied flow i.e. the condition changes over a long distance

(ii) Rapidly varied flow – the change takes place abruptly

*DETAIL DISCUSSION IN THE CLASS AFTER THE READING ASSIGNMENTS.*

## COMPUTING THE GRADUALLY VARIED FLOW PROFILE

There are several methods of solution; (i) a case of uniform x-section and slope (ii) natural channels having irregular x-section and slope

**METHODS AVAILABLE**

(i) The numerical methods i.e Standard Step method, distance calculated from depth

(ii) Direct integration methods

(iii) Graphical methods

**STANDARD STEP METHOD**

As demonstrated in the last lecture

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + (S - S_0)L$$

$$E_1 = E_2 + (S - S_0)L$$

$$L = \frac{E_1 - E_2}{S - S_0} = \frac{\Delta E}{S - S_0}$$

Which means that $\frac{\Delta E}{\Delta x} = S - S_0$

$$\frac{\Delta E}{S - S_0}\Delta x \text{ distance}$$

**EXAMPLE**

4. A rectangular channel 3.5m wide n=0.014 runs on a slope of 0.001 from a lake whose surface level is 3.5m above the channel bed at the lake outlet. A free overfall is to be located at some downstream section, how far should it be from the lake outlet so as to make the depth at the outlet 1% less than it would have been if no overfall were present.
5. Water flows uniformly at a steady rate of 0.4m3/s in a very long triangular flume which has a side slopes of 1:1. The bottom of the flume is on a slope of 0.006 and n=0.012. Is the flow sub-critical or supercritical?
6. A channel has a trapezoidal cross section with a base width of 0.6m and sides sloping at $45^o$. When the flow along the channel is $20m^3$/minute determine the critical depth.
7. A smooth transition is to be made from a trapezoidal channel with bottom width 2m and side slope 2H:1V, to a rectangular channel 2m wide. The flow rate is 18m3/s and the depth in the trapezoidal channel is 2m. Calculate the change in bottom elevation of the rectangular channel to avoid choking. Neglect energy losses.
8. (a) A storm water drainage channel is 12m wide and has steep banks (assume rectangular) it slopes 1:2500 in the direction of flow, if Chezy C=50 for the channel boundary and its design discharge equals $35m^3/s$ what is the corresponding depth of flow? (b) A broad crested weir 3m high with ann 18.5m

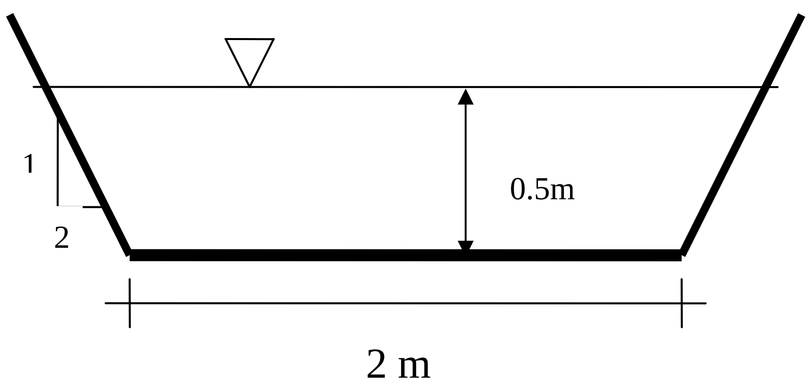
crest length is built across the channel what would be it's effect on the water surface elevation 4km upstream from the weir. Weir formula $Q = 1.85BH^{3/2}$

9. A trapezoidal drainage channel have the following characteristics.b=6m, side slopes 2H:1V, So=0.0016, n=0.025, Q=12m3/s, $\alpha = 1.1$. A dam backs the water up to a depth of 1.5m immediately behind the dam. Calculate the distance upstream from the dam to a point where the depth of flow is 1% greater than normal depth, note that $\frac{\alpha V^2}{2g} = \frac{1.1V^2}{2g}$.

**ASSIGMENTS**

**Problem 1**

Given : Trapezoidal earth channel B = 2m, sideslope = 1V:2H, **n=0.02,** S = 0.003m/m, normal depth y = 0.5m .

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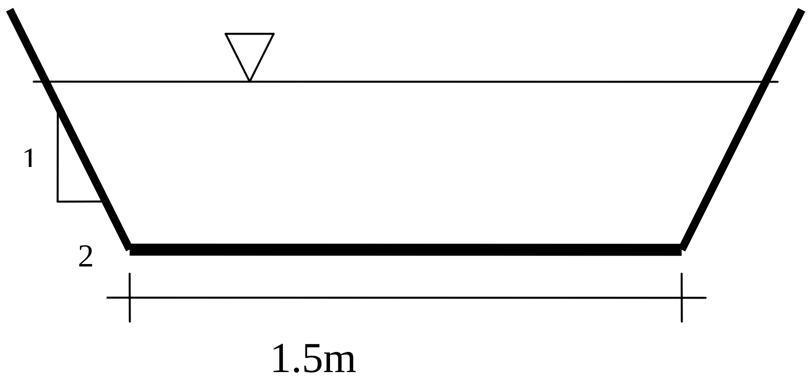
Find : Velocity V and discharge Q

**Problem 2**

Given : A concrete trapezoidal channel B = 1.5m, sideslope = 1V:2H, n=0.013, slope = 0.002, Q = 3 $m^3$/s

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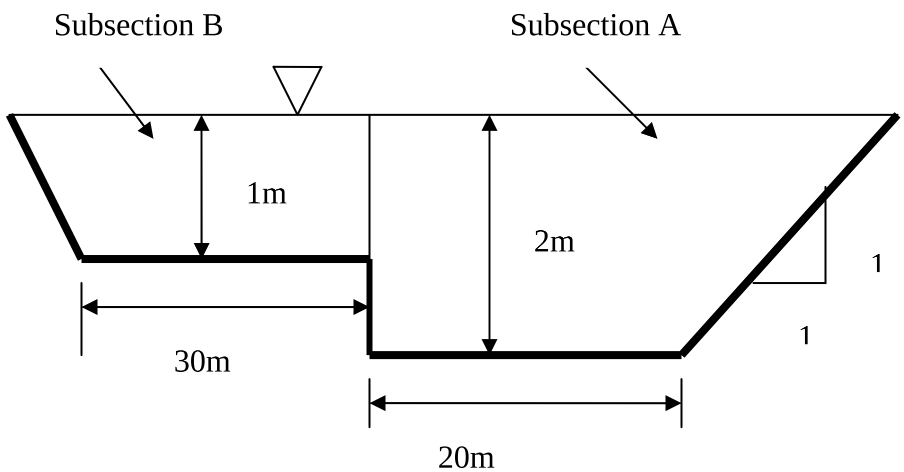
Find : Depth y and velocity v

**Problem 3**

Given: A compound channel as illustrated, with an n value of 0.03 and a longitudinal slope of 0.002m/m

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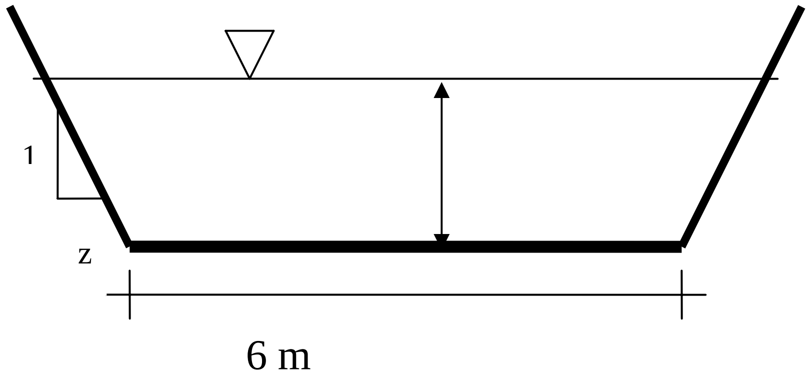
Find: Discharge Q

**Problem 4**

Given : Determine the critical depth in a trapezoidal shaped swale with z = 1, given a discharge of $9.2m^3/s$ and bottom Width = 6m. Also, determine the critical velocity.

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Find : Critical depth and Velocity at critical depth

# WATER HAMMER

Water hammer is the term used to express the resulting shock caused by the sudden decrease in the motion (velocity) of a fluid.

Simply put if the velocity of a liquid in a pipeline is abruptly decreased by a valve movement the phenomenon encountered is called WATER HAMMER.

- It is a very important problem in the case of hydroelectric plants where the flow of water must be rapidly varied in proportion to the load changes on turbine.
- Water hammer occurs in liquid flow pressure systems whenever a valve is closed.
- ***Note: The terminology water hammer may be misleading since the phenomenon can occur in any liquid.***
- In a pipeline the time of travel of the pressure wave up and back (round trip) is given by: $T_r = \dfrac{2L}{C_p}$
- $T_r =$ time of roundtrip in seconds
- L=Length of pipe in meter
- $C_p =$ Celerity of pressure wave in (m/s)
- **FOR RIGID PIPES (Non-Elastic):** The velocity of pressure wave (sound wave) is commonly referred to as ACOUSTIC VELOCITY.
- $C = \sqrt{\dfrac{gE_v}{\gamma}} = \sqrt{\dfrac{E_v}{\rho}} kN/m^2$
- $E_v =$ Volume modulus of the medium. For water a typical value is $2.o7x10^6 kN/m^2$, thus the velocity of pressure wave in water is C=1440m/s.

- **ELASTIC PIPES:** But for water in an elastic pipe this value is modified by the stretching of the pipe walls. $E_v$ is replaced by K such that
- $K = \dfrac{E_v}{1 + \left(\dfrac{D}{t}\right)\left(\dfrac{E_v}{E}\right)}$
- D= diameter of the pipe
- t=thickness of the pipe
- E=the modulus of elasticity of the pipe material.

Therefore the velocity of a pressure wave in an **<u>elastic pipe</u>** is;

$$C_p = \sqrt{\frac{gK}{\gamma}} = C\sqrt{\frac{1}{1 + \frac{DE_v}{tE}}}$$

***For normal pipe dimensions the velocity of a pressure wave in a water pipe usually ranges between <u>600 and 1200m/s</u> but it will always be less than <u>1440m/s.</u>***

- The increase in pressure caused by the **sudden closing of a valve** is calculated by;

  $d_p = \rho C_p dV$

- Change in pressure = density x celerity x change in velocity.

**<u>INSTANTANEOUS CLOSURE:</u>**

In case of instantaneous and complete closure of a valve, the velocity is reduced from V to zero, i.e. $\Delta V = V$ , $\Delta p$ then represents the increase in pressure due to valve closure,

- The water hammer pressure $P_h = \rho C_p V$

**For Instantaneous Valve closure** $P_h = \rho C_p V$

**<u>RAPID CLOSURE:</u>** $(t_c < \frac{2L}{C_p})$

It is physically impossible for a valve to be closed instantaneously. Letus consider a real case where the valve is closed in a finite time $t_c$ which is more than zero but less than $\frac{2L}{C_p}$

i.e. $t_c > 0$ but $t_c < T_r = \frac{2L}{C_p}$

**<u>SLOW CLOSURE</u>:** $(t_c > \frac{2L}{C_p})$

Slow closure will be defined as one in which the time of valve movement is greater than $\frac{2L}{C_p}$

Tests have shown that for slow valve closure, i.e. in a time greater than $\frac{2L}{C_p}$ , the excess pressure produced decreases uniformly from the value at valve to zero at the intake. The water hammer pressure $P_h^{'}$ developed by gradual closure of a valve when $(t_c > \frac{2L}{C_p})$ is given approximately by $P_h^{'} = \frac{2L/C_p}{t_c} P_h = \frac{2LV\rho}{t_c}$ where $t_c$ is the time of closure.

- A pipe can be protected from the effects of high water-hammer pressure through the use of slow-closing valves, the use of automatic relief valves which permit water to escape when the pressure exceeds a certain value, or through application of surge chambers.

**EXAMPLES**

1. Compare the velocities of the pressure waves traveling along a rigid pipe containing (a) Water at $16^o$ C (b) Glycerin at $20^o$ C (c) Oil of relative density 0.8. Use the values of bulk modulus for glycerin and oil of $4.34x10^9$ and $1.38x10^9$ $N/m^2$. If the liquids are flowing in a 0.3m pipe at 1.2m/s and the flow stopped suddenly, what increase in pressure could be expected assuming the pipe to be rigid?
2. A 1.2m steel pipe 10mm thick carries water at $16^o$ C at a velocity of 1.8m/s. If the pipe line is 3km long and if the valve at the discharge end is shut in 2.5seconds. What increase in stress in the pipe could be expected? E for steel= $207x10^9$ $N/m^2$, $E_v$ water at $16^o$ C $=2.16x10^9$ $N/m^2$.
3. A valve is suddenly closed in a 75mm pipe carrying glycerin at $20^o$ C. The increase in pressure is 7bar. What is the probable flow in $m^3/s$? Use $\rho = 1262,\ E_v = 4.39x10^9\, N/m^2$

# DIMENSIONAL ANALYSIS AND SIMILARITY

- Dimensional analysis plays an important role in the organization of experimental work and the presentation of its results.
- The application of fluid mechanics in design relies on the use of empirical results built up from an extensive body of experimental research.
- Together dimensional analysis, similarity and model testing technique allow the design engineer to **predict accurately** and **economically the performance** of the prototype system, e.g. an aircraft wing, ship, dam, spillway, harbor construction.
- **Dimensional Analysis:** is a useful technique for the investigation of problems in all braches of engineering and particularly in fluid mechanics.
- If it is possible to identify the factors involved in a physical situation, dimensional analysis can usually establish the form of the relationship between them.
- The technique does not appear to be as precise as the usual algebraic analysis which seems to provide exact solutions but these are usually obtained by making a series of simplifying assumptions which do not always correspond with the real facts.
- The qualitative solution obtained by dimensional analysis can usually be converted into a quantitative result determining any unknown factors experimentally.
- **DIMENSIONS:** Any physical situation whether it involves a single object or a complete system can be described in terms of a number of ***RECOGNISABLE*** properties which the object or system possesses.

**EXAMPLE:**

A moving object could be described in terms of its;

(i) Mass

(ii) Length

(iii) Area

(iv) Volume

(v) Velocity

(vi) Acceleration

- It's Temperature or electrical properties, density, viscosity of the medium through which it moves would also be of importance, since they would affect its motion.
- These MEASURABLE properties used to describe PHYSICAL STATE of the body or systems are known as its DIMENSIONS.

**UNITS:** To complete the description of the physical situation, it is also necessary to know the MAGNITUDE of each dimension. Length=(meter).

- The distinction between units and dimensions is that dimensions are properties that can be measured and units are standard elements in terms of which these dimensions can be described quantitatively and assigned numerical values.

**DIMENSIONAL REASONING:**

Analyzing any physical situation it is necessary to decide what factors are involved;

- Then try to determine a quantitative relationship between them.
- The factors involved can be assessed from OBSERVATION, EXPERIMENT or INTUITION.

- 1+3=4 numerically correct but in physical terms it may be untrue, for example 1elephant + 3 aero planes =4 days is untrue.
- An equation describing a physical situation will only be true if all the terms are of the same kind and have the same dimensions.
- Area = LxL=$L^2$ area has the dimension of $L^2$ . The corresponding unit of area will be the unit of length squared $m^2$ in SI units.
- $$V = \frac{L}{T} = LT^{-1}$$
  $$a = \frac{V}{T} = \frac{LT^{-1}}{T} = LT^{-2}$$

In practice for any given system of units, the constant of proportionality is made UNITY.

Force $\alpha$ Mass x Acceleration

Force = Mass x Acceleration

$$F = ma$$
$$F = MLT^{-2}$$
$$\Pr essure = \frac{Force}{Area} = \frac{F}{L^2} = FL^{-2}$$
$$Density = \frac{Mass}{Volume} = \frac{M}{L^3} = ML^{-3}$$
$$M = FT^2L^{-1}$$
$$\therefore Density = ML^{-3} = FT^2L^{-4}$$

***SEE PAGE 669 DOUGLAS FOR DIMENSIONS OF QUANTITIES COMMONLY OCCURING IN MECHANICS TABLE 24.1.***

- The SI system is a rationalized system of metric units in which the units for all physical quantities can be derived from SIX basic arbitrarily defined units which are:

(i) Length meter

(ii) Mass kilogram

(iii) Time second

(iv) Electric current ampere

(v) Absolute temperature kelvin

(vi) Luminous Intensity candela

***Details of the basic and derived SI units are given in Table 24.3 page 673 (Douglas)***

# SIMILITUDE AND DIMENSIONAL ANALYSIS

It is usually impossible to determine all the essential facts for a given fluid flow by pure theory and hence dependence must often be placed upon experimental investigation. The number of tests to be made can be greatly reduced by;

(i) Systematic program based on dimensional analysis

(ii) Application of the laws of similitude or similarity

The laws of similitude make it possible to determine the performance of the **prototype**, which means the full-size device from tests made with **model**.

- ***Important hydraulic structures are now designed and built only after extensive model studies have been made.***

- Application of DIMENSIONAL ANALYSIS and hydraulic SIMILITUDE enable the engineer to **organize** and **simplify** the experiments and to **analyze** the results.

# HYDRAULIC MODELS

Hydraulic models in general may be either (i) TRUE MODELS (ii) DISTORTED MODELS.

True models have all significant characteristics of the prototype reproduced to scale i.e. (geometrically similar) and satisfy design restrictions (kinematic and dynamic similitude)

**I . GEOMETRIC SIMILARITY (SIMILITUTDE)** The model and its prototype be identical in **shape** but differ only in **size.** Geometric similitude exists between model and prototype if the ratios of all corresponding dimensions in model and prototype are equal.

- $L_r = Scale\ \text{ratio} = \frac{L_p}{L_m}$ , $L_r^{\ 2} = \frac{A_p}{A_m} = \frac{L_p^{\ 2}}{L_m^{\ 2}}$

  - Note that $L_r = \frac{L_p}{L_m}$ scale ratio ,

  - the reciprocal of this $\lambda = \frac{L_m}{L_p}$ will be referred to as the **model ratio or model scale**.

**II.** **KINEMATIC SIMILARITY:** Kinematic similarity implies geometric similarity and in addition it implies that the ratio of the **velocities** at all corresponding points in the flow is the same.

Velocity ratio $= V_r = \dfrac{V_p}{V_m} = \dfrac{L_r}{T_r}$ and its value in terms of $L_r$ will be determined by dynamic considerations. T is dimensionally L/V .

The time scale is $T_r = \dfrac{L_r}{V_r}$

Acceleration scale is $a_r = \dfrac{L_r}{T_r^{\,2}}$

Discharge $= \dfrac{Q_p}{Q_m} = Q_r = \dfrac{L_r^{\,3}}{T_r}$

**III DYNAMIC SIMILARITY:** If two systems are dynamically similar, corresponding FORCES must be in the same ratio in the two. Forces that may act on a fluid element are:

(i) Gravity $F_G = mg = \rho L^3 g$

(ii) Pressure $F_P = (\Delta p)L^2$

(iii) Viscosity $F_V = \mu\left(\dfrac{du}{dy}\right)A = \mu\left(\dfrac{V}{L}\right)L^2 = \mu VL$

(iv) Elasticity $F_E = E_V A = E_V L^2$

(v) Surface Tension $F_T = \sigma L$

(vi) Inertia $FI = ma = \rho L^3 \dfrac{L}{T^2} = \rho L^4 T^{-2} = \rho V^2 L^2$

The conditions required for complete similitude are developed from Newton's second law of motion $\sum F_x = ma_x$

**REYNOLDS NUMBER**

Considering the ratio of Inertia forces to viscous forces the parameter obtained is called Reynolds Number $R_e$ or $N_R$. The ratio of these two forces is:

$R_e$ or $N_R = \frac{F_I}{F_V} = \frac{InertiaForces}{ViscousForces} = \frac{L^2V^2\rho}{LV\mu} = \frac{LV\rho}{\mu} = \frac{LV}{\upsilon}$ is a dimensionless number.

**EXAMPLE**

1. If the Reynolds number of a model and its prototype are the same find an expression for $V_r$, $T_r$, and $a_r$.
2. Let us consider the drag force $F_D$ exerted on a sphere as it moves through a viscous liquid

*The relationship of variables is our concern our approach is to* ***satisfy dimensional homogeneity, i.e. Dimensions on LHS=Dimensions on the RHS***

**TWO METHODS ARE AVAILABLE**

Let us consider the drag force $F_D$ exerted on a sphere as it moves through a viscous liquid

1. **RAYLEIGH METHOD**
2. **BUCKINGHAM $\pi$ THEOREM**

## SOLUTION TO PROBLEM 2

(i) Visualize the physical problem

(ii) Consider what physical factors influence the drag force

(a) the size of the sphere

(b) the velocity of the sphere

(c) fluid properties , density and viscosity

## RAYLEIGH METHOD

$F_D = f\left(D, V, \rho, \mu\right)$ could be written as power equation

$F_D = CD^a V^b \rho^c \mu^d$ where C is a dimensionless constant. Using MLT system and substituting the proper dimensions.

$$F = ma$$

$$\frac{ML}{T^2} = L^a \left(\frac{L}{T}\right)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{LT}\right)^d$$

To satisfy dimensional homogeneity the exponents of each dimension must be identical on both sides of the equation.

| | |
|---|---|
| M: | $1 = c + d$ |
| L: | $1 = a + b - 3c - d$ |
| T: | $-2 = -b - d$ |

We have 3 equations and 4 unknowns

Express three of the unknowns in terms of the fourth.

Solving for a, b, c, in terms of d

$$a = 2 - d$$
$$b = 2 - d$$
$$c = 1 - d$$
$$F_D = CD^{2-d}V^{2-d}\rho^{1-d}\mu^d$$

Grouping variables according to their exponents

$$F_D = C\rho D^2V^2\left(\frac{VD\rho}{\mu}\right)^d$$

The quantity $\frac{VD\rho}{\mu} = R_e$

The power equation can be expressed as $F_D = f(R_e)\rho D^2V^2$ or $\left(\frac{F_D}{\rho D^2V^2}\right) = f(R_e)$

## BUCKINHAM $\pi$-THEOREM

This theorem states that if there are n dimensional variables I a dimensionally homogeneous equation, described by m fundamental dimensions, they may be grouped in n-m (n minus m) dimensionless groups.

$$F_D = f(D,V,\rho,\mu)$$
$$n = 5 = F_D, D, V, \rho, \mu$$
$$m = 3 = M, L, T$$
$$n - m = 2$$

Buckingham referred to these dimensionless groups as $\pi$terms. The advantage of the $\pi$ theorem is that it tells one ahead of time how many dimensionless groups are to be expected.

$$f'(F_D, D, V, \rho, \mu) = 0$$
$$n = 5$$
$$m = 3$$
$$n - m = 2$$
$$\phi(\pi_1, \pi_2) = 0$$

Arrange the five parameters into 2 dimensionless groups, ***taking $\rho$ , D and V as the primary variables.***

***It is generally advantageous to choose primary variables that relate to Geometry, Kinematics and Mass.

$$\pi_1 = \rho^{a1} D^{b1} V^{c1} \mu^{d1}$$
$$\pi_2 = \rho^{a2} D^{b2} V^{c2} F_D^{\ d2}$$

Since the $\pi$ 's (pi's) are dimensionless they can be replaced with $M^0 L^0 T^0$

Working with $\pi_1$

$$M^0 L^0 T^0 = \left(\frac{M}{L^3}\right)^{a1} L^{b1} \left(\frac{L}{T}\right)^{c1} \left(\frac{M}{LT}\right)^{d1}$$
$$M : 0 = a1 + d1$$
$$L : 0 = -3a1 + b1 + c1 - d1$$
$$T : 0 = -c1 - d1$$

Solving for a1, b1 and c1 in terms of d1

$$a1 = -d1$$
$$b1 = -d1$$
$$c1 = -d1$$
$$\pi_1 = \rho^{-d1} D^{-d1} V^{-d1} \mu^{d1} = \left(\frac{\mu}{\rho D V}\right)^{d1} = \left(\frac{\rho D V}{\mu}\right)^{d1} = N_R = R_e = \mathrm{Re}\, ynoldsNumber$$

Working in a similar fashion with $\pi_2$

$$\pi_2 = \frac{F_D}{\rho D^2 V^2}$$

$$\frac{F_d}{\rho D^2 V^2} = \phi''(N_R)$$

$$F_D = \phi''(N_R)\rho D^2 V^2$$

**<u>Take Note</u>**

(a) That dimensional analysis does not provide a complete solution to fluid problems

(b) It provides a partial solution only

(c) That the success of dimensional analysis depends entirely on the ability of the individual using it to define the parameters that are applicable.

EXAMPLES

1) The discharge through a horizontal capillary tube is thought to depend upon the pressure per unit length, the diameter, and the viscosity. Find the form of the equation.

2) The losses per unit length $\left(\frac{\Delta h}{L}\right)$ of pipe in turbulent flow through a smooth pipe depend upon velocity V, diameter D, gravity g, dynamic viscosity $\mu$ and density $\rho$. With dimensional analysis determine the general form of the equation.

NOTE:

1. ALL PROBLEMS AND EXERCISES WILL BE SOLVED IN THE CLASS AND SOME WILL BE TAKEN AT TUTORIAL CLASS
2. THIS CLASS NOTE WILL NOT REPLACE THE RECOMMENDED TEXTS
3. SOME OF THE BOOKS ARE AVAILABLE IN THE MAIN LIBRARY AND COLLEGE LIBRARY