COURSE CODE:	*ELE 403*
COURSE TITLE:	*Servomechanism and Control*
NUMBER OF UNITS:	*3 Units*
COURSE DURATION:	*Three hours per week*

COURSE DETAILS:

Course Coordinator:	**NUGA O.O.**
Email:	nugaoo@unaab.edu.ng
Office Location:	Civil Engineering Building
Other Lecturers:	None

COURSE CONTENT:

Control system concept: open and closed loop control systems, block diagrams. Resume of Laplce transform. Transfer functions of electrical and control systems. Electromechanical devices: Simple and multiple gear trains, electrical and mechanical analysis. Error detector and transducer in control systems. The amplidyne: AC and DC tachogenerator and servomotors, rotary and translational potentiometers. Hydraulic and pneumatic servomotors and controllers. Dynamics of simple servomechanism. Steady state error and error constants, the use of non-dimensional notations and the frequency response test. Log and polar plots of control systems. Basic stability concepts in control systems.

COURSE REQUIREMENTS:

This is a compulsory course for all 200 level students in the College of Engineering. In view of this, students are expected to participate in all the course activities and have minimum of 75% attendance to be able to write the final examination.

READING LIST:

- John J. D'Azzo and Constanttine H. Houpis Linear Control system Analysis and Design with MATLAB New York Marcel Dekker, Inc 2003

- Katsuhiko Ogata Modern Control Engineering United State of America Prentice Hall. 1997
- Paraskevopoulos P. N. Modern Control Engineering New York Marcel, Dekker, Inc 2002
- Norman S. Nise Control Systems Engineering Fourth Edition John Wiley and Sons, Inc 2004

LECTURE NOTES

CONTROL SYSTEM CONCEPT

An automatic control system is a combination of components that act together in such a way that the overall system behaves automatically in a prespecified desired manner.

A close examination of the various machines and apparatus that are manufactured today leads to the conclusion that they are partially or entirely automated, e.g., the refrigerator, the water heater, the clothes washing machine, the elevator, the TV remote control, the worldwide telephone communication systems, and the Internet.

Industries are also partially or entirely automated, e.g., the food, paper, cement, and car industries. Examples from other areas of control applications abound: electrical power plants, reactors (nuclear and chemical), transportation systems (cars, airplanes, ships, helicopters, submarines, etc.), robots (for assembly, welding, etc.), weapon systems (fire control systems, missiles, etc.), computers (printers, disk drives, magnetic tapes, etc.), farming (greenhouses, irrigation, etc.), and many others, such as control of position or velocity, temperature, voltage, pressure, fluid level, traffic, and office automation, computer-integrated manufacturing, and energy management for buildings. All these examples lead to the conclusion that automatic control is used in all facets of human technical activities and contributes to the advancement of modern technology.

The distinct characteristic of automatic control is that it reduces, as much as possible, the human participation in all the aforementioned technical activities. This usually results in decreasing labor cost, which in turn allows the production of more goods and the construction of more works. Furthermore, automatic control reduces work hazards, while it contributes in reducing working hours, thus offering to give people a better quality of life (more free time to rest, develop hobbies, have fun, etc.).

Automatic control is a subject which is met not only in technology but also in other areas such as biology, medicine, economics, management, and social sciences.

In particular, with regard to biology, one can claim that plants and animals owe their very existence to control. To understand this point, consider for example the human body, where a tremendous number of processes take place automatically: hunger, thirst, digestion, respiration, body temperature, blood circulation, reproduction of cells, healing of wounds, etc. Also, think of the fact that we don't even decide when to drink, when to eat, when to go to sleep, and when to go to the toilet. Clearly, no form of life could exist if it were not for the numerous control systems that govern all processes in every living organism.

It is important to mention that modern technology has, in certain cases, succeeded in replacing body organs or mechanisms, as for example in replacing a human hand, cut off at the wrist, with an artificial hand that can move its fingers automatically, as if it were a natural hand. Although the use of this artificial hand is usually limited to simple tasks, such as opening a door, lifting an object, and eating, all these functions are a great relief to people who were unfortunate enough to lose a hand.

The Basic Structure of A Control System

A system is a combination of components (appropriately connected to each other) that act together in order to perform a certain task. For a system to perform a certain task, it must be excited by a proper input signal. Figure 1.1 gives a simple view of this concept, along with the scientific terms and symbols. Note that the response $y(t)$ is also called system's behavior or performance.

Symbolically, the output $y(t)$ is related to the input $u(t)$ by the following equation

$$y(t)=Tu(t) \qquad 1.1$$

where T is an operator. There are three elements involved in Eq. (1.1): the input $u(t)$, the system T, and the output $y(t)$. In most engineering problems, we usually know (i.e., we are given) two of these three elements and we are asked to find the third one. As a result, the following three basic engineering problems arise:

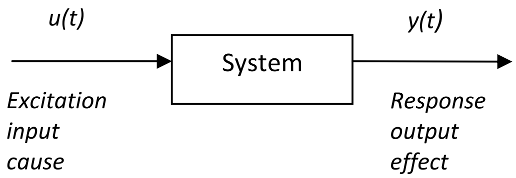

Figure 1.1 Schematic diagram of a system with its input and output.

1. The analysis problem. Here, we are given the input *u(t)* and the system T and we are asked to determine the output *y(t)*
2. The synthesis problem. Here, we are given the input *u(t)* and the output *y(t)* and we are asked to design the system T.
3. The measurement problem. Here, we are given the system T and the output *y(t)* and we are asked to measure the input *u(t)*.

Control systems can be divided into two categories: the open-loop and the closed-loop systems.

An open-loop system (Figure 1.2a) is a system whose input *u(t)* does not depend on the output *y(t)*, i.e., *u(t)* is not a function of *y(t)*.
A closed-loop system (Figure 1.2b) is a system whose input *u(t)* depends on the output *y(t)*, i.e., *u(t)* is a function of *y(t)*.

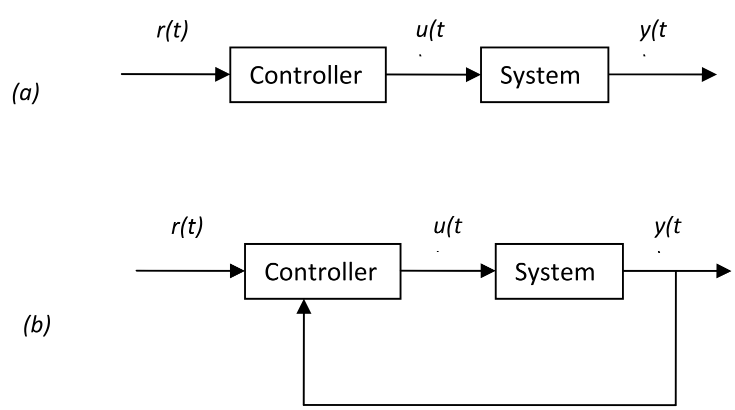

Figure 1.2 Two types of systems: (a) open-loop system; (b) closed-loop system.

In control systems, the control signal *u(t)* is not the output of a signal generator, but the output of another new additional component that is added to the system under control. This new component is called controller (and in special cases regulator or compensator). Furthermore, in control systems, the controller is excited by an external signal *r(t)*, which is called the reference or command signal. This reference signal *r(t)* specifies the desired performance (i.e., the desired output *y(t)*) of the open- or closed-loop system. That is, in control systems, we aim to design an appropriate controller such that the output *y(t)* follows the command signal *r(t)* as close as possible. In particular, in open-loop systems (Figure 1.2a) the controller is excited only by the reference signal *r(t)* and it is designed such that its output *u(t)* is the appropriate input signal to the system under control, which in turn will produce the desired output *y(t)*. In closed-loop systems (Figure 1.2b), the controller is excited not only by reference signal *r(t)* but also by the output *y(t)*. Therefore, in this case the control signal *u(t)* depends on both *r(t)* and *y(t)*. To facilitate better understanding of the operation of open-loop and closed-loop systems the following introductory examples is presented below:

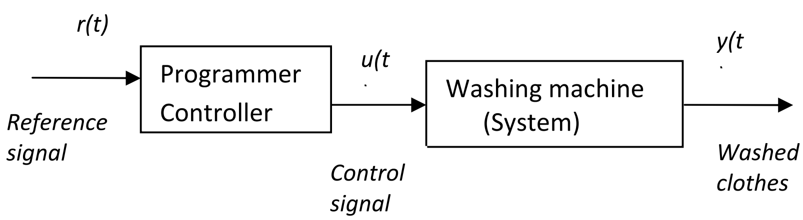

Figure 1.3 The clothes washing machine as an open-loop system.

A very simple introductory example of an open-loop system is that of the clothes washing machine (Figure 1.3). Here, the reference signal *r(t)* designates the various operating conditions that we set on the ''programmer,'' such as water temperature, duration of various washing cycles, duration of clothes wringing, etc. These operating conditions are carefully chosen so as to achieve satisfactory clothes washing.

The controller is the ''programmer,'' whose output *u(t)* is the control signal. This control signal is the input to the washing machine and forces the washing machine to execute the desired operations assigned in the reference signal *r(t)*, i.e., water heating, water changing, clothes wringing, etc. The output of the system *y(t)* is the ''quality'' of washing, i.e., how well the clothes have been washed. It is well known that during the operation of the washing machine, the output (i.e., whether the clothes are well washed or not) it not taken into consideration. The washing machine performs only a series of operations contained in *u(t)* without being influenced at all by *y(t)*. It is clear that here *u(t)* is not a function of *y(t)* and, therefore, the washing machine is a typical example of an open-loop system. Other examples of open-loop systems are the electric stove, the alarm clock, the elevator, the traffic lights, the worldwide telephone communication system, the computer, and the Internet.

A very simple introductory example of a closed-loop system is that of the water heater (Figure 1.4). Here, the system is the water heater and the output *y(t)* is the water temperature. The reference signal *r(t)* designates the desired range of the water temperature. Let this desired temperature lie in the range from 65 to 70^{o}C. In this example, the water is heated by electric power, i.e., by a resistor that is supplied by an electric current. The controller of the system is a thermostat, which works as a switch as follows: when the temperature of the water reaches 70^{o}C, the switch opens and the electric supply is interrupted. As a result, the water temperature starts falling and when it reaches 65^{o}C, the switch closes and the electric supply is back on again. Subsequently, the water temperature rises again to 70^{o}C, the switch opens again, and so on. This procedure is continuously repeated, keeping the temperature of the water in the desired temperature range, i.e., between 65 and 70^{o}C.

A careful examination of the water heater example shows that the controller (the thermostat) provides the appropriate input $u(t)$ to the water heater. Clearly, this input $u(t)$ is decisively affected by the output $y(t)$, i.e., $u(t)$ is a function of not only of $r(t)$ but also of $y(t)$. Therefore, here we have a typical example of a closed-loop system.

Other examples of closed-loop systems are the refrigerator, the voltage control system, the liquid-level control system, the position regulator, the speed regulator, the nuclear reactor control system, the robot, and the guided aircraft. All these closed-loop systems operate by the same principles as the water heater presented above.

It is remarked that in cases where a system is not entirely automated, man may act as the controller or as part of the controller, as for example in driving, walking, and cooking. In driving, the car is the system and the system's output is the course and/or the speed of the car. The driver controls the behavior of the car and reacts accordingly: he steps on the accelerator if the car is going too slow or turns the steering wheel if he wants to go left or right. Therefore, one may argue that driving a car has the structure of a closed-loop system, where the driver is the controller.

Similar remarks hold when we walk. When we cook, we check the food in the oven and appropriately adjust the heat intensity. In this case, the cook is the controller of the closed-loop system.

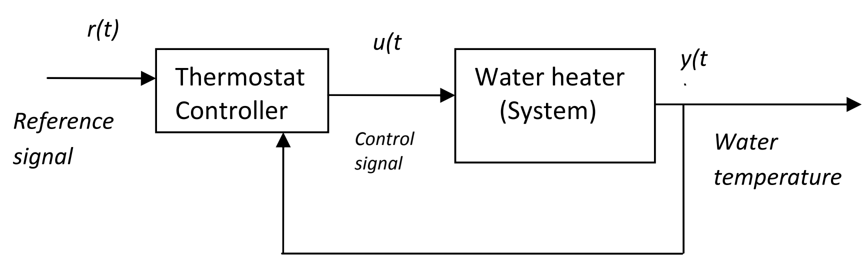

Figure 1.6 The water heater as a closed-loop system.

From the above examples it is obvious that closed-loop systems differ from open-loop systems, the difference being whether or not information concerning the system's output is fed back to the system's input. This action is called feedback and plays the most fundamental role in automatic control systems.

Indeed, it is of paramount importance to point out that in open-loop systems, if the performance of the system (i.e., y(t)) is not satisfactory, the controller (due to the lack of feedback action) does nothing to improve it. On the contrary, in closed loop systems the controller (thanks to the feedback action) acts in such a way as to keep the performance of the system within satisfactory limits.

Closed-loop systems are mostly used when the control specifications are highly demanding (in accuracy, in speed, etc.), while open-loop systems are used in simple control problems. Closed-loop systems are, in almost all cases, more difficult to design and implement than open-loop systems.[2]

2.0 DEFINITION OF THE LAPLACE TRANSFORM

The direct Laplace transformation of a function of time f (t) is given by

$$L[f(t)] = \int_0^\infty f(t)\, e^{-st} dt = F(s) \qquad 2.1$$

where $L[f(t)]$ is a shorthand notation for the Laplace integral. Evaluation of the integral results in a function *F(s)* that has *s* as the parameter. This parameter s is a complex quantity of the form $\sigma + j\omega$. Since the limits of integration are zero and infinity, it is immaterial what value f (t) has for negative or zero time.

There are limitations on the functions f (t) that are Laplace transformable. Basically, the requirement is that the Laplace integral converge, which means that this integral has a definite functional value. To meet this requirement the function f (t) must be (1) piecewise continuous over every finite interval $0 \leq t_1 \leq t \leq t_2$ and (2) of exponential order. A function is piecewise continuous in a finite interval if that interval can be divided into a finite number of subintervals, over each of which the function is continuous and at the ends of each of which f (t) possesses finite right- and left-hand limits. A function f (t) is of exponential order if there exists a constant *a* such that the product $e^{-at}|f(t)|$ is bounded for all values of t greater than some finite value *T*. This imposes the restriction that s, the real part of s, must be greater than a lower bound σ_a for which the product $e^{-\sigma_a t}|f(t)|$ is of exponential order. A linear differential equation with constant coefficients and with a finite number of terms is Laplace transformable if the driving function is Laplace transformable.

LAPLACE TRANSFORM THEOREMS

The Laplace transform is a special case of the generalized integral transform presented below.

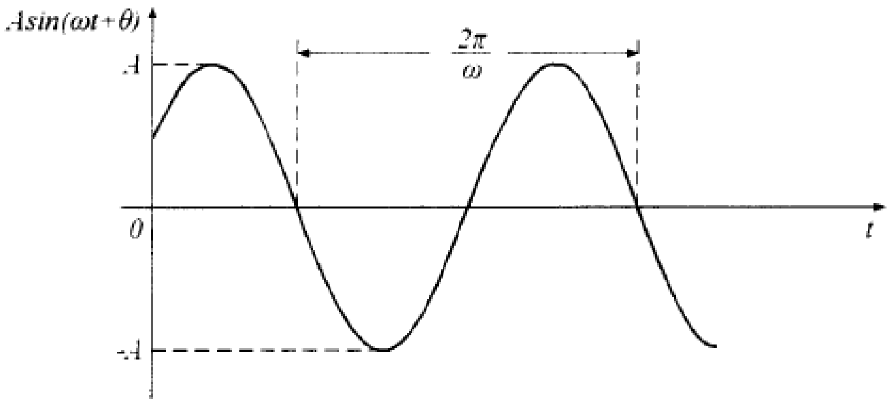

Figure 2.1 The sinusoidal function

Introduction to Laplace Transform

The Laplace transform is a linear integral transform with kernel $k(s, t)= e^{-st}$ and time interval (0,1). Therefore, the definition of the Laplace transform of a function f (t) is as follows:

$$L\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

where L designates the Laplace transform and s is the complex variable defined as $s = \sigma + j\omega$. Usually, the time function $f(t)$ is written with a small f, while the complex variable function $F(s)$ is written with a capital F.

For the integral to converge, $f(t)$ must satisfy the condition

$$\int_0^\infty |f(t)|e^{-\sigma t}\,dt \le M$$

Where σ and M are finite positive numbers.

Let $L\{f(t)\} = F(s)$. Then, the inverse Laplace transform of $F(s)$ is also a linear integral transform, defined as follows:

$$L^{-1}\{F(s)\} = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(s)e^{st}ds = f(t)$$

where L^{-1} designates the inverse Laplace transform, $j = \sqrt{-1}$, and c is a complex constant.

The most important properties and theorems of the Laplace transform are presented below. In general, they are helpful in evaluating transforms.

Theorem 1: Linearity. If a is a constant or is independent of s and t, and if f (t) is transformable, then

$$L[af(t)] = aL[f(t)] = aF(s) \qquad 2.2$$

Theorem 2: Superposition. If $f_1(t)$ and $f_2(t)$ are both Laplace-transformable, the principle of superposition applies:

$$L[f_1(t) \pm f_2(t)] = L[f_1(t)] \pm L[f_2(t)] = F_1(s) \pm F_2(s) \qquad 2.3$$

Theorem 3: Translation in time. If the Laplace transform of f (t) is F(s) and a is a positive real number, the Laplace transform of the translated function

$f(t-a)u_{-1}(t-a)$ is

$$L[f(t-a)u_{-1}(t-a)] = e^{-as}F(s) \qquad 2.4$$

Translation in the positive t direction in the real domain becomes multiplication by the exponential e^{-as} in the s domain.

Theorem 4: Complex differentiation. If the Laplace transform of $f(t)$ is $F(s)$, then

$$L[tf(t)] = -\frac{d}{ds}F(s) \qquad 2.5$$

Multiplication by time in the real domain entails differentiation with respect to s in the s domain.

Example 1. Using $L[cos\omega t]$

$$L[tcos\omega t] = -\frac{d}{ds}\left(\frac{s}{s^2+\omega^2}\right) = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$$

Example 2. Using $L[e^{-\alpha t}]$

$$L[te^{-\alpha t}] = -\frac{d}{ds}L[e^{-\alpha t}] = -\frac{d}{ds}\left(\frac{1}{s+\alpha}\right) = \frac{1}{(s+\alpha)^2}$$

Theorem 5: Translation in the s Domain. If the Laplace transform of $f(t)$ is $F(s)$ and a is either real or complex, then

$L[e^{at}f(t)] = F(s-a)$ 2.6

Multiplication of e^{at} in the real domain becomes translation in the s domain.

Example 3. Starting with $L[\sin \omega t] = \omega/(s^2+\omega^2)$ and applying Theorem 5 gives

$$L[e^{-\alpha t}\sin \omega t = \frac{\omega}{(s+\alpha)^2+\omega^2}]$$

Theorem 6: Real Differentiation. If the Laplace transform of *f (t)* is *F(s)*, and if the first derivative of f (t) with respect to time *Df (t)* is transformable, then

$L[Df(t)] = sF(s) - f(0^+)$ 2.7

The term $f(0^+)$ is the value of the value of the right-hand limit of the function *f (t)* as the origin $t=0$ is approached from the right side (thus through positive values of time). This includes functions, such as the step function, that may be undefined at $t=0$. For simplicity, the plus sign following the zero is usually omitted, although its presence is implied.

The transform of the second derivative $D^2f(t)$ is

$L[D^2f(t)] = s^2F(s) - sf(0) - Df(0)$ 2.8

where *Df (0)* is the value of the limit of the derivative of *f (t)* as the origin $t=0$, is approached from the right side.

The transform of the nth derivative $D^nf(t)$ is

$L[D^nf(t)] = s^nF(s) - s^{n-1}f(0) - s^{n-2}Df(0) - \cdots - sD^{n-2}f(0) - D^{n-1}f(0)$ 2.9

Note that the transform includes the initial conditions, whereas in the classical method of solution the initial conditions are introduced separately to evaluate the coefficients of the solution of the differential equation. When all initial conditions are zero, the Laplace transform of the nth derivative of *f (t)* is simply $s^nF(s)$.

Theorem 7: Real Integration. If the Laplace transform of f (t) is F(s), its integral

$$D^{-1}f(t) = \int_0^t f(t)dt + D^{-1}f(0^+)$$

Is transformable and the value of its transform is

$L[D^{-1}f(t)] = \frac{F(s)}{s} + \frac{D^{-1}f(0^+)}{s}$ 2.10

The term $D^{-1}f(0^+)$ is the constant of integration and is equal to the value of the integral as the origin is approached from the positive or right side. The plus sign is omitted in the remainder of this text.

The transform of the double integral $D^{-2}f(t)$ is

$$\mathcal{L}[D^{-2}f(t)] = \frac{F(s)}{s^2} + \frac{D^{-1}f(0)}{s^2} + \frac{D^{-2}f(0)}{s} \qquad 2.11$$

The transform of the *n*th-order integral $D^{-n}f(t)$ is

$$\mathcal{L}[D^{-n}f(t)] = \frac{F(s)}{s^n} + \frac{D^{-1}f(0)}{s^n} + \cdots + \frac{D^{-n}f(0)}{s} \qquad 2.12$$

Theorem 8: Final Value. If $f(t)$ and $Df(t)$ are Laplace transformable, if the Laplace transform of $f(t)$ is $F(s)$, and if the limit $f(t)$ as $t \rightarrow \infty$ exists, then

$$\lim_{s\rightarrow 0} sF(s) = \lim_{t\rightarrow\infty} f(t) \qquad 2.13$$

This theorem states that the behavior of $f(t)$ in the neighborhood of $t=\infty$ is related to the behavior of $sF(s)$ in the neighborhood of $s=0$. If $sF(s)$ has poles [values of s for which $|sF(s)|$ becomes infinite] on the imaginary axis (excluding the origin) or in the right-half s plane, there is no finite final value of $f(t)$ and the theorem cannot be used. If $f(t)$ is sinusoidal, the theorem is invalid, since $\mathcal{L}[\sin\omega t]$ has poles at $s = j\omega$ and $\lim_{t\rightarrow\infty} \sin\omega t$ does not exist. However, for poles of $sF(s)$ at the origin, $s=0$, this theorem gives the final value of $f(\infty)=\infty$. This correctly describes the behavior of $f(t)$ as $t \rightarrow \infty$.

Theorem 9: Initial Value. If the function $f(t)$ and its first derivative are Laplace transformable, if the Laplace transform of $f(t)$ is $F(s)$, and if $\lim_{s\rightarrow\infty} sF(s)$ exists, then

$$\lim_{s\rightarrow\infty} sF(s) = \lim_{t\rightarrow 0} f(t)) \qquad 2.14$$

This theorem states that the behavior of f (t) in the neighborhood of $t=0$ is related to the behavior of $sF(s)$ in the neighborhood of $|s| =\infty$.There are no limitations on the locations of the poles of $sF(s)$.

Theorem 10: Complex Integration. If the Laplace transform of $f(t)$ is $F(s)$ and if $f(t)/t$ has a limit as $t \rightarrow 0^+$, then

$$\mathcal{L}\left[\frac{f(t)}{t}\right] = \int_0^{\infty} F(s)\,ds \qquad 2.15$$

This theorem states that division by the variable in the real domain entails integration with respect to *s* in the *s* domain.[1]

APPLICATIONS OF THE LAPLACE TRANSFORM

This section presents certain applications of the Laplace transform in the study of linear systems.

Example 1

Determine the voltage across the capacitor of the circuit shown in Figure 2.2. The switch S closes when t = 0. The initial condition for the voltage capacitor is zero, i.e.

Vc (0) = 0.

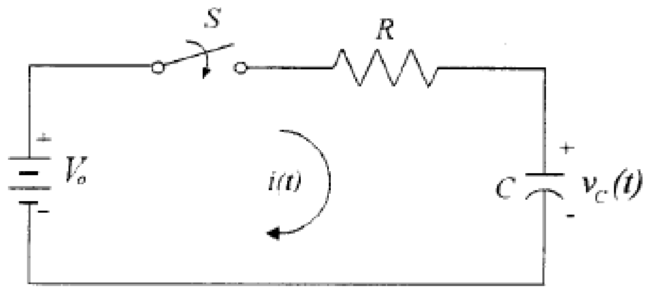

Figure 2.2 RC network

Solution

From Kirchhoff's voltage law we have

$$Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)dt = V_0$$

Applying the Laplace transform to both sides of the integral equation, we get the following algebraic equation

$$RI(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{i^{(-1)}(0)}{s}\right] = \frac{V_0}{s}$$

Where $I(s)=L\{i(t)\}$ and $i^{(-1)}(0) = \int_{-\infty}^{0} i(t)dt = Cv_c(0) = 0$. Replacing $i^{(-1)}(0) = 0$ in the above equation, we have

$I(s)\left[\frac{1}{Cs} + R\right] = \frac{V_0}{s}$ or $I(s) = \frac{V_0/R}{s+1/RC}$

The inverse laplace transform of *I(s)* is as follows

$$i(t) = L^{(-1)}\{I(s)\} = \frac{V_0}{R}e^{-t/RC}$$

Hence, the voltage $v_c(t) = V_0 - Ri(t) = V_0 - V_0e^{-t/RC} = V_0[1 - e^{-t/RC}]$

Transfer Functions

In contrast to the differential equation method which is a description in the time domain, the transfer function method is a description in the frequency domain and holds only for a restricted category of systems, i.e., for linear time-invariant systems having zero initial conditions. The transfer function is designated by H(s) and is defined as follows:

The transfer function H(s) of a linear, time-invariant system with zero initial conditions is the ratio of the Laplace transform of the output y(t) to the Laplace transform of the input u(t), i.e.,

$$H(s) = \frac{L\{y(t)\}}{L\{u(t)\}} = \frac{Y(s)}{U(s)} \qquad 1$$

Example: Transfer function for a differential equation

Find the transfer function represented by:

$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

Solution: Take the Laplace transform of both sides, assuming zero initial conditions

$$sC(s) + 2C(s) = R(s)$$

The transfer function, G(s), is

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$

Find he transfer function, $G(s) = C(s)/R(s)$, corresponding to the differential equation

$$\frac{d^3c}{dt^3} + 3\frac{d^2c}{dt^2} + 7\frac{dc}{dt} + 5c = \frac{d^2r}{dt^2} + 4\frac{dr}{dt} + 3r$$

Electric Network Transfer Function

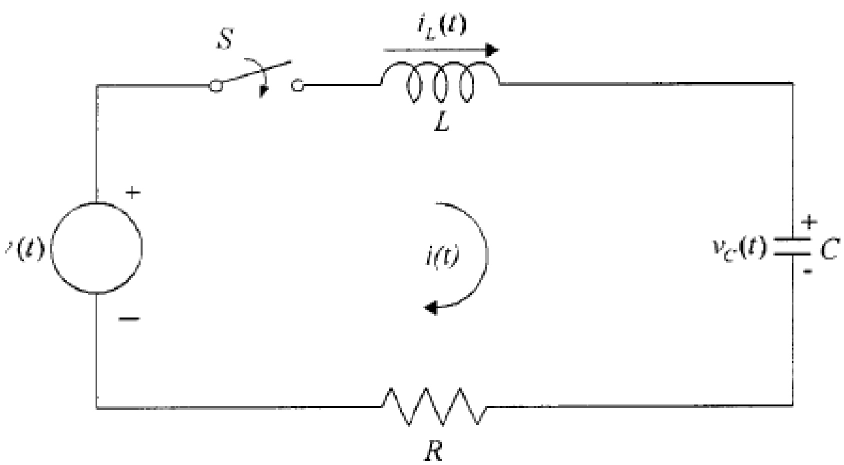

Figure 1.1 RLC network

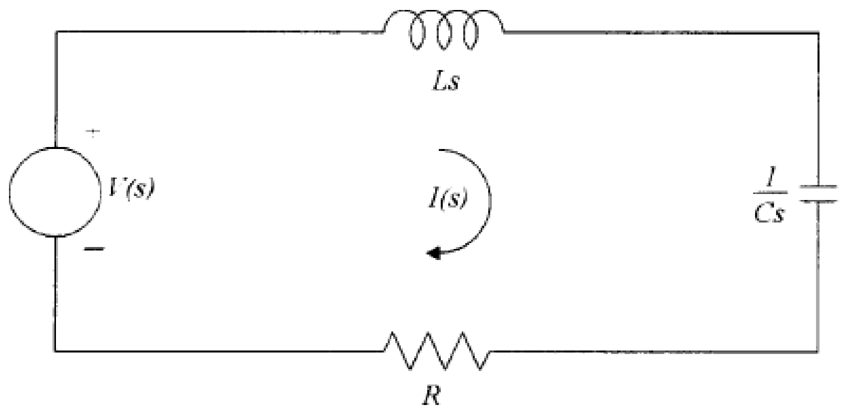

Figure 1.2 RLC circuit

Consider the network shown in figure 1.1. Derive the transfer function *H(s)=I(s)/V(s)*.

Solution

This network, in the frequency domain and with zero initial conditions I_0 and V_0, is as show in figure 1.2. To determine the transfer function H(s)=I(s)/V(s), we work as follows: From Kirchhoff's voltage law, we have

$$LsI(s) + RI(s) + \frac{I(s)}{Cs} = V(s)$$

Therefore, the transfer function sought is given by

$$H(s) = \frac{I(s}{V(s)} = \frac{Cs}{LCs^2 + RCs + 1}$$

Furthermore, consider as circuit output the voltage $V_R(s)$ across the resistor. In this case, the transfer function becomes

$$H(s) = \frac{V_R(s)}{V(s)} = \frac{RI(s)}{V(s)} = \frac{RCs}{LCs^2 + RCs + 1}$$